1) $\lim_{h \to 0} \frac{2(-3+h)^2 - 18}{h}$

$\rightarrow$ $\lim_{h \to 0} \frac{2(-3+h)^2 - 18}{h} = \lim_{h \to 0} \frac{2(9-6h+h^2)-18}{h}$

$= \lim_{h \to 0} \frac{18 - 12h + h^2 - 18}{h}$

$= \lim_{h \to 0} \frac{h^2 - 12h}{h}$

$= \lim_{h \to 0} \frac{h(h-12)}{h}$

$= \lim_{h \to 0} h - 12$

$\lim_{h \to 0} \frac{2(-3+h)^2 - 18}{h} = -12$

2) $g(x) = \begin{cases} 2x, & x < 6 \\ x-1, & x \geq 6 \end{cases}$

(a) $x = 4$, (b) $x = 6$

$\rightarrow$ a) at $x = 4$

$x = 4 < 6$, so,

$g(x) = 2x$

$\lim_{x \to 4} g(x) = \lim_{x \to 4} 2x$

$= 2(4)$

$\lim_{x \to 4} g(x) = 8$

$g(4) = 2(4)$

$g(4) = 8$

$g(4) = \lim_{x \to 4} g(x)$, So, $g(x)$ is continuous at $x = 4$.

b) at $x = 6$

Here, Left hand limit and Right hand limit are different.

| | |
|---|---|
| $LHL = \lim_{x \to 6^-} g(x)$ | $RHL = \lim_{x \to 6^+} g(x)$ |
| $= \lim_{x \to 6^-} 2x$ | $= \lim_{x \to 6^+} x - 1$ |
| $= 2(6)$ | $= 6 - 1$ |
| $LHL = 12$ | $= 5$ |

The $LHL \neq RHL$, hence, the given function $g(x)$ is discontinuous at $x = 6$.

3) $\lim_{x \to 9} \frac{x-9}{3-\sqrt{x}} = \lim_{x \to 9} \left( \frac{x-9}{3-\sqrt{x}} \times \frac{3+\sqrt{x}}{3+\sqrt{x}} \right)$

$$= \lim_{x \to 9} \frac{(x-9)(3+\sqrt{x})}{9-x}$$

$$= \lim_{x \to 9} \frac{(-1)(9-x)(3+\sqrt{x})}{(9-x)}$$

$$= -1 \lim_{x \to 9} 3+\sqrt{x}$$

$$= -1\,(3+\sqrt{9})$$

$$= -1\,(3+3)$$

$$\lim_{x \to 9} \frac{x-9}{3-\sqrt{x}} = -6$$

4) $f(x) = \frac{4x+5}{9-3x}$

(a) $x=-1$, (b) $x=0$, (c) $x=3$

→ (a) $f(x) = \frac{4x+5}{9-3x}$, $x=-1$

$$\lim_{x\to -1} f(x) = \lim_{x\to -1} \frac{4x+5}{9-3x}$$

$$= \lim_{x\to -1} \frac{4(-1)+5}{9-3(-1)}$$

$$= \frac{-4+5}{9+3}$$

$$\lim_{x\to -1} \frac{4x+5}{9-3x} = \frac{1}{12}$$

$f(x)$ at $x=-1$

$$f(-1) = \frac{4(-1)+5}{9-3(-1)}$$

$$f(-1) = \frac{1}{12}$$

$\lim_{x\to -1} \frac{4x+5}{9-3x} = f(-1)$, so, $f(x)$ is continuous at $x=-1$.

b) $x=0$

$$\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{4x+5}{9-3x}$$

$$= \frac{4(0)+5}{9-3(0)}$$

$$\lim_{x\to 0} \frac{4x+5}{9-3x} = \frac{5}{9}$$

$f(x)$ at $x=0$

$$f(0) = \frac{4(0)+5}{9-3(0)}$$

$$f(0) = \frac{5}{9}$$

$$\lim_{x \to 0} \frac{4x+5}{9-3x} = f(0), \text{ so, } f(x) \text{ is continuous at } x = 0.$$

c) $x = 3$

$$\lim_{x \to 3} f(x) = \lim_{x \to 3} \frac{4x+5}{9-3x}$$

$$= \lim \frac{4(3)+5}{9-3(3)}$$

$$= \frac{12+5}{9-9}$$

$$= \text{Not defined.}$$

$\lim_{x \to 3} f(x)$ is discontinuous at $x = 3$, because the limit does not exist. It is an infinite discontinuity.

5) $$\lim_{x \to \infty} \frac{6e^{4x} - e^{-2x}}{8e^{4x} - e^{2x} + 3e^{-x}}$$

→ $$\lim_{x \to \infty} \frac{6e^{4x} - e^{-2x}}{8e^{4x} - e^{2x} + 3e^{-x}} = \lim_{x \to \infty} \frac{e^{2x-2}(6e^{4x}) - e^{-2}}{(8e^{4x} - e^{2x})e^{x} + 3}$$

$$= \lim_{x \to \infty} \frac{e^{-2}(6e^{6x} - 1)}{(8e^{5x} - e^{3x}) + 3}$$

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5) $\lim_{x\to\infty} \dfrac{6e^{4x}-e^{-2x}}{8e^{4x}-e^{2x}+3e^{-x}}$

$\rightarrow \lim_{x\to\infty} \dfrac{6e^{4x}-e^{-2x}}{8e^{4x}-e^{2x}+3e^{-x}} = \lim_{x\to\infty}\left(\dfrac{6e^{6x}-1}{e^{2x}} \times \dfrac{e^{x}}{8e^{5x}-e^{3x}+3}\right)$

$= \lim_{x\to\infty} \dfrac{6e^{6x}-1}{8e^{6x}-e^{4x}+3e^{x}}$

$= \dfrac{6e^{6(\infty)}-1}{8e^{6(\infty)}-e^{4(\infty)}+3e^{(\infty)}}$

$= \dfrac{-1}{0}$

$\lim_{x\to\infty} \dfrac{6e^{4x}-e^{-2x}}{8e^{4x}-e^{2x}+3e^{-x}}$ = Not defined $(\infty)$

The limit does not exist. It is an infinite limit.

6) $g(y) = \begin{cases} y^2+5 & , y < -2 \\ 1-3y & , y \geq -2 \end{cases}$

(a) $\lim_{y\to 6} g(y)$ (b) $\lim_{y\to -2} g(y)$

$\rightarrow$ a) $\lim_{y\to 6} g(y) = \lim_{y\to 6} 1-3y$ [Because $y=6 > -2$]

$= 1-3(6)$

$= 1-18$

$\lim_{y\to 6} g(y) = -17$

b) $\lim_{y\to -2} g(y) = \lim_{y\to -2} 1-3y$ [Because $y=-2=-2$]

$= 1-3(-2)$

$= 1+6$

$\lim_{y\to -2} g(y) = 7$

7) $f(t) = (4t^2 - t)(t^3 - 8t^2 + 12)$

$\rightarrow$ $f(t) = (4t^2 - t)(t^3 - 8t^2 + 12)$

Let $u = 4t^2 - t$

$u' = 8t - 1 \rightarrow ①$

Let $v = t^3 - 8t^2 + 12$

$v' = 3t^2 - 16t \rightarrow ②$

$f(t) = u.v.$

$f'(t) = u.v' + v.u'$ [By Product Rule]

$= (4t^2 - t)(3t^2 - 16t) + (t^3 - 8t^2 + 12)(8t - 1)$ [From ① & ②]

$= 12t^4 - 64t^3 - 3t^3 + 16t^2 + 8t^4 - 64t^3 + 96t - t^3 + 8t^2 - 12$

$f'(t) = 20t^4 - 132t^3 + 24t^2 - 12$

8) $R(w) = \dfrac{3w + w^4}{2w^2 + 1}$

$\rightarrow$ $R(w) = \dfrac{3w + w^4}{2w^2 + 1}$

Let $u = 3w + w^4$.

$u' = 3 + 4w^3 \rightarrow ①$

Let $v = 2w^2 + 1$

$v' = 4w \rightarrow ②$

$R(w) = \dfrac{u}{v}$

$R'(w) = \dfrac{-u.v' + v.u'}{v^2}$ [By Quotient Rule]

$= \dfrac{-(3w + w^4)(4w) + (2w^2 + 1)(3 + 4w^3)}{(2w^2 + 1)^2}$ [From ① & ②]

$$= \frac{-12w^2 \mp 4w^5 + 6w^2 + 8w^5 + 3 + 4w^3}{(2w^2+1)^2}$$

$$R'(w) = \frac{4w^5 + 4w^3 \mp 6w^2 + 3}{(2w^2+1)^2}$$

9) $f(x) = 2e^x - 8^x$

$\rightarrow$ $f(x) = 2e^x - 8^x$

Let $u = 2e^x$

$u' = 2e^x \rightarrow (1)$

Let $v = 8^x$

$v' = 8^x \log 8 \rightarrow (2)$

$f(x) = u - v$

$f'(x) = u' - v'$

$f'(x) = 2e^x - 8^x \log 8$. [From (1) & (2)]

10) $y = z^5 - e^z \ln(z)$

$\rightarrow$ $y = z^5 - e^z \ln(z)$

$$\frac{dy}{dx} = 5z^4 - \left(\frac{e^z}{z} + e^z \ln(z)\right)$$

$$\frac{dy}{dx} = 5z^4 - \frac{e^z}{z} + - e^z \ln(z)$$

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11) a) $f(x) = (6x^2+7x)^4$

$\rightarrow$ $f(x) = (6x^2+7x)^4$

Diff. wrt $x$

$f'(x) = 4(6x^2+7x)(12x+7)$

$= 4(72x^3+42x^2+84x^2+49x)$

$= 4(72x^3+126x^2+49x)$

$f'(x) = 288x^3+504x^2+196x$

b) $f(t) = 5+e^{4t+t^7}$

$\rightarrow$ $f(t) = 5+e^{4t+t^7}$

Diff. wrt $t$

$f'(t) = (e^{4t+t^7})(4+7t^6)$

$f'(t) = (4+7t^6)e^{4t+t^7}$

12) a) $z = \ln(7-x^3)$

$\rightarrow$ $z = \ln(7-x^3)$

Diff. wrt $x$

$$\frac{dz}{dx} = \frac{1}{7-x^3} \times (-3x^2)$$

Diff. wrt $x$

$$\frac{d^2z}{dx^2} = \frac{(-3x^2)(-3x^2) - (7-x^3)(-6x)}{(7-x^3)^2}$$

$$= \frac{9x^4+42x-6x^4}{(7-x^3)^2}$$

$$\frac{d^2z}{dx^2} = \frac{3x^4+4\cancel{2}x}{(7-x^3)^2}$$

b) $Q(v) = \dfrac{2}{(6+2v-v^2)^4}$

$\rightarrow$ $Q(v) = \dfrac{2}{(6+2v-v^2)^4}$

Diff. wrt v

$$Q'(v) = \frac{2\left(4(6+2v-v^2)^3(2-2v)\right) - \cdot}{\left((6+2v-v^2)^4\right)^2}$$

$$= \frac{8(2-2v)\cancel{(6+2v-v^2)^3}}{(6+2v-v^2)^{\cancel{8}\,5}}$$

$$Q'(v) = \frac{16-16v}{(6+2v-v^2)^5}$$

Diff. wrt x

$$Q''(x) = \frac{(16-16v)\left(5(6+2v-v^2)^4(2-2v)\right) - (6+2v-v^2)^5(-16)}{\left((6+2v-v^2)^5\right)^2}$$

$$= \frac{(16-16v)(10-10v)(6+2v-v^2)^4 - (6+2v-v^2)^5(-16)}{(6+2v-v^2)^{10}}$$

$$= \frac{\cancel{(6+2v-v^2)^4}\left(160-160v-160v+160v^2+16(6+2v-v^2)\right)}{(6+2v-v^2)^{\cancel{10}\,6}}$$

$$= \frac{160-320v+160v^2+96+32v-16v^2}{(6+2v-v^2)^6}$$

$$Q''(x) = \frac{144v^2-288v+256}{(6+2v-v^2)^6}$$

c) $f(t) = \ln(1+t^2)$

$\rightarrow$ $f(t) = \ln(1+t^2)$

Diff. wrt t

$$f'(t) = \frac{1}{1+t^2} \times (2t)$$

Diff. wrt t

$$f''(t) = \frac{(2t)(2t) - (1+t^2)(2)}{(1+t^2)^2}$$

$$f''(t) = \frac{4t^2 - 2 - 2t^2}{(1+t^2)^2}$$

$$f''(t) = \frac{2t^2 - 2}{(1+t^2)^2}$$

13) $f(x) = x^3 - 3x^2 - 9x + 12$

Diff. wrt $x$

$f'(x) = 3x^2 - 6x - 9$

Let $f'(x) = 0$

$0 = 3(x^2 - 2x - 3)$

$0 = x^2 - 2x - 3$

$0 = x^2 + x - 3x - 3$

$0 = x(x+1) - 3(x+1)$

$0 = (x+1)(x-3)$

$x = -1, 3$

$f'(x) = 3x^2 - 6x - 9$

Diff. wrt $x$

$f''(x) = 6x - 6$

Substitute values of $x$ in $f''(x)$

$f''(-1) = 6(-1) - 6$

$= -6 - 6$

$= -12 < 0$

So, $-1$ is the Maxima

$f''(3) = 6(3) - 6$

$= 18 - 6$

$= 12 > 0$

So, $3$ is the Minima

Substitute $x$ in $f(x)$

$f(-1) = (-1)^3 - 3(-1)^2 - 9(-1) + 12 = -1 - 3 + 9 + 12 = 17$

$f(3) = (3)^3 - 3(3)^2 - 9(3) + 12 = 27 - 27 - 27 + 12 = -15$

Hence, the Maximum is 17 and the Minimum is $-15$

14) $f(x) = 4x^3 - 18x^2 + 24x - 7$

Diff. wrt $x$

$f'(x) = 12x^2 - 36x + 24$

Let $f'(x) = 0$

$0 = 12(x^2 - 3x + 2)$

$0 = (x-1)(x-2)$

$x = 1, 2$

$f'(x) = 12x^2 - 36x + 24$

Diff. wrt $x$

$f''(x) = 24x - 36$

Substitute values of $x$ in $f''(x)$

$f''(1) = 24(1) - 36$

$= 24 - 36$

$= -12 < 0$

So, 1 is the Maxima ~~point~~

$f''(2) = 24(2) - 36$

$= 48 - 36$

$= 12 > 0$

So, 2 is the Minima

Substitute the values of $x$ in $f(x)$

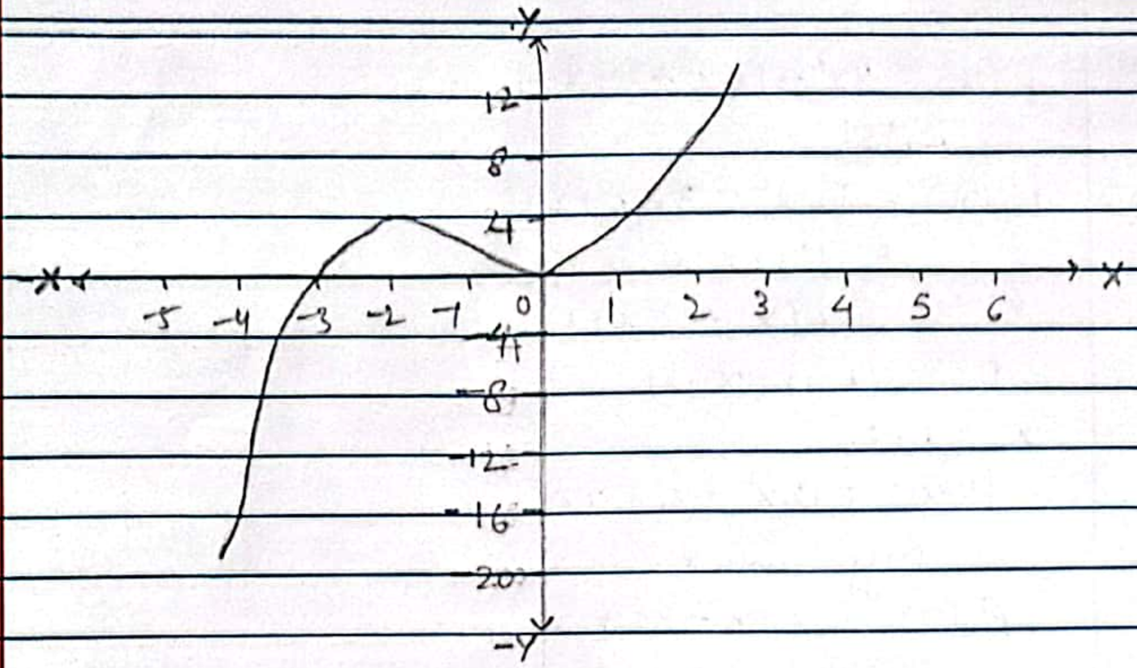
$f(1) = 4(1)^3 - 18(1)^2 + 24(1) - 7 = 4 - 18 + 24 - 7 = 3$

$f(2) = 4(2)^3 - 18(2)^2 + 24(2) - 7 = 32 - 72 + 48 - 7 = 1$

Hence, the Maximum is 3 and the Minimum is 1.

15) $f(x) = x^2(x+3)$

| | | | | | |
|---|---|---|---|---|---|
| $x$ | 0 | 1 | -2 | -3 | -4 |
| $y$ | 0 | 4 | 4 | 0 | -16 |




16) $f(x) = 3x^2 - 6x$

$f(x) = 3x(x-2)$

| | | | | | |
|---|---|---|---|---|---|
| x | 0 | 2 | 3 | -1 | 1 |
| y | 0 | 0 | 9 | 9 | -3 |