1. $$\begin{bmatrix} 1 & 2 \\ a & 0 \end{bmatrix}\begin{bmatrix} 3 & b \\ -4 & 1 \end{bmatrix} = \begin{bmatrix} -5 & 6 \\ 12 & 16 \end{bmatrix}$$

To find: a, b and c.

$$\begin{bmatrix} 1 & 2 \\ a & 0 \end{bmatrix}\begin{bmatrix} 3 & b \\ -4 & 1 \end{bmatrix} = \begin{bmatrix} 3-8 & b+2 \\ 3a & ab \end{bmatrix} = \begin{bmatrix} -5 & 6 \\ 12 & 16 \end{bmatrix}$$

$\Rightarrow$ $b+2=6$

$3a = 12$, and

$ab = 16$

$\Rightarrow$ $a = \frac{16}{b} = \frac{12}{3}$

$\Rightarrow$ $a=4$ and $b=4$

2. $$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

To find A^{20}

Soln $A^{20} = (A^2)^{10}$

$$\Rightarrow A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

$$\Rightarrow A^{20} = (A^2)^{10} = I^{10} = I$$

3. The matrix is not invertible if the determinant is zero.

$$\begin{vmatrix} 1 & \lambda & 0 \\ 3 & 2 & 0 \\ 1 & 1 & 1 \end{vmatrix} = 1(2) - \lambda(3-1) = 0$$

$$= 2 - 2\lambda = 0$$

$$\Rightarrow \lambda = 1$$

4.

Let the sales matrix for parts be S

$$S = \begin{array}{c} \\ \text{Mumbai} \\ \text{Pune} \end{array} \begin{array}{c} \begin{array}{ccc} \text{W.B} & \text{C.F} & \text{FM} \end{array} \\ \begin{bmatrix} 20 & 10 & 8 \\ 15 & 12 & 4 \end{bmatrix} \end{array}$$

Let the price matrix for parts be P

$$P = \begin{array}{c} \\ \text{WB} \\ \text{C.F} \\ \text{F.M} \end{array} \begin{array}{c} \begin{array}{cc} \text{usual} & \text{A-Plus} \end{array} \\ \begin{bmatrix} 700 & 600 \\ 300 & 200 \\ 1200 & 1000 \end{bmatrix} \end{array}$$

Revenue = $S \times P$

$$= \begin{bmatrix} 20 & 10 & 8 \\ 15 & 12 & 4 \end{bmatrix} \begin{bmatrix} 700 & 600 \\ 300 & 200 \\ 1200 & 1000 \end{bmatrix}$$

$$= \begin{bmatrix} 14000 + 3000 + 9600 & 12000 + 2000 + 8000 \\ 10500 + 3600 + 4800 & 9000 + 2400 + 4000 \end{bmatrix}$$

$$\begin{array}{c} \\ \text{Mumbai} \\ \text{Pune.} \end{array} \begin{array}{c} \begin{array}{cc} \text{Undiscounted} & \text{Discounted} \end{array} \\ \begin{bmatrix} 26600 & 22000 \\ 18900 & 15400 \end{bmatrix} \end{array}$$

5. Let the first term be a common difference be d.

The nth term $= a + (n-1)d$

According to the given condition we have

$$a + 3d = 3a$$

$$a + 6d = 2(a + 2d) + 1$$

$$\Rightarrow \quad 2a = 3d$$

$$a = 2d - 1$$

$$4d - 2 = 3d$$

$$\Rightarrow \quad d = 2$$

$$\Rightarrow \quad \underline{\underline{a = 3}}$$

6. $7(a + 6d) = 11(a + 10d)$

$4a + 68d = 0$

$a + 17d = 0$; 18^{th} term.

Hence proved.

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7. The sum to n terms is given be $\frac{n}{2}\left(2a+(n-1)d\right)$

$\therefore\ 10(2a+19d) = 11(2a+21d)$

$20a+190d : 22a+231d$

$-2a = 41d$

$-2a = 41(-2)$ $(d=-2)$

$\underline{a=41}$

8. The sum to n terms is given be $\frac{n}{2}\left(2a+(n-1)d\right)$

$= na + \frac{n^2 d}{2} - \frac{nd}{2}$

$= n^2\left(\frac{d}{2}\right) + n\left(a-\frac{d}{2}\right)$

Comparing coefficients of n and n^2 we get,

$\frac{d}{2} = 2$ and $a - \frac{d}{2} = 5$

Solving, we get $d=4$ and $a=7$

9. $1111\ldots1$ (91 times) $= 1+10+100+1000+\ldots 10^{90}$

This is the sum of a G.P with $a=1$, $n=91$, $r=10$

$\text{Sum} = \frac{1(10^{91}-1)}{10-1}$

$= \frac{(10^7)^{13}-1}{10^7-1} \times \frac{10^7-1}{10-1}$

$(1+10^7+10^{14}+\ldots 10^{91}) \times (1+10+100+\ldots 10^7)$

Since both the terms are natural numbers, the given number is a multiple of two numbers

Therefore the given number is composite and not a prime.

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10. Let the number be a, ar and ar^2

$a+ar+ar^2=31$

$a(1+r+r^2)=31$

We also have,

$a^2+(ar)^2+(ar^2)^2=651$

$a^2(1+r^2+r^4)=651$

Squaring equation, we get:

$a^2(1+r^2+r^4+2r+2r^2+2r^3)=961$

or

$a^2(1+r^2+r^4)+a^2(2r+2r^2+2r^3)=961$

$a^2(1+r^2+r^4)+(2ar)(a)(1+r+r^2)=961$

$651+(2ar)(31)=961$

$2ar(31)=310$

$ar=5$

$a=\frac{5}{r}$ ...... (1)

Substituting the value of a in (1), we get

$\frac{5}{r}(1+r+r^2)=31$

$\frac{5}{r}+5+5r=31$

$\Rightarrow \underline{\underline{r=5}}$

$\Rightarrow a=25$ or $a=1$