PAGE No. Submission
DATE 28/11/2021

FM. No.1

1) The nominal rate of discount per annum convertible quarterly is 8%.

i. Calculate the equivalent force of interest.

→ $d^{(4)} = 8\%$ p.a

$\therefore d = 1 - \left[1 - \frac{d^{(4)}}{4}\right]^4$

$= 1 - [1 - 0.02]^4$

$= 7.76318\%$ p.a

$\therefore \delta = -\ln(1-d)$

$= -\ln(1 - 0.0776318)$

$\therefore \delta = 8.08108\%$ p.a

ii. Equivalent effective rate of interest.

→ $i = \frac{d}{1-d}$

$= \frac{0.0776318}{1 - 0.0776318} = 8.416573\%$

iii) Equivalent nominal rate of disc. per annum convertible monthly.

→ $\therefore d^{(12)} = 12\left[1 - (1-d)^{1/12}\right]$

$= 12[1 - (1 - 8.08108$

2) F.O.I $\delta(t) = 0.004t + 0.0002t^2$ for all t.

i. PV of sum of rupees 1000 payable at the end of 10 years from now.

→ $\delta(t) = 0.004t + 0.0002t^2$

PAGE No.
DATE / /

$\therefore PV_{10} = 1000 \times V(0,10)$

$= 1000 \times e^{-\int_0^{10} 0.004t + 0.0002t^2}$

$= 1000 \times e^{-\left[\frac{0.004t^2}{2} + \frac{0.0002t^3}{3}\right]_0^{10}}$

$= 1000 \times e^{-0.26667}$

$\therefore PV_{10} = 765.92834$

ii. constant annual effective r.o.i over 10yrs.

$\rightarrow \because PV_{10} = 1000 \times V(0,10)$

$765.92834 = 1000 \times \frac{1}{A(0,10)}$

$\therefore A(0,10) = \frac{1000}{765.92834}$

$\therefore (1+i)^{10} = 1.305605169$

$\therefore i = \sqrt[10]{1.305605169} - 1$

$= 1.027025 - 1$

$= 0.027025$

$\therefore i = 2.7025\%$

3) i) Explain the relationship $d = iv$ by general reasoning where 'd' is the effective annual r.o. discount and 'i' is eff. annual r.o. interest.

$\rightarrow$ We know that,

$1+i = \frac{1}{1-d}$

$\therefore 1-d = \frac{1}{1+i}$

PAGE No.
DATE / /

$$\therefore -d = \frac{1}{1+i} - 1$$

$$\therefore d = \frac{-1}{1+i} + 1$$

$$= \frac{-\cancel{1}+\cancel{1}+i}{1+i}$$

$$\therefore d = \frac{i}{1+i}$$

Also, since $v = \frac{1}{1+i}$

$$\therefore d = iv$$

ii. Nominal r.o.d per annum convertible half yearly which is equivalent to

a) An eff. r.o.d of 2.25% per quarter.

→ Given that $\frac{d^{(4)}}{4} = 2.25\%$

$$\therefore d^{(2)} = 2\left[1-(1-d)^{1/2}\right]$$

$$= 2\left[1-\left(1-1-\left(1-\frac{d^{(4)}}{4}\right)^4\right]^{1/2}$$

$$= 2\left[1-\left(\frac{d^{(4)}}{4}-1\right)^4\right]^{1/2}$$

$$= 2\left[1-\left(\frac{d^{(4)}}{4}-1\right)^2\right.$$

$$= 2\left[1-0.95550625\right]$$

$$= 0.0889875$$

$$\therefore d^{(2)} = 8.89875$$

PAGE No.
DATE / /

b> Nominal r.o.d of 5% conv. every 2yrs.

→ Given that $d^{(1/2)} = 5\%$

$\therefore d^{(2)} = 2\left[1-(1+i)^{-1/2}\right]$

$= 2\left[1-\left(1+\frac{0.05}{0.5}\right)^{-1/2}\right]$

$= 2\left[1+1.3545900\right]^{*}$

$= 4.70918\%$

$\therefore d^{(2)} = 4.7091\%$

iii> A 91-day gov. bill provides the investor with an annual eff. rate of return of 5%. calculate annual simple disc. rate at which bill is discounted.

→ $i = 5\%$. $n = 91$ days

$\therefore A.V = 100(1.05)^{91/365}$

$= 101.22384$

$\therefore 100\left(1+\frac{91d}{365}\right) = 101.22384$

$\therefore d = 4.8494619\%$

4> i> Explain the relationship ~~

⇒ this question is repeated!

DATE / /

ii. Given $i = 0.08$ find

a) $d^{(12)}$

$\therefore d^{(12)} = 12[1-(1.08)^{-1/12}]$

$= 12[0.006392898]$

$= 0.076714776$

$\therefore d^{(12)} = 7.67148\%$

b) $i^{(365)}$

$\therefore i^{(365)} = 365[(1.08)^{1/365} - 1]$

$= 365[0.000210874]$

$= 0.076969155$

$\therefore i^{(365)} = 7.6969155\%$

c) $\delta = ?$

We know that $d = \frac{i}{1+i} = \frac{0.08}{1.08} = 0.0740740$

$\therefore \delta = -\ln(1-d) = -\ln(1-0.0740740)$

$= -\ln(0.9259259)$

$= 0.07696104$

$\therefore \delta = 7.6961\%$

PAGE No.
DATE / /

d) $i^{(0.5)}$

$\rightarrow \therefore i^{(0.5)} = 0.5\left[(1.08)^2 - 1\right]$

$= 0.5\left[0.1664\right]$

$= 0.0832$

$\therefore i^{(0.5)} = 8.32\%$

5) i) The rate of discount p.a convertible quarterly is 6%. Calculate
a) Equivalent r.o.i p.a convertible half yrly.

$\rightarrow d^{(4)} = 6\%$

$\therefore d = 1 - \left[1 - \frac{d^{(4)}}{4}\right]^4$

$= 1 - \left[1 - \frac{6\%}{4}\right]^4 = 1 - (1 - 0.015)^4$

$= 0.058663$

$\therefore d = 5.8663\%$

$\therefore i = \frac{d}{1-d} = 6.23193\%$

$\therefore i^{(2)} = 2\left[(1+i)^{1/2} - 1\right]$

$= 2\left[\sqrt{1.0623193} - 1\right]$

$\therefore i^{(2)} = 6.13775\%$

PAGE No.
DATE / /

b) Equi. rate of discount p.a con. monthly

$d^{(12)} = 12\left[1-(1-d)^{1/12}\right]$

$= 12\left[1-(1-0.0586635)^{1/12}\right]$

$= 0.0603025$

$\therefore d^{(12)} = 6.03025\%$

ii) On 15th April 2005, Amit borrowed ₹2 lakh to be repaid a year later by single payment of ₹2,20,000. He repaid it early on 17th July, 2005.

a) Find sum paid by Amit to terminate the contract assuming that interest is reduced proportionately for early settlement.

→ $C = 200000$ at $t=0$ & $AV = 220000$

$\therefore A \cdot V = C \cdot A(0,1)$

$\therefore 220000 = 200000\ (1+i)$

$\therefore \frac{11}{10} = 1+i$

$\therefore i = \frac{1}{10} = 0.1 = 10\%$

We know that "$\frac{A.V}{P.V} = (1+i)^n$

$\therefore A \cdot V = 200000\,(1.1)^{93/365}$

$= 2,00,470.5755$

$\therefore A \cdot V = 200470.5755$

PAGE No.
DATE / /

6) i) The manufacturer of a certain toy sells to retailers on either of the following terms:-

a) cash payment: 30% below recommended retail price.

b) six months credit: 25% below -11-.

Find effective annual r.o.d offered by the manu. to retailers who pay cash as opposed to those retailers who accept the credit terms.

→ Let retail price be taken as ₹100.

∴ For condition a)

$= 100 - 30\%$ of 100

$= 70$

and for b) $= 100 - 25\%$ of 100

$= 75$

$\therefore \frac{70}{75} = (1-d)^{1/2}$

$\therefore d = -\left(\frac{70}{75}\right) + 1$

$= -0.871111 + 1$

$= 0.12888$

$\therefore d = 12.888\%$

ii) Find $d^{(12)}$, $i^{(2)}$ and δ

→ We know that,

$$(1-d) = \left[1 - \frac{d^{(12)}}{12}\right]^{12}$$

PAGE No.
DATE / /

$\therefore (1-0.12888)^{1/12} = 1 - \frac{d^{(12)}}{12}$

$\therefore d^{(12)} = 13.71954 39\%$

$\rightsquigarrow \therefore i^{(2)} \Rightarrow \left[1+\frac{i^{(2)}}{2}\right]^2 = \frac{1}{1-d}$

$\therefore 1 + \frac{i^{(2)}}{2} = \left(\frac{1}{1-d}\right)^{1/2}$

$\therefore i^{(2)} = 14.28571429\%$

$\rightsquigarrow \delta = -\ln(1-d)$

$= -\ln(1-0.128888)$

$= 13.7985756\%$

7) If an investment fund offers to increase INR 20,000 to INR 26,000 in 17 months. Find

i) Nominal r.o.i p.a conv. quarterly.

$\rightarrow \therefore 26000 = 20000(1+i)^{17/12}$

$(1.3)^{12/17} = 1+i$

$\therefore i = 0.20346$

$\therefore i^{(4)} = 4\left[(1+i)^{1/4} - 1\right]$

$= 4\left[(1.20346)^{1/4} - 1\right]$

$i^{(4)} = 18.9555\%$

ii) Nominal r.o.d p.a conv. half yearly.

$\rightarrow$ $20000 = \frac{26000}{(1+i)^{17/12}}$

$i = 0.20346$

$\therefore d = \frac{0.20346}{1.20346}$

$\therefore d = 0.16906$

$\therefore d^{(2)} = 2[1-(1-d)^{1/2}]$

$= 2[1-(1-0.16906)^{1/2}]$

$\therefore d^{(2)} = 17.68845\%$

iii) Simple r.o.i p.a

$\rightarrow$ $26000 = 20000\left[1+\frac{17i}{12}\right]$

$\frac{17(i)}{12} = 0.3$

$\therefore i = 21.17647\%$

iv) Comment

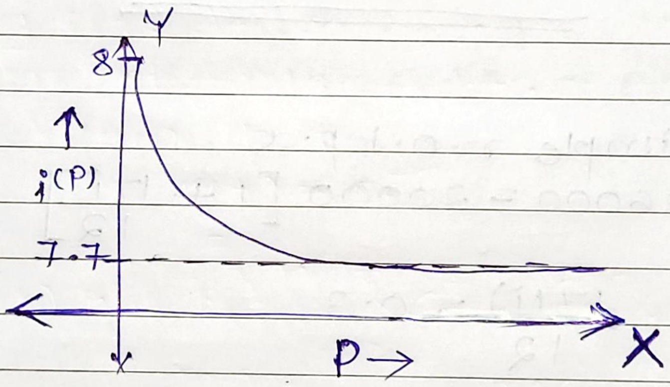
$\rightarrow$ As we can see, in this question S.I will be greater than C.I.

PAGE No.
DATE / /

8) Explain with the help of a graph, for an eff. interest rate of 8% p.a. the relationship betn equivalent nominal rates of interest convertible pthly ($i^{(p)}$) and p.

→ $i = 8$; $i^{(p)} = p[(1+i)^{1/p} - 1]$

∴ p	$i^{(p)}$
1	0.08
2	0.07846
4	0.07770
5	0.07755
10	0.07725



This graph is exponential as well as shows inverse relation between 'p' and 'i'

It is limiting at $p \to \infty$ and it is on X axis, whereas for Y-axis it is limiting to $i \to 8$.

PAGE No.
DATE / /

g) i) Define force of interest.

→ The measure of the interest at individual moment of time is called the Force of Interest. It is denoted by 'δ'. (delta)

ii) If $i^{(2)} = 0.0775$

Find $i, \delta, \alpha, d^{(12)}, i^{(1/2)}$

→ a) $i = \left[1 + \frac{i^{(p)}}{p}\right]^p - 1$

$= \left[1 + \frac{0.0775}{2}\right]^2 - 1$

$= 1.079001563 - 1$

$= 0.079001563$

$\therefore i = 0.079001\%$

b) $\delta = \ln(1+i)$

$= \ln(1 + 0.079001)$

$= 0.0760356$

$\therefore \delta = 7.60356\%$

c) $d^{(12)} = 12\left[1 - (1+i)^{-1/12}\right]$

$= 12\left[1 - (1.079)^{-1/12}\right] = 0.07579$

$\therefore d^{(12)} = 7.579\%$

d) $i^{(0.5)} = 0.5[(1+i)^{1/0.5} - 1]$

$= 0.5[(1.079)^2 - 1]$

$= 0.08212$

$\therefore i^{(0.5)} = 8.212\%$

10) $\delta(t)$ the f.o.i p.a at time t yrs is :-

$$\delta(t) = \begin{cases} 0.08 & \text{for } 0 \le t \le 5 \\ 0.06 & \text{for } 5 \le t \le 10 \\ 0.04 & \text{for } t \ge 10 \end{cases}$$

Using $\delta(t)$, derive the expression for $v(t)$, the present value of 1 due at time t.

→ We know that

$v(t) = e^{-\delta}$

$\therefore v(t) = e^{-\int_0^5 0.08\,dt} \times e^{-\int_5^{10} 0.06\,dt} \times e^{\int_{10}^{t} 0.04\,dt}$

$= e^{-[0.08t]_0^5} \times e^{-[0.06t]_5^{10}} \times e^{-[0.04t]_{10}^{t}}$

$= e^{-0.4} \times e^{-[0.6-0.3]} \times e^{-[0.04t\ 0.4]}$

$= e^{-0.4} \times e^{-0.3} \times e^{-0.04t+0.4}$

$= e^{[(-0.4-0.3-0.04t+0.4)]}$

$= e^{-[0.3+0.04t]}$

$\boxed{\therefore v(t) = e^{-[0.3+0.04t]}}$

Required Expression.

PAGE No.
DATE / /

11) a) A 9-month loan, repayable by a single payment of 50,000 is issued at a rate of commercial discount of 18% pa. what amt. was initially lent to the borrower?

→ $d = 18\%$ pa

$i = 0.21951$

$\therefore AV = 50000 \cdot v^{\cancel{9}3/\cancel{12}4}$

$\therefore PV = \dfrac{50000}{(1+i)^{3/4}}$

$\therefore PV = 43085.48439$

b) i) If $\delta = 6\%$. find $d^{(12)} = ?$

→ $(1-d) = e^{-\delta}$

$\therefore \left(1 - \dfrac{d^{(12)}}{12}\right)^{12} = e^{-\delta}$

$= e^{-0.06}$

$= 0.941764534$

$\therefore 1 - \dfrac{d^{(12)}}{12} = 0.995012479$

$\therefore \dfrac{d^{(12)}}{12} = 0.004987521$

$\therefore d^{(12)} = 5.9814\%$

ii) If $d^{(4)} = 6\%$, find $i^{(12)}$, $i^{(4)}$

DATE / /

$\rightarrow d = 1 - \left[1 - \frac{0.06}{4}\right]^4$

$= 0.058663449$

$\therefore i = \frac{d}{1-d} = 0.062319315$

$\therefore i^{(12)} = 12\left[(1+i)^{1/12} - 1\right]$

$= 12\left[(1.062319315)^{1/12} - 1\right]$

$= 0.060607089$

$\therefore i^{(12)} = 6.0607\%$

f

$i^{(4)} = 4\left[(1+i)^{1/4} - 1\right]$

$= 4\left[(1.062319315)^{1/4} - 1\right]$

$= 0.060913705$

$\therefore i^{(4)} = 6.0913\%$

PAGE No.
DATE / /

c) Arrange the foll. qtys in ascending order of numerical value, giving brief reason for it assuming same eff. rate:
$i, d, i^{(4)}, \delta$

→ Order ⇒ $d < i^{(4)} < \delta < i$

→ Finding the values for the above, the order comes out to be the same as above!

d) Explain in words why $d = iv$ holds true.

→ We know that

$$d = iv$$

where d = discount rate
i = interest rate
V = discount

∴ when interest rate is discounted we get discounting rate, thus it holds true.

12) A deposit of 7049 is accumulated at the foll. rates of interest: 6% p.a nominal convertible monthly for first 2 yrs followed by 1.5% per quarter simple for next two and a half years followed by a rate of disc. of 6% p.a conv. monthly for next $1\frac{1}{2}$ yrs. Find Accumulated amt after 6 years.

→ Amount invested at $t=0$ ⇒ 7049.

Given that $i^{(12)} = 6\%$ p.a. for 2 yrs.

and $i^{(4)} = 2.5\%$ p.a for $2\frac{1}{2}$ yrs
and $d^{(12)} = 6\%$ p.a for $1\frac{1}{2}$ yrs.

$\therefore AV_6 = 7049 \times A(0,2) \times A(2,4.5) \times A(4.5,6)$

$\therefore A(0,2) = \left[1 + \frac{i^{(12)}}{12}\right]^{24}$

$= 1.127159776$ — ①

$\therefore A(2,4.5) = \left[1 + \frac{i^{(4)}}{4}\right]^{10}$

$= 1.160540825$ — ②

$\therefore A(4.5,6) = \frac{1}{1-d} = \frac{1}{\left[1 - \frac{d^{(12)}}{12}\right]^{18}}$

$= 1.094421325$ — ③

$\therefore AV_6 = 7049 \times 1.127159 \times 1.160540 \times 1.094421$

$\therefore AV_6 = 10091.55199$

13) Calculate the time in yrs for an investment to double at:

a) An eff. r-o-i of 10% pa

→ Let the amt. invested at start is x.

$\therefore 2x = x(1.1)^t$

$\therefore t = \frac{\ln(2)}{\ln(1.1)}$

PAGE NO.
DATE / /

$\therefore t = 7.27254$ years

b) Nominal discount rate of 10% pa conv. monthly.

→ $d^{(12)} = 10\%$ p.a

$$d = 1 - \left[1 - \frac{0.1}{12}\right]^{12}$$

$$= 0.09554$$

$$\therefore i = 0.10564$$

$$\ln(2) = t \ln(1.10564)$$

$$\therefore t = \frac{\ln(2)}{\ln(1.10564)}$$

$\therefore t = 6.90218$ years

—— x —— o —— x ——