# NMA ... Assignment - 2

1. $r' = 12.65$ , $r = 12.5$ cm

$\Delta r = 0.15$ cm

Area $= \pi r^2$

$A' = \pi (12.65)^2 = 502.725 \text{ cm}^2$

$A = \pi (12.5)^2 = 490.874 \text{ cm}^2$

$\Delta A = A' - A = 11.8517 \text{ cm}^2$

i) Absolute error $= |502.725 - 490.874|$

$= 11.851$

ii) Relative error $= \dfrac{\Delta A}{A} \approx \cancel{\dfrac{11.851}{502.725}}$

$= \dfrac{11.851}{490.874} = 0.02414$

iii) Percentage error $= 2.414\ \%$

PAGE No

DATE

2\.

| $x$ | $y=f(x)$ |
|---|---|
| 4.0 | 0.60206 |
| 4.5 | 0.65321 |
| 5.5 | 0.74036 |
| 6.0 | 0.77815 |

| $x$ | $y$ | $\Delta y$ | $\Delta^2 y$ | $\Delta^3 y$ |
|---|---|---|---|---|
| 4 | 0.60206 | | | |
| 4.5 | 0.65321 | 0.05115 | | |
| 5.5 | 0.74036 | 0.08715 | 0.036 | |
| 6 | 0.77815 | 0.03779 | -0.049 | -0.085 |

$$f(5) = \frac{(5-4)(5-4.5)(5-5.5)}{(6-4)(6-4.5)(6-5.5)}(0.77815) + \frac{(5-4)(5-4.5)(5-6)}{(5.5-4)(5.5-4.5)(5.5-6)}(0.74036) + \frac{(5-4)(5-5.5)(5-6)}{(4.5-4)(4.5-5.5)(4.5-6)}(0.65321) + \frac{(5-4.5)(5-5.5)(5-6)}{(4-4.5)(4-5.5)(4-6)}(0.60206)$$

$$= (-0.1667)(0.77815) + (0.667)(0.74036) + (0.667)(0.65321) + (-0.1667)(0.60206)$$

$$= 0.6994302$$

(b)

| $x$ | $y$ |
|---|---|
| 4.5 | 0.65321 |
| 5.5 | 0.74036 |

$$f(5) = \frac{(5-4.5)(0.74036)}{(5.5-4.5)} + \frac{(5-5.5)(0.65321)}{(4.5-5.5)}$$

$$= 0.696785$$

PAGE No.
DATE / /

(5)($\cancel{4}$) $A^2 = A$

$7A - (I + A)^3 = ?$

$\therefore 7A - (I^3 + 3I^2A + 3A^2I + A^3)$

$= 7A - (I + 3A + 3A + A)$

$\because A^3 = A^2 \cdot A = A \cdot A = A^2 = A.$

$= 7A - (I + \cancel{3}\,7A)$

$= -I$

(6) Inverse of

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix}$$

| | | |
|---|---|---|
| $A_{11} = 1$ | $A_{12} = -1$ | $A_{13} = 2$ |
| $A_{21} = 4$ | $A_{22} = 0$ | $A_{23} = 6$ |
| $A_{31} = 0$ | $A_{32} = 1$ | $A_{33} = 1$ |
| $M_{11} = -6$ | $M_{12} = +4$ | $M_{13} = 4$ |
| $M_{21} = +3$ | $M_{22} = 1$ | $M_{23} = -1$ |
| $M_{31} = -6$ | $M_{32} = +2$ | $M_{33} = 4$ |

$|A| = (1)(-6) - (-1)(-4) + \cancel{4}\,2(4)$

$= -6 - 4 + 8$

$= -2.$

$$A^{-1} = \frac{-1}{2}\begin{bmatrix} -6 & 3 & -6 \\ 4 & 1 & 2 \\ 4 & -1 & 4 \end{bmatrix}$$

7) $\vec{A} = 8\hat{i} - 9\hat{j}$ $\qquad |\vec{A}| = \sqrt{64+81} = 12.042$

$\vec{B} = 7\hat{i} - 9\hat{j}$ $\qquad |\vec{B}| = \sqrt{49+81} = 11.4017$

$\vec{A} \cdot \vec{B} = (8\hat{i} - 9\hat{j}) \cdot (7\hat{i} - 9\hat{j})$

$= 56 + 81$

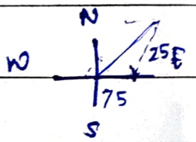
$AB \cos\theta = 137$

$\cos\theta = \dfrac{137}{\sqrt{130}\sqrt{145}} = 0.9978$

$\theta = \cos^{-1}(0.9978)$

$= 3.75868°$

8)




10) $t_6 = 32$

$t_8 = 128$

$t_n = ar^{n-1}$

$t_6 = ar^5 \qquad t_8 = ar^7$

$\dfrac{t_6}{t_8} = r^2$

$\therefore \dfrac{128}{32} = r^2$

$\therefore 4 = r^2$

$\boxed{r = 2}$

9. $= 101, 104 \ldots\ldots 998$

$\therefore a = 101, \quad d = 3$

$t_n = a + (n-1)d$

$998 = 101 + (n-1)(3)$

$897 = (n-1)3$

$299 = (n-1)$

$\therefore n = 300$

$S_{300} = \frac{n}{2}(2a + (n-1)d)$

$= 150(202 + (299)(3))$

$= 164850.$

11. $(4+3x)^5$

$= {}^5C_0(4)^5 + {}^5C_1(4)^4(3x) + {}^5C_2(4)^3(3x)^2 + {}^5C_3(4)^2(3x)^3 + {}^5C_4(4)(3x)^4 + {}^5C_5(3x)^5$

$= (4)^5 + 5(4)^4(3x) + 10(4)^3(3x)^2 + 10(4)^2(3x)^3 + (5)(4)(3x)^4 + (3x)^5$

$= 1024 + 3840x + 5760x^2 + 4320x^3 + 1620x^4 + 243x^5$

12. $\begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

$$\begin{bmatrix} 2-\lambda & -3 & 0 \\ 2 & -5-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{bmatrix} = 0$$

$(2-\lambda)((-5-\lambda)(3-\lambda)) + 3(2(3-\lambda)) = 0$

$(3-\lambda)(-(2-\lambda)(5+\lambda) + 6) = 0$

$(3-\lambda)(6 - (10 - 3\lambda - \lambda^2)) = 0$

$(3-\lambda)(3\lambda + \lambda^2 - 4) = 0$

$9\lambda + 3\lambda^2 - 12 - 3\lambda^2 - \lambda^3 + 4\lambda = 0$

$13\lambda - \lambda^3 - 12 = 0$

$\cancel{\lambda(13-\lambda^2}$ $a = -1$, $b = 0$, $c = 13$, $d = -12$

$a + b + c + d = 0$

$\therefore \lambda = 1$.

| | | | | |
|---|---|---|---|---|
| 1 | $-1$ | $0$ | $13$ | $-12$ |
| | | $-1$ | $-1$ | $+12$ |
| | $-1$ | $-1$ | $+12$ | $0$ |

$\therefore -\lambda^2 - \lambda + 12 = 0$

$(\lambda - 4)(\lambda + 3) = 0$

Hence the roots are $(+4, -3, 1)$

~~(4a) f(x)~~ for eigen vectors

for $\lambda = 3$

$(A - \lambda I)\, X = 0$

$$\left[\begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} - (3)\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\right]\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\begin{bmatrix} -1 & -3 & 0 \\ 2 & -8 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$R_2 \rightarrow R_2 + 2R_1$

$$\begin{bmatrix} 1 & -3 & 0 \\ 0 & -14 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_1 \rightarrow R_1 - \frac{3}{14} R_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -14 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2 \rightarrow R_2 / -14$

$$\begin{bmatrix} +1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1\cancel{0} \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_1 = 0$, $x_2 = 0$, $x_3 =$ ~~det~~ indeterminate $= \alpha$

for $\lambda = 3$, ~~eq~~ eigen vector is $\begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$

(ii) for $\lambda = 1$

$$(A - \lambda I)X = 0$$

$$\left( \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & -3 & 0 \\ 2 & -6 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$R_2 \to R_2 - 2R_1$

$$\begin{bmatrix} 1 & -3 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x - 3y \\ 0 \\ 2z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

hence the eigen vector are

$$\begin{bmatrix} 3y \\ y \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} y$$

PAGE No.
DATE / /

(iii) for $\lambda = -4$

$(A - \lambda I) X = 0$

$$\left(\begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right) \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & -3 & 0 \\ 2 & -6 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$R_2 \to R_2 / -14$

$$\left(\begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} + 4 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right) \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 6 & -3 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_1 \to R_1 - 3R_2$

$$\begin{bmatrix} 0 & 0 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 2x - y \\ 7z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$2x - y = 0$ $\qquad$ $y = 2$

$7z = 0$ $\qquad$ $z = 0$

Hence the eigen vector is

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ 2x \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} x$$

3. $x^4 - x - 10 = 0$

| $x$ | $f(x)$ |
|---|---|
| 1.5 | -6.437 |
| 2 | 4 |

$$x_1 = \frac{2+1.5}{2} = 1.75$$

$$f(x_1) = -2.371$$

$$x_2 = \frac{3.75}{2} = 1.875$$

$$f(x_2) = 0.4846$$

$$x_3 = \frac{1.875 + 1.75}{2} = 1.8125$$

$$f(x_3) = -1.020$$

$$x_4 = \frac{1.8125 + 1.875}{2} = 1.84375$$

$$f(x_4) = -0.2877$$

$$x_5 = \frac{1.84375 + 1.875}{2} = 1.8594$$

$$f(x_5) = 0.093378$$

$\therefore$ 1.8594 is root of given eqn

8.


$\therefore$ displacement vector d

$|d| = \sqrt{(75)^2+(25)^2+2(75)(25)\cos(30)}$

$\approx \cancel{\sqrt{7794}\ m}$

$= 97.456$ m

4. $f(x) = x^3 - x - 1$ $\qquad f'(x) = 3x^2 - 1$

| $x$ | $f(x)$ | $f'(x)$ |
|---|---|---|
| 1 | -1 | 2 |
| 2 | 5 | 11 |

$x_1 = x - \dfrac{f(x)}{f'(x)} = 1 - \dfrac{(-1)}{2} = 1.5$

$f(x_1) = 0.875$ $\qquad f'(x_1) = 5.75$

$x_2 = 1.5 - \dfrac{(0.875)}{(5.75)} = 1.3478$

$f(x_2) = 0.10068$ $\qquad f'(x_2) = 3.0434$

$x_3 = 1.3478 - \dfrac{(0.10068)}{(3.0434)}$

$= 1.31472$

$f(x_3) = -0.042247$ $\qquad f'(x_3) = 2.94416$

$x_4 = 1.31472 - \dfrac{(-0.042247)}{2.94416}$

$= 1.329069$

$f(x_4) = 0.0186327$ $\qquad f'(x_4) = 2.9872$

$x_5 = 1.32283$ $\qquad f(x_5) = -8.0309 \times 10^{-3}$

root of given eqn is 1.32283.

13. $f(x) = x^3 - 7x^2 + 8x - 3$ $\quad f'(x) = 3x^2 - 14x + 8$

$x_0 = 5$

$f(x_0) = -13$ $\quad f'(x_0) = 13$.

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 5 + 1 = 6$$

$f(x_1) = 9$ $\quad f'(x_1) = 32$

$$x_2 = 6 - \frac{9}{32} = 5.71875.$$

14. $(1 - x + x^2)^4$

$$= {}^4C_0 (1-x)^4 (x^2)^0 + {}^4C_1 (1-x)^3 (x^2) + {}^4C_2 (1-x)^2 (x^2)^2 + {}^4C_3 (1-x)(x^2)^3 + {}^4C_4 (1-x)^0 (x^2)^4$$

$$= (1-x)^4 + 4(1-x)^3(x^2) + 12(1-x)^2(x^4) + 4(1-x)(x^6) + (x^8).$$

~~15.~~

PAGE No.

DATE / /

15. $f(x) = 3\log x - e^{-x}$

| $x$ | $f(x)$ |
| --- | --- |
| 1 | $-0.367$ |
| 2 | $0.7677$ |

$x_1 = \cancel{\frac{x_1+x_2}{2}}\ \frac{a+b}{2} = 1.5$

$\cancel{f(x_3)}\ f(x_1) = 0.3051$

$x_2 = \frac{1.5+1}{2} = 1.25$

$f(x_2) = 4.2 \times 10^{-3}$

$x_3 = \frac{1+1.25}{2} = 1.125$

$f(x_3) = -0.171$

$x_4 = \frac{1.25+1.125}{2} = \cancel{1.0625}\ 1.1875$

$f(x_4) = -0.08108$