

$$\textcircled{1} d^4 = 8\%$$

$$d = 1 - \left(1 - \frac{d^4}{4}\right)^4$$

$$= 1 - \left(1 - \frac{0.08}{4}\right)^4$$

$$d = 0.07763184$$

$$d \approx \frac{d}{1-d} = 0.084165784 \approx 8.4165784\%$$

$$\textcircled{i)} d \approx \log(1 + 0.084165784)$$

$$d \approx 0.080810828$$

$$\approx 8.0810828\%$$
~~$$d \approx 1.084165783$$~~

$$\textcircled{ii)} d \approx 0.084165784 \approx 8.4165784\%$$

$$\textcircled{iii)} d^{12} \approx P \left[1 - (1-d)^{1/P} \right]$$

$$= 12 \left[1 - (1 - 0.07763184)^{1/12} \right]$$

$$= 0.080539339$$

$$\approx 8.0539339\%$$

$$② \delta(t) = 0.004t + 0.0002t^2$$

$$i) 1000 e^{\int_0^{10} 0.004t + 0.0002t^2 dt}$$

$$= 1000 \cdot e^{-[0.004t^2/2 + 0.0002t^3/3]_0^{10}}$$

$$= 1000 \cdot e^{-[0.004(10)^2/2 + 0.0002(10)^3/3 - 0]}$$

$$= 1000 \cdot e^{-[0.2 + 0.06666]}$$

$$= 1000 \cdot e^{-[0.26666]}$$

$$= 765.9334446$$

✓

$$ii) PV_2 = 1000 \cdot V(0, 10)$$

$$765.9334446 = 1000 \cdot V(0, 10)$$

$$V(0, 10) = \frac{765.9334446}{1000}$$

$$V(0, 10) = 0.765933444$$

$$\frac{1}{(1+i)^{10}} = 0.765933444$$

$$1.305596469 = (1+i)^{10}$$

$$i = 0.027024719$$

✓

③ i) Discounting i from the discounting factor V , we obtain d

$$d = \frac{i}{1+i}$$

ii) $d^2 = ?$

$$a) \frac{d^4}{4} = 2.25\%$$

$$d = 1 - \left(1 - \frac{d^4}{4}\right)^{1/4}$$

$$= 1 - \left(1 - 0.0225\right)^{1/4}$$

$$d = 0.087007806$$

$$d^2 = 2(1 - (1 - 0.087007806)^{1/2})$$

$$d^2 = 0.088987499$$

$$b) d = 1 - \left(1 - \frac{d^{1/2}}{1/2}\right)^{1/2} = 1 - \left(1 - \frac{0.05}{1/2}\right)^{1/2} = 5.1316701\%$$

$$d^2 = 2(1 - (1 - 0.051316701)^{1/2})$$

$$= 5.1992506\%$$

$$\text{iii)} \frac{1}{(1+i)^{91/365}} = \left(1 - \frac{91}{365} d \right)$$

$$d = \frac{365}{91} \left(1 - \frac{1}{(1.05)^{91/365}} \right)$$

$$= 0.048494619$$

$$= 4.8494619\%$$

$$4) \text{ ii) } i = 0.08$$

$$a) d^{12} =$$

$$\frac{d}{1-d} = \frac{d \cdot i}{1+i} = 0.074074074$$

$$d^{12} = 12 \left[1 - (1 - 0.074074074)^{1/12} \right]$$

$$= 7.6714776\%$$

$$b) i^{365} = \left(365(1+0.08)^{1/365} - 1 \right)$$

$$= 0.076969155 \text{ or } 7.6969155\%$$

$$c) d = \log(1+i)$$

$$= \log(1+0.08)$$

$$= 0.076961041 = 7.6961041\%$$

$$d) i^{(0.5)} = 0.5 [(1 + 0.08)^{1/0.5} - 1] = 8.32\%$$

$$5) i) d^4 = 6\%$$

$$a) i^2 =$$

$$d_2 = 1 - \left(\frac{1 - d^4}{4} \right)^4$$

$$d_2 = 1 - \left(\frac{1 - 0.06}{4} \right)^4$$

$$d_2 = 5.8663449\%$$

$$i_2 = \frac{d}{1-d} = \frac{0.058663449}{1 - 0.058663449} = 6.2319336\%$$

$$i^2 = 2 \left((1 + 0.062319336)^{1/2} - 1 \right) = 6.1377535\%$$

$$b) d^{12} = 12 \left[1 - (1 - 0.058663449)^{1/12} \right] = 0.060302524 = 6.0302524\%$$

$$ii) 200000 \quad AV(0,1) = 220000$$

$$AV(0,1) = 220000 / 200000$$

$$AV(0,1) = 1.1$$

$$(1+i)^n = 1.1$$

$$i = 0.1$$

$$a) 200,000 \times (1 + 0.1)^{98/365} = ₹ 204916.3564$$

$$7) 20000 \cdot A(0,17) = 26000$$

$$A(0,17) = 1.3$$

$$(1+i)^{17/12} = 1.3$$

$$(1+i) = 1.3^{(12/17)}$$

$$i = 20.3457067\%$$

$$i) d^4 = 4 \left((1 + 0.0203457067)^{1/4} - 1 \right)$$

$$= 18.9552545\%$$

$$ii) d^2 =$$

$$d = \frac{i}{1+i} = 0.169060511$$

$$d^2 = P \left(1 - \left(1 - \frac{dP}{P} \right)^P \right)$$

$$= 2 \left(1 - \left(1 - 0.169060511 \right)^{1/2} \right)$$

$$= 0.176882351$$

$$iii) i = 20.3457067\%$$

$$iv) d > d^4 > d^2 > d$$

9) i) Force of interest is interest earned during a very short time period. It is not possible in real life.

$$ii) i^2 = 0.0775$$

$$i = \left(1 + \frac{0.0775}{2} \right)^2 - 1$$

$$i = (1.03875)^2 - 1$$

$$i = 0.079001562 \approx 7.9001562\%$$

$$s = \log(1+i) \approx 0.076036133$$
$$\approx 7.6036133\%$$

$$d^{12}_z$$

$$d_z \frac{j}{1+j} = 0.073217282$$

$$\begin{aligned} d^{12}_z &= 12 \left(1 - (1 - 0.073217282)^{1/2} \right) \\ &= 0.075795745 \\ &= 7.5795745\% \end{aligned}$$

$$\begin{aligned} j^{1/2} &= \frac{1}{2} \left((1 + 0.079001562)^2 - 1 \right) \\ &= 0.082122185 \\ &= 8.2122185\% \end{aligned}$$

$$b) \quad 0 \leq t \leq 5 \quad \int_0^t 0.08$$

$$v(t) = e^{0.08t}$$

$$5 \leq t \leq 10 \quad \int_0^t 0.08$$

$$v(t) = e^{0.08t}$$

$$i) a) PV \cdot (1 + 0.18)^{3/4} = 50000$$

$$\frac{50000}{(1.18)^{3/4}} = PV = \frac{50000}{1.132169642}$$

$$= 44162.99302 //$$

$$b) i) d = 6\%$$

$$d = 1 - e^{-d}$$

$$d = 0.058235466$$

$$d^{12} = 12 \left(1 - (1 - 0.058235466)^{1/12} \right)$$

$$= 5.9850249\%$$

$$ii) d = 6\%$$

$$d = 1 - \left(1 - \frac{d^4}{4} \right)^4$$

$$= 1 - \left(1 - \frac{0.06}{4} \right)^4$$

$$d = 0.058663449$$

$$i = \frac{d}{1-d} = 0.062319314$$

$$i^{12} = 12 \left((1 + 0.062319314)^{1/12} - 1 \right)$$

$$= 0.06060691$$

$$= 6.060691\%$$

$$i^4 = 4 \left((1 + 0.062319314)^{1/4} - 1 \right)$$

$$= 0.060913704$$

$$= 6.0913704\%$$

c) a) $d^4 > d > d$

12)

$$\begin{array}{c} d^{12} = 6\% \quad d^4 = 1.5\% \quad d^{12} = 6\% \\ 0 \quad 2 \quad 4.5 \end{array}$$

$$\begin{aligned} d^{12} &= 6\% \\ d &= \left(1 + \frac{0.06}{12}\right)^{12} - 1 \\ &= 0.061677811 \\ &= 7049(1 + 0.061677811)^2 \\ &= 7945.34925 \end{aligned}$$

$$\begin{aligned} \frac{d^4}{4} &= 1.5 \\ d &= 0.0613635 \\ &= 6.13635\% \\ &= 7945.34925(1 + 2.5 \times 0.0613635) \\ &= 9164.235347 \end{aligned}$$

$$\begin{aligned} d^{12} &= 6\% \\ d &= 1 - \left(1 - \frac{d^{12}}{12}\right)^{12} \\ &= 1 - \left(1 - \frac{0.06}{12}\right)^{12} = 1 - (0.941622806) \\ &= 0.058377194 \end{aligned}$$

$$\begin{aligned} &= \frac{9164.235347}{(1 - 0.058377194)^{1.5}} \\ &= \frac{9164.235347}{0.913724884} \\ &= 10029.53461 \end{aligned}$$

$$13) \quad x \cdot (1 + 0.10)^n = 2x$$

$$(1.10)^n = 2$$

$$\log (1.10)^n = \log 2$$

$$n \log (1.1) = 0.69314718$$

$$n \times 0.095310179 = 0.69314718$$

$$n = 0.69314718 / 0.095310179 = 7.2754$$

$$ii) \quad d^{12} = 10 \cdot 1$$

$$d = 1 - \left(\frac{1 - d^{12}}{12} \right)^{12}$$

$$= 1 - \left(\frac{1 - 0.10}{12} \right)^{12}$$

$$= 0.095541699$$

$$x / (1 - 0.095541699)^n = 2x$$

$$n \log 0.904458301 = 0.69314718$$

$$\frac{n}{(1 - 0.095541699)^n} = 2$$

$$1 \times (1 - 0.095541699)^{-n} = 2$$

$$-n \log 0.904458301 = \log 2$$

$$-n = \frac{0.69314718}{0.100419076}$$

$$n = 6.90254891$$