PS Assignment 1 :-

Q1] i) 4, 7, 9, 9, 11, 15, 18, 18, 18, 25

~~Arranging them in ascending order :-~~

We can see from the given data that :-

Mode of the Data = Number which occurs highest no of times

$= \underline{18}$

N = No of Elements = 10

Thus Median will be :-

$$\frac{\left(\frac{N}{2}\right)th + \left(\frac{N+1}{2}\right)th}{2} \Rightarrow \frac{5th\ \text{Value} + 6th\ \text{Value}}{2}$$

$$\Rightarrow \frac{11+15}{2} \Rightarrow \frac{26}{2} \Rightarrow \boxed{13}$$

$\Rightarrow \boxed{\text{Median} = 13}$

$$\text{Mean} = \frac{\sum x_i}{N} = \frac{4+7+9+9+11+15+3\times18+25}{10}$$

$$= \frac{22+18+18\times3+15+25}{10}$$

$$= \frac{37+4\times18+25}{10}$$

$= 13.4$

~~Variance = $E(x^2) - [E(x)]^2$~~

ii) Variance $= \frac{1}{n-1}\left\{\sum x^2 - n(\bar{x})^2\right\}$

$\sum x^2 = 2170$

Variance $= \frac{1}{9}\left\{2170 - 10(13.4)^2\right\}$

$= \frac{1}{9}\left\{2170 - 1795.6\right\} = \underline{\underline{41.6}}$

$\therefore S.D = \sqrt{Variance} = \sqrt{41.6} = 6.449\cancel{8}1$

ii) Table :-

x	0	1	2	3	4
frequency (n)	5	11	26	15	3

$$Mean = \frac{\sum x_i f_i}{\sum f_i} = \frac{123}{60}$$

$$Median = \left(\frac{N}{2}\right)th\ Value = (30th)\ Value = 2$$

$$Mode = 2$$

$$Variance = \frac{1}{n-1}\left(\sum x_i^2 f_i - n(\sum f_i x_i)^2\right)$$

$$= \frac{1}{n-1}\left\{298 - \cancel{2}60(4.2025)\right\}$$

$$= \frac{1}{59}\left\{298 - 252.15\right\}$$

$$= \frac{45.85}{59} = 0.77711$$

~~= 0.8~~

$$\boxed{S.D = \sqrt{Var(X)} = 0.88153}$$

Q2) Let x = No. of claims incurred in a year.

$\boxed{\lambda = 0.5}$

~~[illegible]~~ Let $\exists$ a Random Variable X s.t

$\boxed{X : \{x\}.}$

i) $P(X=0) = \frac{\lambda^0 e^{-\lambda}}{0!} = e^{-0.5} = \frac{1}{\sqrt{e}}$

ii) ~~Required~~

Required Probability $= P(X \geq 1) \cdot P(X=0) \cdot P(X=0)$

$$= \{1 - P(X<1)\}\{P(X=0)\}^2$$

$$= \frac{1 - P(X<1)}{e}$$

$$= \frac{1 - 1/\sqrt{e}}{e}$$

$$= \frac{1 - 0.60653}{e}$$

$$= \frac{0.39347}{e}$$

$$= \underline{\underline{0.1447495}}$$

Q8] $X \sim Ga(\overset{\alpha}{2}, \overset{\lambda}{1/2})$

Range (x) $(0, \infty)$

i) Mean of the Distribution $= \frac{\alpha}{\lambda} = \frac{2}{1/2} = \underline{4}$

$$SD = \frac{\alpha}{\lambda^2} = \frac{2}{1/4} = \underline{8}$$

ii) As the inherent property of gamma function, the graph will be highly skewed. Also, as the shape parameter is 2 and scale parameter is 1/2, thus it increases and decreases the height and spread of the graph respectively.

iii) ~~CDF~~

$$PDF = \frac{\lambda^{\alpha} x^{\alpha-1} e^{-\lambda x}}{\Gamma(\alpha)}$$

$$= \frac{(1/2)^2 x e^{-x/2}}{\Gamma(2)}$$

$$= \underline{\frac{1}{4 \times 2}\left(x e^{-x/2}\right)} = \underline{\frac{1}{8}\{x e^{-x/2}\}}$$

$$CDF = \int_0^x \frac{x e^{-x/2}}{8} dx$$

$$= \frac{1}{8} \int_0^x x e^{-x/2} dx$$

$$= \frac{1}{8}\left[e^{-x/2} \int_0^x x\,dx - \int \frac{e^{-x/2}}{-1/2} \int_0^x x\,dx \right]$$

$$= \frac{1}{8}\left[\left(\frac{x^2 e^{-x/2}}{2}\right)_0^x + 2 \int_0^x x^2 e^{-x/2} dx \right]$$

$$= \frac{1}{8} \int_0^x x e^{-x/2} dx$$

$$= \frac{1}{8}\left[x \int_0^x e^{-x/2} dx - \int_0^x \left(\int_0^x e^{-x/2} dx \right) dx \right]$$

$$= \frac{1}{8}\left[\left(\frac{x e^{-x/2}}{-1/2} \right)_0^x - \left(\int_0^x \frac{e^{-x/2}}{-1/2} dx - \frac{e^0}{-1/2} dx \right) \right]$$

$$= \frac{1}{8}\left[-2xe^{-x/2} + 0 + 2\int_0^x e^{-x/2} dx - 2 \int_0^x dx \right]$$

$$= \frac{1}{8}\left[-2xe^{-x/2} + 2\left(\frac{e^{-x/2}}{-1/2} \right)_0^x - 2x \right]$$

$$= \frac{1}{8}\left[-2xe^{-x/2} - 4\left(e^{-x/2} - 1\right) - 2x \right]$$

$$= \frac{-1}{4}\left[xe^{-x/2} + 2\left(e^{-x/2} - 1\right) + 2x \right]$$

4] There are 10 male and 8 female staff

$P(\text{required}) = P\left(\frac{\text{It consists 2 qualified females}}{\text{It has atleast one female}} \right)$

By Conditional Probability

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$10, 8 \Rightarrow {}^{18}C_2$ $\frac{8\times7}{2}$

Let A = Both female are fully qualified
B = Atleast one of them ~~are~~ female

$$P(B) = \frac{{}^8C_1 \cdot {}^{10}C_1 + {}^8C_2 \cdot {}^{10}C_0}{{}^{18}C_2}$$

$$= \frac{8\times10 + 28\times1}{\frac{18\times17}{2}}$$

$$= \frac{80+28}{9\times17} = \frac{\cancel{108}\ 12}{\cancel{153}\ 17} = \frac{12}{17}$$

$$P(A\cap B) = \frac{{}^7C_2}{{}^{18}C_2} = \frac{7\times\cancel{6}\,\cancel{2}}{6\,\cancel{18}\times17} = \frac{7}{51}$$

Now, from ①,

$$P(A/B) = \frac{P(A\cap B)}{P(B)}$$

$$= \frac{\frac{7}{\cancel{51}\ 3}}{\frac{12}{\cancel{17}}} = \frac{7}{3\times12} = \frac{7}{36}$$

Q5] Let A - Packages delivered by level 1
B - Packages delivered by level 2
C :- Packages delivered by level 3

$P(A) = 0.4$, $P(B) = 0.5$, $P(C) = 1 - P(A) - P(B)$
$= 0.1$

Let Q_1 :- Late Arrival ~~using~~ given L-1
Q_2 :- Late Arrival ~~using~~ given L-2
Q_3 :- Late Arrival ~~using~~ given L-3

$P(Q_1) = 0.2$, $P(Q_2) = 0.4$, $P(Q_3) = 0.1$,

[Note, Here a doubt :- These probabilities are not summing up to 1]

By Bayes Theorem:-

$$P\left(\text{sent by L-2} / \text{Late}\right) = \frac{P(B)P(a_2)}{P(A)P(a_1)+P(B)P(a_2)+P(C)P(a_3)}$$

$$= \frac{0.5 \times 0.4}{0.4\times 0.2 + 0.5\times 0.4 + 0.1\times 0.1}$$

$$= \frac{5\times 4}{8+20+1} = \frac{20}{29}$$

6] $X \sim U(1,2)$, $\boxed{Y = \frac{3}{6-X}}$ $\Rightarrow \frac{3}{Y} = 6-X$

$PDF(X) = \frac{1}{1} = 1$ $\Rightarrow X = 6 - \frac{3}{Y}$

$$F_Y(t) = P(Y \le t)$$

$$= P\left(\frac{3}{6-X} \le t\right)$$

$$= P\left(\frac{6-X}{3} \ge \frac{1}{t}\right)$$

$$= P\left(2 - \frac{X}{3} \ge \frac{1}{t}\right)$$

$$= P\left(\frac{X}{3} \le 2 - \frac{1}{t}\right)$$

$$= P\left(X \le 3\left(2 - \frac{1}{t}\right)\right)$$

$$= F_X\left(6 - \frac{3}{t}\right)$$

$$= \frac{6 - \frac{3}{t} - 1}{1} = 5 - \frac{3}{t}$$

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$F f^{\prime}_{X}(t) = 5 - \frac{3}{t}$

~~Integrating~~ Differentiating :

$f_X(t) = 0 - 3\left[\frac{-1}{t^2}\right] = \frac{3}{t^2}$

Q7) i) $F(3) = P(X \le 3)$

$= P(X=1) + P(X=2) + P(X=3)$

$= 0.05 + 0.15 + 0.35$

$= 0.55$

ii) $E(X) = \sum x_i p_i$

$= 0.05 + 0.30 + 1.05 + 1.6 + 0.25$

$= 0.60 + 2.65$

$= 3.25$

iii) $V(X) = E(X^2) - [E(X)]^2$

$E(X^2) = \sum x_i^2 p_i$

$= 0.05 + 4 \times 0.15 + 9 \times 0.35 + 16 \times 0.4 + 25 \times 0.05$

$= 0.05 + 0.60 + 3.15 + 6.4 + 1.25$

$= \cancel{7.45}\ 11.45$

$V(X) = 10.5625 - 11.45$

$= 0.8875$

iv) Coeff of skewness $= \frac{\sum (x_i - \bar{x})^3}{\sigma^3} \sqrt{\frac{N(N-1)}{N-2}}$

x	1	2	3	4	5
$x - \bar{x}$	-2.25	-1.25	-0.25	0.75	1.75
$(x-\bar{x})^3$	-11.39	-1.95	-0.015	0.421	5.359

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Prob (p) = 0.2
n = 15

X :- No of claims exceeding 1000
∴ $X \sim Bin(n = 15, p = 0.2)$
⇒ Required Prob = $P(X<3)$

$= P(X=0) + P(X=1) + P(X=2)$

$= {}^{15}C_0 p^0 q^{15} + {}^{15}C_1 p^1 q^{14} + {}^{15}C_2 p^2 q^{13}$

$= (0.8)^{15} + 15(0.2)(0.8)^{14} + \frac{15\times 14}{2}(0.2)^2(0.8)^{13}$

$= (0.8)^{13}\left[0.64 + 0.3\times 0.8 + 15\times 7\times 0.04\right]$

$= (0.8)^{13}\left[0.64 + 0.24 + 4.2\right]$

$= 5.08\times(0.8)^{13}$

$= 0.27928$

9] X :- No of weeks until 3rd success occurs
Prob of winning = 0.01

~~fx(x) =~~
~~P(X=7) =~~ $P(X=x) = \binom{x-1}{k-1} p^k q^{x-k}$

$P(X=7) = \binom{6}{2} p^3 q^4$

$= \binom{6}{2}(0.01)^3(0.99)^4$

~~= 3×5×0.0~~

$= 0.0000144089$

Q10) $\lambda \to$ mean no of claims per day.
Average for the 5 days (1 week) will be:

$5\lambda = 2$

$\boxed{\lambda = \frac{2}{5}} = 0.4$

$P(X \geq 2) = 1 - P(X \leq 2)$

$= 1 - P(X \leq 2)$
$= 1 - 0.99207$
$= \underline{0.00793}$

Q11] $P(\text{male}) = P(\text{female}) = \frac{1}{2}$

There is no interdependence between the first born and the next born child. Thus :-
$P(A) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

Q12) X: No. of claims per day
$\Rightarrow X \sim Poi(10)$
Y: Time elapsed between 2 claims.
$Y \sim Exp(10)$
$\Rightarrow \cancel{P(Y < 0.1)} \Rightarrow P(Y = y) = \lambda e^{-\lambda y}$

$\cancel{P(X < 0.1) = 10e}$ $P(X \leq$

$P(Y \leq 0.1) = \cancel{P(Y \leq 0.1) - P(Y = 0.1)} = 1 - e^{-1}$
$= \cancel{1 - e^{-1} - (10e^{-1})}$
$= \cancel{1 - 11e^{-1}}$
$= \cancel{1 - \frac{11}{e}}$

$= 1 - 0.36787$
$= \underline{0.63213}$

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Q12] $X \sim U(75, 120)$

~~$P(80 < X < 100) =$~~

$$f_X(x) = \frac{1}{120-75} = \frac{1}{45} \quad \text{for } 75 < x < 120$$

$$P(80 < X < 100) = P(X > 100) = \frac{20}{45} = \frac{4}{9}$$

Q14] $X \sim Ga(5, 0.4)$

$$P(X < 22.89) = ?$$

Lemma:- If $X \sim Ga(\alpha, \lambda)$
then $2\alpha X \sim \chi^2_{2\lambda}$, Here $\alpha = 5, \lambda = 0.4$

Thus,

$$P(X < 22.89) = P(10X < 228.9)$$
$$= P(\chi^2_{0.8} < 228.9)$$