FM Homework-1
[Unit-1, 2]

1) $d^{(4)} = 8\%$

$$d = 1 - \left(1 - \frac{d^{(p)}}{p}\right)^p$$

$$= 1 - \left(1 - \frac{8\%}{4}\right)^4$$

$$= 1 - (1 - 2\%)^4$$

$$= 1 - (1 - 0.02)^4$$

$$= 0.07763184 \quad \text{OR} \quad 7.763184\%$$

(i) $\delta = -\ln(1-d)$

$$= \ln(1 - 0.07763184)$$

$$= -(-0.080810829)$$

$$= 0.080810829 \quad \text{OR} \quad 8.0810829\%$$

(ii) $$i = \frac{1}{\left(1 - \frac{d^{(p)}}{p}\right)^p} - 1$$

$$= 1.084165784 - 1$$

$$= 0.084165784 \quad \text{OR} \quad 8.4165784\%$$

(iii) $d^{(12)} = p\left[1-(1-d)^{1/p}\right]$

$= 8.653934\%$

Q.2) $\delta(t) = \cancel{0.04}\ 0.004t + 0.0002t^2$

(i) Amount due $= C = 1000$

$PV_0 = C \cdot V(t)$

$= 1000 \times \left(e^{-\int_0^{10} 0.004t + 00002t^2\, dt}\right)$

$= 1000\left(e^{-\int_0^{10} 0.002t^2 + 0.0002t^3/3\, dt}\right)$

$= 1000\left(e^{-0.266667}\right)$

$= 1000\,(0.76593)$

$= 765.93$

(ii) $(1+i)^t = e^{\delta}$

$(1+i) = e^{\delta/10}$

$i = e^{\delta/10} - 1$

$i = 1.027025438 - 1.$

$i = 2.7025438\%$

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3) (i) The relationship $d = iv$ has an intresting interpretation. Intrest earned on an investment, paid at the ~~begning~~ beginning of the period is discount(d). Intrest earned on an investment paid at the end of the period is intrest(i). Thus, when we discount from i at the end of the period with the discounting factor (v) we get (d).

(ii) a) $\frac{d^{(4)}}{4}$ = effectiverate of discount quaterly = 2.25%

$$\text{Nominal discount} = 1-(1-2.25\%)^4$$

$$= 8.7007806\%$$

$$d^{(2)} = 2\left[1-(1-8.7007806)^{1/2}\right]$$

$$= 0.088987499$$

$$= 8.8987499\%.$$

b) $d^{(1/2)} = .5\%$

$$d = 1-\left(1-\frac{d^{(p)}}{p}\right)^p$$

$$= 1-\left(1-\frac{0.05}{1/2}\right)^{1/2}$$

$$= 1-(1-0.1)^{1/2} = 0.05131670]$$

$$d^{(2)} = p\left[1-(1-d)^{1/p}\right]$$

$$= 2\left[1-(1-0.051316701)^{1/2}\right]$$

$$= 0.05199\,2506$$

$$= 5.1992506\%$$

(iii) $V(0,1) = \dfrac{1}{(1+i)^{91/365}} = \dfrac{91}{365}\, d$

$i = 5\%$

$$= \frac{1}{(1.05)^{91/365}} = \frac{91}{365}\, d$$

$$= \cancel{0.98790956}$$

$$d = \left[1 - \frac{1}{(1.05)^{91/365}}\right]\frac{365}{91}$$

$$= 0.04849\,4618$$

$$= 4.84946\cancel{20}18\%$$

4) i) same as 3(i)

(ii). $i = 0.08$

a) $d^{(12)} = 12\left[1-(1+0.08)^{-1/12}\right]$

$= 7.6714776\%$

b) $i^{(365)} = 12\left[\cancel{(1+i)}(1+0.08)^{1/12}\right.$

c) $\delta = \ln(1+i)$

$= 7.6961041\%$

d) $i^{(0.5)} = 0.5\left[(1+0.08)^{1/0.5} - 1\right]$

$= 8.32\%$

5)(i) $d^{(4)} = 6\%$.

$$d = 1-\left(1-\frac{d^{(p)}}{p}\right)^{p}$$

$$= 1-\left(1-\frac{0.06}{4}\right)^{4}$$

$= 5.8663449\%$

(a) $i = \frac{d}{(1-d)}$

$= 0.062319314$

$i^{(2)} = 2\left[(1+0.62319314)^{1/2} - 1\right]$

$= 6.139752\%$

b) $d^{(12)} = 12\left[1-(1-d)^{1/12}\right]$

$= 12\left[1-(1-0.05866344 9)^{1/12}\right]$

$= 6.0362524\%$

(ii) $i = \frac{20000}{200000} \times 100$

$= 10\%$.

No. of days = 93 days.

AV $= 200000 \times (1.1)^{93/365}$

$= 2,04,916.3564$.

6) Let the retail price be 'x'

If retail ~~price~~ pays cash = 70% x

PV = 0.7x.

~~the he has to pay =~~

If he uses 6 month credit period = 75% x

P.V. = 0.75x in 6 months time.

We need to find discounting factor for both cases.

Time = 6months = 1/2 yr.

$P.V_0 = (1-d)^n \times A.V$

$0.7x = (1-d)^{1/2} \times 0.75x$

~~0.7x~~

$\frac{0.7}{0.75} = (1-d)^{1/2}$

$1-\left(\frac{0.7}{0.75}\right)^2 = d$

$1 - 0.871112 = d$

$= 12.89\%$

(ii)(a) $(d)^{12} = 12\left(1-(1-12.88889\%)^{1/12}\right)$

$= 12\left(1-(1-0.1288889)^{1/12}\right)$

$= 13.7195451\%$

b) $i = \frac{d}{1-d} = 15.9011042$.

$i^{(2)} = 2\left[(1+i)^{1/2} - 1\right]$

$= 2\left[(1+0.159011042)^{1/2} - 1\right]$

$= 15.3147502\%$

(c) $\delta = -\ln(1-d)$
$= -(-\cancel{14.7}\ 0.147567092)$
$= 14.7567092\%$

7 Amt invested $= C = 20000$.

$A.V_{17\,months} = 26000$.

$t = 17\,months = 1 + 5/12$ ~~months~~ years
$= 1.4167$.

$A.V_n = C(1+i)^n$

$26000 = 20000(1+i)^{1.4166667}$

$i = 20.3457067\%$

(i) $i^{(4)} = 4((1+0.203457067)^{1/4} - 1)$.
$= \cancel{18.995}\ 18.9552545\%$.

(ii) ~~$d^{(12)}\ 12(12(1+0.203457067)^{1/12} - 1)$~~

$d^{(12)} = p(1-(1+i)^{-1/p})$.

~~18.3776527~~ $= 17.6882352\%$

(iii) $A.V = C(1+in)$
$26000 = 20000(1+(0.203457067)(1.466666$
$26000 = 20000(1+1.4166667i)$
$= \cancel{24.7646169}\ 21.1764705\%$.

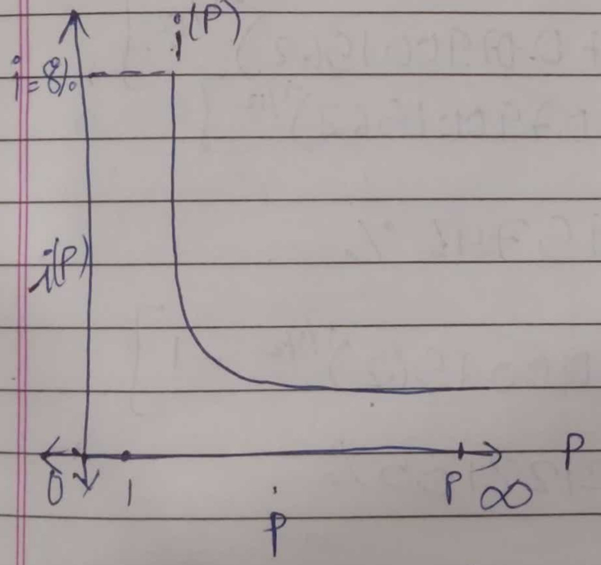
(iv) Numerically the intrest rate in compound interest highest and discounting rate is the lowest But they both yield the same AV and PV at t_0.

8) $i = 8\%$

$i^{p} = p[(1+i)^{1/p} - 1]$

$=$

p	i^{p}
1	8%
2	7.84609%
3	7.79567%
4	7.77062%
12	~~7.72084%~~ 7.72084%
100	7.69907%
365	~~7.66~~ 7.69691%
∞	7.696104%



→ ~~More frequently~~ $i^{(p)} \propto \frac{1}{p}$

→ When continuous compounding approaches ∞ $i^{(p)}$ has a limiting value known as force of intrest or δ

9) i) Force of intrest - The measure of intrest at individukl moments of time or at any instan of time is known as force of intrest

ii) $i^{(2)} = 0.0775$.

$$i = \left(1 + \frac{i^{(p)}}{p}\right)^{p} - 1$$

$$= \left(1 + \frac{0.0775}{2}\right)^{2} - 1$$

$$= 7.9001562\%$$

$$\delta = \ln(1+i)$$

$$= 7.6036133\%$$

$$d^{(12)} = 12\left[1 - (1 + 0.079001562)^{-1/12}\right]$$

$$= 12\left[1 - (1.079001562)^{-1/12}\right]$$

$$= 7.5795746\%$$

$$i^{(1/2)} = \frac{1}{2}\left[(1 + 0.079001562)^{1/1/2} - 1\right]$$

$$= 8.2122185\%$$

10) $\delta(t) =$

$V(t) =$

$=$

when

$V(t) =$

when

$V(t)$

11) a)

n

b)

9) i) Force of intrest - The measure of intrest at individukl moments of time or at any insta of time is known as force of interest

ii) $i^{(2)} = 0.0775$.

$$i = \left(1 + \frac{i^{(p)}}{p}\right)^p - 1$$

$$= \left(1 + \frac{0.0775}{2}\right)^2 - 1$$

$$= 7.9001562\%$$

$$\delta = \ln(1+i)$$

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$$d^{(12)} = 12\left[1 - (1 + 0.079001562)^{-1/12}\right]$$

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$$i^{(1/2)} = \frac{1}{2}\left[(1 + 0.079001562)^{1/1/2} - 1\right]$$

$$= 8.2122185\%$$

10) $\cancel{\delta(t)}$ when $0 \leq t \leq 5$

$V(t) = e^{-\int 0.08dt}$

$= e^{-0.08t}$

when $5 \leq t \leq 10$

$V(t) = e^{-0.06t - 0.1}$

when $t \leq 10$.

$V(t) = e^{-\int_0^5 0.08\,dt - \int_5^{10} 0.06\,dt - \int_{10}^{t} 0.04\,dt}$

$= e^{-0.4 + 0.3 - 0.6 + 0.4 - 0.04t}$

$= e^{-0.04t - 0.3}$

11) a) $C = 50000$ = Amt due.

$d = 18\%$

$n = 9/12 = 3/4$ years

$P.V = C(1-d)^{t}$

$= 50000(1 - 0.18)^{3/4}$

$= 50000(0.82)^{3/4}$

$= 43{,}085.42623$

b) i) $\delta = 6\%$

$d = 1 - e^{-\delta}$

$= 5.8235466\%$

$d^{12} = p(1-(1-d)^{1/p})$

$= 12(1-(1-0.058235466)^{1/12})$

$= 5.9850249\%$

ii) $d^4 = 6\%$

$$i = \frac{1}{\left(1 - \frac{0.06}{4}\right)^4} - 1$$

$= 6.2319315\%$

$$i^4 = \cancel{4(1+i)} \cdot p\left((1+i)^{1/p} - 1\right)$$

$= 6.0913705\%$

$$i^{12} = p\left((1+i)^{1/p} - 1\right)$$

$$= 12\left((1 + 0.062319315)^{1/12} - 1\right)$$

$\approx 6.060688\%$

c) Ascending order $\rightarrow d < \delta < i^{(4)}$

d will always be lowest ~~than the~~ numerical value as it is charged at the begning of the term. So due to time value of money $d < \delta < i^{(4)}$. δ refers to continuos compounding i.e more often than $i^{(4)}$ (quarterly Compounding). Due to more frequent compounding than $i^{(4)}$, δ will yield ~~more~~ higher accumulated value than $i^{(4)}$, if $\delta = i^{(4)}$. Thus δ has to be smaller than $i^{(4)}$ to yield the same accumulating value.

d) same as 3(i)

7049 $\qquad i_s^{(4)} = 1.5\%$

12Y $\vdash$ 0 $\quad i^{(12)} = 6\%$ $\quad$ 2 $\qquad$ 4.5 $\quad d^{(12)} = 6\%$ $\quad$ 6

$C = 7049 =$ Amt invested

$$A.V_6 = C \cdot A(0,6)$$
$$= C \cdot A(0,2) \cdot A(2,4.5) \cdot A(4.5,6)$$

$\Rightarrow$ for $A(0,2)$

$$i^{(12)} = 6\%. \quad \frac{i^{(12)}}{12} = 0.5\%$$

$$= A(0,2) = (1.005)^{24}.$$

$\Rightarrow$ for $A(2,4.5)$

$$= i_s^{(4)} = 1.5\%$$

$$= A(2,4.5) = (1 + 10(0.015)) = 1.15$$

$\Rightarrow$ $A(4.5,6)$

$$d^{(12)} = 6\% = \frac{d^{(12)}}{12} = 0.5\%.$$

$$A(4.5,6) = \frac{1}{(1-0.005)^{18}}$$

$$= 9999.8936$$

$$AV_6 = 7049 \cdot (1.005)^{24} \times 1.15 \times \frac{1}{(1-0.005)^{18}}$$

13) Amt. invested $= C = x$

$AV_n = 2x$.

Time $= n$

i) $i = 10\%$

$AV_n = C(1+i)^n$

$2x = = x(1+10\%)^n$

$2 = 1.1^n$

$\ln 2 = n \ln 1.1$

$n = 7.27$ years

ii) $d^{(12)} = 10\%$ $\frac{d^{(12)}}{12} = 0.83333334$

$$AV_n = \frac{C}{\left(1 - \frac{d^{(12)}}{12}\right)^{n12}}$$

$$2x = \frac{x}{(1 - 0.83333334)^{n \cdot 12}}$$

$12n(1 - 0.83333334\%) = \ln 1/2$

$n = 6.902$ years