HOMEWORK

ASSIGNMENT

Ans 1) $\lim_{h \to 0} \dfrac{2(-3+h)^2 - 18}{h}$

$= \dfrac{2(9+h^2-6h)-18}{h}$

$= \dfrac{18+2h^2-12h-18}{h}$

$= \dfrac{2h^2-12h}{h}$

$= \dfrac{2h(\cancel{2}h-6)}{h}$

$= 2(h-6)$

$\Rightarrow \lim_{h \to 0} 2(h-6)$

$= 2(0-6) = -12$

Ans 2) $g(x) = \begin{cases} 2x & x<6 \\ x-1 & x \geq 6 \end{cases}$

(a) $x = 4$

It is continuous at $x = 4$

(b) $x = 6$

$LHL = \lim_{x \to 6^-} 2x = 2(6) = 12$

$= \lim_{h \to 0} 2(-6+h) = 2(-6+0) = 12$

$f(6) = x - 1$

$= 6 - 1$

$= 5$

$RHL = \lim_{x \to 6^+} f(x)$

$= \lim_{x \to 6^+} x - 1$

$= 6 - 1$

$= 5$

$\because$ $LHL \neq f(6) = RHL$

Thus, the limit does not exist at $x = 6$.

Date: / /
Page:

Ans 3) $\lim_{x \to 9} \dfrac{x-9}{3-\sqrt{x}}$

$\Rightarrow \dfrac{x-9}{3-\sqrt{x}}$ —— (1)

Rationalising ~~eq~~ (1)

$= \dfrac{x-9}{3-\sqrt{x}} \times \dfrac{3+\sqrt{x}}{3+\sqrt{x}}$

$= \dfrac{(x-9)(3+\sqrt{x})}{(9-x)}$

$= \cancel{3x + x\sqrt{x} - 27 - 9\sqrt{x}}$

$= -1(3+\sqrt{x})$

$= \lim_{x \to 9} -(3+\sqrt{x})$

$= -(3+\sqrt{9})$

$= -(3+3)$

$= -6$

Ans 4) $f(x) = \dfrac{4x+5}{9-3x}$

a) $x = -1$

$f(-1) = \dfrac{4(-1)+5}{9-3(-1)} = \dfrac{-4+5}{9+3} = \dfrac{1}{12} = 0.0833$

$\lim_{x \to -1} f(x) = \lim_{x \to -1} \left(\dfrac{4x+5}{9-3x}\right) = \dfrac{4(-1)+5}{9-3(-1)} = \dfrac{1}{12} = 0.083$

Date: / /
Page:

$\therefore$ f(x) is continuous at $x = -1$

b) $x = 0$

$f(0) = \frac{4(0)+5}{9-3(0)} = \frac{5+0}{9-0} = \frac{5}{9} = 0.55$

$\lim_{x \to 0} f(x) = \lim_{x \to 0} \left(\frac{4x+5}{9-3x}\right) = \frac{4(0)+5}{9-3(0)} = \frac{5}{9} = 0.55$

$\therefore$ f(x) is continuous at $x = 0$

c) $x = 3$

$f(3) = \left(\frac{4(3)+5}{9-3(3)}\right) = \frac{12+5}{9-9} = \frac{17}{0} \Rightarrow$ not defined

$\therefore$ F(x) is not continuous / discontinuous at $x = 3$

Ans 5) $\lim_{x \to \infty} \frac{6e^{4x} - e^{-2x}}{8e^{4x} - e^{2x} + 3e^{-x}}$

$= \frac{6e^{\infty} - e^{-\infty}}{8e^{\infty} - e^{\infty} + 3e^{-\infty}}$ = not defined

Ans 6) $g(y) = \begin{cases} y^2+5 & y < -2 \\ 1-3y & y \geq -2 \end{cases}$

b) $\lim_{y \to 2^-} g(y) = \lim_{y \to 2^-} (y^2+5) = (-2)^2+5 = 9$

$f(-2) = 1-3(-2) = 1+6 = 7$

Date: / /
Page:

$\lim_{y \to -2^+} f(x) = \lim_{y \to -2^+} (1-3y) = 1-3(-2) = 7$

LHL $\neq f(-2) =$ RHL.

Ans 7) $f(t) = (4t^2 - t)(t^3 - 8t^2 + 12)$

$\frac{dy}{dt} = \left(\frac{d}{dt}(4t^2 - t)\right) \cdot (t^3 - 8t^2 + 12) + \left(\frac{d}{dt}(t^3 - 8t^2 + 12)\right)(4t^2 - t)$

$= (8t - 1)(t^3 - 8t^2 + 12) + (3t^2 - 16t)(4t^2 - t)$

$\Rightarrow 8t^4 - 64t^3 + 96t - t^3 + 8t^2 - 12 + 12t^4 - 3t^3 - 64t^3 + 16t^2$

$= 20t^4 - 132t^3 + 24t^2 + 96t - 12$

Ans 8) $y = R(\omega) = \frac{3\omega + \omega^4}{2\omega^2 + 1}$

$\frac{dy}{d\omega} = \frac{2\omega^2 + 1 \cdot \left(\frac{d}{d\omega} 3\omega + \omega^4\right) - 3\omega + \omega^4 \left(\frac{d}{d\omega} 2\omega^2 + 1\right)}{(2\omega^2 + 1)^2}$

$= \frac{(2\omega^2 + 1)(4\omega^3 + 3) - (3\omega + \omega^4)(4\omega)}{(2\omega^2 + 1)^2}$

$= \frac{8\omega^5 + 6\omega^2 + 4\omega^3 + 3 - 12\omega^2 - 4\omega^5}{(2\omega^2 + 1)^2}$

$= \frac{4\omega^5 \cancel{+10\omega^3} - 12\cancel{\omega^2} + \cancel{3}\omega}{(2\omega^2 + 1)^{\cancel{2}}}$

$= \frac{4\omega^5 + 4\omega^3 - 6\omega^2 + 3}{4\omega^4 + 1 + 4\omega^2} = \frac{4\left(\omega^5 + \omega^3 - \frac{3\omega^2}{2} + 3/4\right)}{\omega^4 + 1/4 + \omega^2}$

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Date: / /

Page:

Ans 9) $y = f(x) = 2e^x - 8^x$

$dy/dx = 2e^x - 8^x \log 8$ $\qquad [\frac{d}{dx} a^x = a^x \log a]$

$= \cancel{2e^x - (8^x . 0 + \log 8.)}$

$= 2e^x - 8^x \ln(8)$

Ans 10) $y = z^5 - e^z \ln(z)$.

Diff. on both sides.

$\frac{dy}{dz} = 5z^4 - (e^z . \frac{1}{z} + \ln(z) . e^z)$ (Product rule)

$\frac{dy}{dz} = 5z^4 - \left(\frac{e^z}{z} + e^z \ln(z)\right)$

Ans 11) a) $f(x) = (6x^2 + 7x)^4$

Diff. on both sides

$\frac{dy}{dx} = 4(6x^2 + 7x)^3 . (12x + 7)$

$= (48x + 28)(6x^2 + 7x)^3$

b) $f(t) = 5 + e^{4t+7}$

$\frac{dy}{dt} = 0 + e^{4t+7} \frac{d}{dx}(4t + 7)$

$\frac{dy}{dt} = 4(e^{4t+7})$.

Ans 12) a) $z = \ln(7 - x^3)$

$\frac{dz}{dx} = \frac{1}{7 - x^3} . \frac{d}{dx}(7 - x^3)$

$= \frac{-3x^2}{7 - x^3}$

Date: / /
Page:

$$\frac{d^2z}{dx^2} = \frac{(7-x^3)\cdot\frac{d}{dx}(-3x^2) - (-3x^2)\cdot\frac{d}{dx}(7-x^3)}{(7-x^3)^2}$$

$$= \frac{-6x(7-x^3) + (3x^2 \cdot -3x^2)}{(7-x^3)^2}$$

$$= \frac{-42x + 6x^4 - 9x^4}{(7-x^3)^2}$$

$$= \frac{-(3x^4+42x)}{(x^3-7)^2}$$

$$= -\frac{(3x^4+42x)}{(-x^3+7)^2} = -\frac{3x(x^3+14)}{(-x^3+7)^2}.$$

b) $y = Q(v) = \frac{2}{(6+2v-v^2)^4}$

Diff. w.r.t to v

$$\frac{dy}{dv} = \frac{(6+2v-v^2)^4\cdot\frac{d}{dx}(2) - 2\cdot\frac{d}{dx}(6+2v-v^2)^4}{[(6+2v-v^2)^4]^2}$$

$$= \frac{0 - 2(4(6+2v-v^2)^3\cdot\frac{d}{dx}(6+2v-v^2)}{[(6+2v-v^2)^4]^2}$$

$$= \frac{-2(4(6+2v-v^2)(2-2v)}{[(6+2v-v^2)^4]^2}$$

$$= \frac{\cancel{-8(}\,8(2v-2)(6+2v-v^2)^3}{[(6+2v-v^2)^4]^2}$$

$$\frac{dy}{dv} = \frac{16v-16}{(6+2v-v^2)^5} = \frac{16(v-1)}{(6+2v-v^2)^5}$$

$$\frac{d^2y}{dx^2} = \frac{(6+2v-v^2)^5 \frac{d}{dx}(16v-16) - (16v-16)\frac{d}{dx}(6+2v-v^2)^5}{((6+2v-v^2)^5)^2}$$

$$= \frac{16(6+2v-v^2)^5 - \cancel{5}(180(v-1)(6+2v-v^2)^4.(2-2v)}{((6+2v-v^2)^{10}}$$

$$= \frac{16(6+2v-v^2)^4[1-5(v-1)(2-2v)]}{(6+2v-v^2)^{10}}$$

$$= \frac{16[1-10v-10v^2-10+10v]}{(6+2v-v^2)^6}$$

$$= \frac{16[10v^2 + 9]}{(6+2v-v^2)^6}$$

c) $y = f(t) = \ln(1+t^2)$

$$\frac{dy}{dt} = \frac{1}{1+t^2} \cdot \frac{d}{dt} t^2$$

$$= \frac{2t}{1+t^2}$$

$$\frac{d^2y}{dt^2} = \frac{(1+t^2)\frac{d}{dt}(2t) - 2t\frac{d}{dt}(1+t^2)}{((1+t^2)^2)}$$

$$= \frac{2(1+t^2) - 2t(2t)}{((1+t^2)^2)}$$

$$= \frac{2+2t^2-4t^2}{(1+t^2)^2}$$

$$= \frac{-2t^2+2}{(1+t^2)^2}$$

Ans 13) $f(x) = x^3 - 3x^2 - 9x + 12$

$f'(x) = 3x^2 - 6x - 9$

Put $f'(x) = 0$

Date:
Page:

$$3x^2 - 6x - 9 = 0$$
$$3(x^2 - 2x - 3) = 0$$
$$(x^2 - 2x - 3) = 0$$
$$(x^2 - 3x + x - 3) = 0$$
$$(x(x-3) + 1(x-3)) = 0$$
$$(x+1)(x-3) = 0$$

$-\infty$ Increasing -1 Decreasing 0 ~~decreasing~~ Decreasing 3 Increasing ∞

-1 (maxima), 0, 3 (minima)

$$f''(x) = 6x - 6$$
$$f''(-1) = 6(-1) - 6 = -6 - 6 = -12$$
$$f''(0) = 6(0) - 6 = 0 - 6 = -6$$
$$f''(3) = 6(3) - 6 = 18 - 6 = 12$$

$f(-1) = (-1)^3 - 3(-1)^2 - 9(-1) + 12 = 17$ & $f(3) = (3)^2 - 3(3)^2 - 9(3) + 12 = -15$

Maximum value occurs at -1 (17) & minimum value occurs at 3 (-15).

Ans 14) $f(x) = 4x^3 - 18x^2 + 24x - 7$

$$f'(x) = 12x^2 - 36x + 24$$

Put $f'(x) = 0$

$$12x^2 - 36x + 24 = 0$$
$$12(x^2 - 3x + 2) = 0$$
$$x^2 - 3x + 2 = 0$$
$$x^2 - 2x - x + 2 = 0$$
$$x(x-2) - 1(x-2) = 0$$
$$(x-2)(x-1) = 0$$

$-\infty$ increasing 1 decreasing 2 increasing ∞

1 (Maxima), 2 (minima)

$f''(x) = 24x - 36$

$f''(1) = 24(1) - 36 = -12$ (Maxima)

$f''(2) = 24(2) - 36 = 12$ (minima)

$y(1) = 4(1)^3 - 18(1)^2 + 24(1) - 7 = 3$; $y(2) = 4(2)^3 - 18(2)^2 + 24(2) - 7 = 1$

Ans 15) $f(x) = x^2(x+3) = x^3 + 3x^2$

$f'(x) = 3x^2 + 6x$

Put $f'(x) = 0$

$3(x^2 + 2x) = 0$

$3x(x+2) = 0$

$3x = 0$ or $x + 2 = 0$

$x = 0, -2$

$-\infty$ increasing, decreasing increasing ∞

-2 0

(Maxima) (Minima)

$f''(x) = 6x + 6$

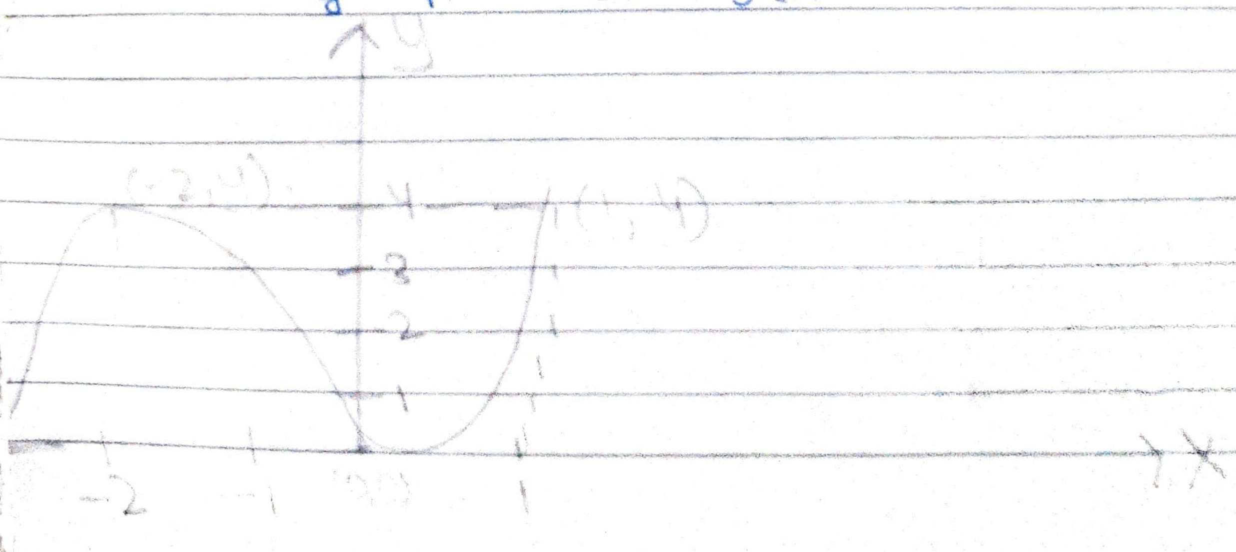
$f''(-2) = 6(-2) + 6 = -12 + 6 = -6$

$f''(0) = 6(0) + 6 = 6 + 0 = 6$

$y = f(0) = (0)^3 + 3(0)^2 = 0$

$y = f(-2) = (-2)^3 + 3(-2)^2 = -8 + 3(4) = 4$

$y = f(1) = (1)^3 + 3(1)^2 = 1 + 3 = 4$



Date:
Page:

Ans 16) $f(x) = 3x^2 - 6x$

$f'(x) = 6x - 6$

Put $f'(x) = 0$

$6x - 6 = 0$

$6x = 6$

$x = 1$

$\leftarrow -\infty$ decreasing | 1 (minima) | increasing $\infty \rightarrow$

$f''(x) = 6$

$f''(1) = 6$

$y = f(2) = 3(2)^2 - 6(2) = 12 - 12 = 0$

$y = f(1) = 3(1)^2 - 6(1) = 3 - 6 = -3$

$y = f(0) = 3(0)^2 - 6(0) = 0$

$y = f(-1) = 3(-1)^2 - 6(-1) = 3 + 6 = 9$