Assignment - 2

1 $\displaystyle\int_{1/50}^{2} \frac{e^{\frac{2}{w}}}{w^2}\, dw$

let $\frac{2}{w} = t$

$-\frac{2}{w^2}\, dw = dt$

$\frac{dw}{w^2} = \frac{-1}{2}\, dt$

$\displaystyle -\frac{1}{2}\int_{100}^{1} e^t\, dt$

$-\frac{1}{2}\left(e^t\right)_{100}^{1}$

$-\frac{1}{2}\left(e^1 - e^{100}\right) = \frac{e^{100} - e^1}{2}$

2) $\displaystyle\int \frac{2t^3+1}{(t^4+2t)^3}\, dt$

$t^4 + 2t = x$

$(4t^3 + 2)\, dt = dx$

$2(2t^3+1)\, dt = dx$

$(2t^3+1)\, dt = \frac{1}{2}\, dx$

$\displaystyle\frac{1}{2}\int \frac{1}{x^3}\, dx = \frac{1}{2}\left(\frac{1}{(x^{2})(-2)}\right) = \frac{-1}{4x^2} + C$

$= \frac{-1}{4(t^4+2t)^4} + C$

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3 $f'(x) = x^4 + 3x - 9$

$f(x) = \int x^4 + 3x - 9 \, dx$

$= \frac{x^5}{5} + \frac{3x^2}{2} - 9x + C$

4) $f(x) = 6 \; , x > 1$

$= 3x^2 \; , x \leq 1$

$\int_{-2}^{3} f(x) \, dx = \int_{-2}^{1} 3x^2 \, dx + \int_{1}^{3} 6 \, dx$

$= \left(\frac{\cancel{3}x^3}{\cancel{3}} \right)_{-2}^{1} + (6x)_{1}^{3}$

$= (1+8) + (18-6)$

$= 9 + 12$

$= 21$

5) $\int 3(8y - 1) e^{4y^2 - y} \, dy$

let $4y^2 - y = t$

$(8y - 1) dy = dt$

$3 \int e^t \, dt$

$3e^t + C$

$3e^{4y^2 - y} + C$

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6) $f''(x) = 15\sqrt{x} + 5x^3 + 6$

$= 15x^{\frac{1}{2}} + 5x^3 + 6$

$f'(x) = \int 15x^{\frac{1}{2}} + 5x^3 + 6 \; dx.$

$= \dfrac{\cancel{15}^{5} x^{\frac{3}{2}}}{\frac{\cancel{3}}{2}} + \dfrac{5x^4}{4} + 6x.$

$= 10x^{\frac{3}{2}} + \dfrac{5x^4}{4} + 6x + C_1$

$f(x) = \int 10x^{\frac{3}{2}} + \dfrac{5x^4}{4} + 6x + C_1 \; dx.$

$= \dfrac{\cancel{10}^{2} x^{\frac{5}{2}}}{\frac{\cancel{5}}{2}} + \dfrac{\cancel{5}x^5}{4\times\cancel{5}} + \dfrac{\cancel{6}^{3} x^2}{\cancel{2}} + C_1x. + C_2$

$f(x) = 4x^{\frac{5}{2}} + \dfrac{x^5}{4} + 3x^2 + C_1x + C_2$

$f(4) = 4(4)^{\frac{5}{2}} + \dfrac{4^{\cancel{5}4}}{\cancel{4}} + 3(4)^2 + 4C_1 + C_2$

$404 = 128 + 256 + 48 + 4C_1 + C_2$

$4C_1 + C_2 = -28 \quad \rightarrow ①$

$f(1) = 4 + \dfrac{1}{4} + 3 + C_1 + C_2$

$-\dfrac{5}{4} - \dfrac{1}{4} - 7 = C_1 + C_2$

$\therefore C_1 + C_2 = -8.5 \quad \rightarrow ②$

① $\rightarrow$ ②

$4C_1 + \cancel{C_2} = -28$

$C_1 + \cancel{C_2} = -8.5$

$- \quad - \quad +$

$3C_1 = -19.5$

$C_1 = -6.5$

$-6.5 + C_2 = -8.5$

$C_2 = -2$

$\therefore f(x) = 4x^{5/2} + \frac{x^5}{4} + 3x^2 - 6.5x - 2$

8) $\frac{dr}{d\theta} = \frac{r^2}{\theta}$

$\frac{1}{r^2} dr = \frac{d\theta}{\theta}$

$\int \frac{1}{r^2} dr = \int \frac{1}{\theta} d\theta$

$\frac{-1}{r} = \log \theta + c$

$r = \frac{-1}{\log \theta + c}$

$2 = \frac{-1}{\log 1 + c}$

$2 = \frac{-1}{c}$

$c = \frac{-1}{2}$

$r = \frac{-1}{\left(\log \theta - \frac{1}{2}\right)}$

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9) $\frac{dv}{dt} = 9.8 - 0.196v$

$\int \frac{dv}{9.8 - 0.196v} = \int dt$

~~$\log|9.8 - 0.196v|$~~

$\frac{\cancel{0.196}\log|9.8 - 0.196v|}{-0.196} = t + C$

$\log|9.8 - 0.196v| = -0.196t + C$

10) $ty' + 2y = t^2 - t + 1$

11) $y' = \frac{xy^3}{\sqrt{1+x^2}}$

$\frac{dy}{dx} = \frac{xy^3}{\sqrt{1+x^2}}$

$\int \frac{dy}{y^3} = \int \frac{x}{\sqrt{1+x^2}} dx$. let $1+x^2 = t$

$dt = 2x$

$-\frac{1}{2y^2} = \frac{1}{2}\int \frac{1}{\sqrt{t}} dt$

~~$\frac{-1}{2y^2} = \frac{1}{2}\left(\frac{t^{-3/2-1}}{-1/2-1}\right)$~~

$$-\frac{1}{2y^2} = \sqrt{1+x^2} + C$$

$y(0) = -1$

$$\frac{-1}{2(-1)^2} = \sqrt{1} + C$$

$$-\frac{1}{2} = 1 + C$$

$$C = -\frac{1}{2} - 1 = -\frac{3}{2}$$

$$\therefore -\frac{1}{y^2} = \sqrt{1+x^2} - \frac{3}{2}$$

12) $f(x) = x^3 - 10x^2 + 6$ $\quad$ $f(3) = 27 - 90 + 6 = -57$

$f'(x) = 3x^2 - 20x$ $\quad$ $f'(3) = 27 - 60 = -33$

$f''(x) = 6x - 20$ $\quad$ $f''(3) = 18 - 20 = -2$

$f'''(x) = 6$ $\quad$ $f'''(3) = 6$

Taylor series $= (-57) + (x-3)(-33) - (x-3)^2(-2) + (x-3)^3(6) + \ldots$

13) $f(x) = (1+x)^{\mu}$ $\quad$ $f(0) = 1^{\mu} = 1$

$f'(x) = \mu(1+x)^{\mu-1}$ $\quad$ $f'(0) = \mu(1)^{\mu-1} = \mu$

$f''(x) = \mu(\mu-1)(1+x)^{\mu-2}$ $\quad$ $f''(0) = \mu(\mu-1)(1)^{\mu-2} = \mu(\mu-1)$

$f'''(x) = \mu(\mu-1)(\mu-2)(1+x)^{\mu-3}$ $\quad$ $f'''(0) = \mu(\mu-1)(\mu-2)$

$$f(x) = 1 + \mu x + \mu(\mu-1)\frac{x^2}{2} + \mu(\mu-1)(\mu-1)(\mu-2)\frac{x^3}{6} + \ldots$$

14) $f(x) = \sqrt{1+x}$ $\qquad f(0) = 1$

$f'(x) = \dfrac{1}{2\sqrt{1+x}}$ $\qquad f'(0) = \dfrac{1}{2}$

$f''(x) = \dfrac{-1}{4(1+x)^{3/2}}$ $\qquad f''(0) = -\dfrac{1}{4}$

$\therefore f(x) = 1 + \dfrac{x}{2} + \dfrac{x^2}{8} + \ldots\ldots$

15) $f(x) = e^{-6x}$ $\qquad f(-4) = e^{24}$

$f'(x) = -6e^{-6x}$ $\qquad f'(-4) = -6e^{24}$

$f''(x) = 36e^{-6x}$ $\qquad f''(-4) = 36e^{24}$

$f(x) = e^{24} - (x+4)6e^{24} + (x+4)^2 18e^{24} + \ldots\ldots$

16) $f(x) = \ln(3+4x)$ $\qquad f(0) = \ln(3)$

$f'(x) = \dfrac{4}{4x+3}$ $\qquad f'(0) = \dfrac{4}{3}$

$f''(x) = \dfrac{-16}{(4x+3)^2}$ $\qquad f''(0) = \dfrac{-16}{9}$

$f(x) = \ln(3) + \dfrac{4x}{3} - \dfrac{8x^2}{9} + \ldots\ldots$