Q1] 1) Arbitrage means buying and selling of ~~2 different~~ an assets in 2 different markets to earn a risk free profit from price differences. Arbitrage oppurtunity arise due to market malfunction caused by overvaluation or undervaluation of an asset between 2 or more markets.

2) law of one price states that the price of an identical asset or commodity will have the same price globally, regardless of location.

3) a) 3-month European put option. $S_0 = Rs\ 125$ $X = Rs\ 120$
Call option Price → Rs 30 $r = 5\%$ p.a.

$$30 + 120e^{-0.05 \times 3/12} = 125 e^{-0.05 \times 3/12} + P$$

$$148.509336 = \cancel{125} + \cancel{P}$$

$$P = \cancel{23.5093369}\ 28.11$$

$$P = \cancel{Rs\ 23.5}\ \ Rs\ 28.11$$

b) If $P = 23$

Q2] $dX_t = \alpha\mu(T-t) + \sigma\sqrt{T-t}\, dZ_t$

Z_t is SBM

~~dZ_t~~

$$G = f(x,t) = e^{m(T-t)-x}$$

By itô's lemma

$$dG = d(f(x,t)) = \frac{df}{dt}dt + \frac{df}{dx}dX_t + \frac{1}{2}\frac{d^2f}{dx^2}(dX_t)^2$$

$$dG = G\cdot\frac{dm}{dt} - G\cdot(\alpha\mu(T-t))dt - G(\sigma\sqrt{(T-t)}dZ_t + \frac{1}{2}G\sigma^2(T-t)dt$$

$$dG = dt\left(G\cdot\frac{dm}{dt} - G(\alpha\mu(T-t)) + \frac{1}{2}G\sigma^2(T-t)\right) - G(\sigma\sqrt{(T-t)}dZ_t$$

As $f(x,t)$ is a martingale, $\therefore$ there is no drift paramater.

$dt = 0$

$$dt\left(G*\frac{dm}{dt} - G(\alpha\mu(T-t)) + \frac{1}{2}G\sigma^2(T-t)\right) = 0$$

$$dt \times G \times \left(\frac{dm}{dt} - (\alpha\mu(T-t)) + \frac{1}{2}\times\sigma^2(T-t)\right) = 0$$

$$\frac{dm}{dt} = \frac{1}{2}\sigma^2(T-t) - (\alpha\mu(T-t))$$

Q 3) $F(t, x) = e^{-t} x^2$

X is SDE

$$\frac{dX_t}{X_t} = 0.25\,dt + \sigma\,dW_t$$

(i) $\frac{df}{dx} = e^{-t} \cdot 2x$ $\qquad$ $\frac{df}{dt} = -e^{-t} x^2 = -f$

By itô's lemma

$$dY_t = \frac{df}{dt} \cdot dt + \frac{df}{dx}\,dX_t + \frac{1}{2} \times \frac{d^2 f}{dx^2}\,\sigma^2 X_t^2\,dt$$

$$= -f\,dt + 2e^{-t} X_t\,dX_t + \frac{1}{2} \times e^{-t} \sigma^2 X_t^2\, 2x\,dt.$$

$$= -Y_t\,dt + 2e^{-t} X_t^2 \frac{dX_t}{X_t} + e^{-t} \sigma^2 X_t^2\,dt$$

$$= -Y_t\,dt + 2Y_t[0.25\,dt + \sigma\,dW_t] + \sigma^2 Y_t\,dt$$

$$\therefore \frac{dY_t}{Y_t} = [2(-1) + \sigma^2]\,dt + 2\sigma\,dWt$$

$$\frac{dY_t}{Y_t} = (\sigma^2 - 0.5)\,dt + 2\sigma\,dW_t$$

$$\therefore dY_t = (\sigma^2 - 0.5) Y_t\,dt + 2\sigma Y_t W_t$$

(ii) $\sigma^2 - 0.5 = 0$

$\sigma^2 = 0.5$

4) If the option is exercised prior to ex-dividend date investor receives $S_t - K$

Not exercised → $S_t - d$

5) $P(S > 258) = ?$

$S_t = S_0 e^{(\mu - \sigma^2/2)t + \sigma W_t}$

$\ln(S_t) \sim N[(\ln(S_0) + (\mu - \sigma^2/2)t + \sigma W_t), \sigma^2 t)$

$\therefore \ln(S_t) = \phi(\ln 254 + (0.16 - 0.35^2/2) \times 0.5, 0.35 \times 0.5^{1/2})$

$= \phi(5.59, 0.247)$

$\therefore P(S > 258) = 1 - N(5.55 - 5.59)/0.247$

$= 1 - N(-0.1369)$

$= 0.5542$

The prob. put option is exercised is the stock price less than 258 in 6 months time $= 1 - 0.5542 = 0.4457$

Q (7) $E(C_t) = S_0\,[p.U + (1-p)d]$

$Var(C_t) = E(C_t^2) - (E(C_t))^2$

$= S_0^2[p.U^2 + (1-p)d^2] - S_0^2\,[pU + (1-p)d]^2$

$= S_0^2\,[p.U^2 + (1-p)d^2 - (pU + (1-p)d)^2]$

$= S_0^2\,p(1-p)(U-d)^2$

By equating

$S_0 e^{rt} = S_0\,p(1-p)(U-d)^2$ — ①

$\sigma^2 S_0^2 t = S_0^2 p(1-p)(U-d)^2$ — ②

from ①

$$p = \frac{e^{rt} - d}{u-d}$$

$$\sigma^2 t = \frac{e^{rt}-d}{U-d}\left(1 - \frac{e^{rt}-d}{u-d}\right)(U-d)^2$$

$= -(e^{-rt} - d)(e^{rt} - U) = (U-d)e^{rt} \times (-(1+e^{2rt}))]$

$\times$ by u

$U^2 e^{rt} - u(1 + e^{2rt} + \sigma^2 t) + e^{rt} = 0$

(ii) a) $\sigma = 0.15$, $t = 0.25$

$\therefore U = e^{0.15\sqrt{.25}} = e^{0.075} = 1.0778840$

$\therefore d = 0.927740$

9) (i) $F = Se^{rt}$ $\quad t = 0.25$

Using put-call parity

$13.334 + 70e^{-0.25r} = 0.12 + S$

$8.869 + 75e^{-0.25r} = 0.568 + S$

$S = 82$ & $r = 7\%$

$F = 83.45$

(ii) $e^{0.5 \times r_6} = e^{0.25 \times r_3} \times e^{0.25 \times r}$

$r_3 = 7\%$ $\quad r_6 = 6\%$ $\quad r = 5\%$

(ii) Put - Call parity

$6.8999 + a \times e^{-7\% \times 0.25} = 1.055 + 82$

$b + 80 \cdot e^{-7\% \times 0.25} = 1.789 + 82$

$2.594 + 85 \times e^{-7\% \times 0.25} = c + 82$

$a = 77.5 \quad b = 5.177 \quad c = 4.119$

9) Forward price $= Se^{(rt)}$

Put - Call parity : $C + Ke^{-rT} = P + S$

c and p = price of European call & put.

$\therefore 13.334 + 70e^{(-0.25r)} = 0.120 + S$

$8.869 + 75e^{-0.25r} = 0.568 + S$

By solving the above equat^n,

$S = 82, \quad r = 0.07$

$\therefore f = 82e^{0.07 \times 0.25}$

$\therefore F = 83.45$

(ii) $e^{(r_6 \times 0.05)} = e^{(r_3 \times 0.25)} \cdot e^{(r_f \times 0.25)}$

$r_3 = 0.07$

By substituting,

$2.569 + 90e^{(0.5 \times r_6)} = 7.909 + 82$

$r_6 = 0.05 \quad \& \quad r_f = 0.05$

(iii) By using put-call parity,

$6.899 + ae^{(-0.07 \times 0.25)} = 1.055 + 82$

$b + 80e^{(-0.07 \times 0.25)} = 1.789 + 82$

$2.594 + 85e^{(-0.07 \times 0.25)} = e + 82$

By solving the equations we get

$a = 77.5$

$b = 5.177$

$c = 4.119$

Q10] $\because$ Rate of int. = 0,

Risk neutral probability will be

$$q = (1-d)/(u-d)$$

$$d = \frac{1}{u}$$

$$q = \left(1-\frac{1}{u}\right) \Big/ \left(u-\frac{1}{u}\right)$$

$$= \frac{1}{u+1}$$

For no arbitrage

$$u > 1 > d$$

Then $u > 1 \Rightarrow u+1 > 2$

$$q = \frac{1}{u+1} < \frac{1}{2}$$

H/P

(ii) from i, $q = \dfrac{1}{u+1}$

$$q = \frac{1}{2}$$

$$\therefore u = 2$$

$$u = e^{(\sigma\sqrt{\delta t})}$$

$$\therefore \sigma = 240.11\%$$

A recombining binomial tree is one in which the sizes of up & down ~~&~~ steps is the same under all steps & across all intervals. Therefore, the risk neutral probability 'q' is also constant at all times & in all states.
It has only $(n+1)$ possible states of time as opposed to second possible states in a non recombining.
It assumes that the volatility & drift parameters of an underlying are constant over time.

$$\begin{tikzcd}[column sep=small, row sep=tiny]
 & & 1320 \\
 & 1100 \arrow[ur] \arrow[dr] & \\
 & & 990 \\
1000 \arrow[uur] \arrow[ddr] & & \\
 & & 1140 \\
 & 950 \arrow[ur] \arrow[dr] & \\
 & & 855
\end{tikzcd}$$

$$q_1 = \frac{e^{(0.0175)} - 0.95}{1.10 - 0.95}$$
$$= 0.4510$$

$$q_2 = \frac{e^{0.025} - 0.90}{1.20 - 0.90}$$
$$= 0.41772$$

Put payoffs at 4 states are 0,0,0,95.
The value of option following an up step over the first 3 months

$$V_1 \cdot e^{0.025} = [0.9172 \times 0] + [0.058228 \times 0]$$
$$\therefore V_1(1) = 0$$

The value of option following a downstep over the first three months is

$$V_2(2) e^{0.025} = [0.41772 \times 0] + [0.58228 \times 95]$$
$$= 53.9502$$

The current value is

$$V_0 e^{(0.0175)} = 29.105 = [0.4510 \times 0] + [0.5490 \times 53.9508]$$

Date

Q12] (i) $Z(t)$ is ~~Stan~~ SBM

(a) $dM(t) = 2dZ_t - 0$

$= 0\,dt + 2dZ_t$

This $M(t)$ has zero drift

(b) $dV(t) = d[Z(t)]^2 - dt$

$d[Z(t)]^2 = 2Z(t) \cdot dZ_t + dt$

Thus, $dV(t) = 2Z_t \cdot dZ_t$. Thus stochastic process $V(t)$ has 0 drift

(c) $dW(t) = d[t^2 Z(t)] - 2tZ(t)\,dt$

$\because d[t^2 Z(t)] = t^2 dZ(t) + 2tZ(t)dt$

$dW(t) = t^2 dZ(t)$

$\therefore$ The process $W(t)$ has 0 drift.

Q13] Let $\frac{S_t}{S_0} \sim LN\left[\left(\mu - \frac{1}{2}\sigma^2\right) \cdot t, \sigma^2 t\right]$

expected value $= q S_0 u + (1-q) S_0 \cdot d$

First time step on the tree:

$S_0 e^{\mu dt} = q S_0 u + (1-q) S_0 d$

$q = (e^{\mu dt} - d) / (u - d)$

S.D. of stock price $= \sigma\sqrt{\delta t}$

Variance,

$q u^2 + (1-q)d^2 - [qu + (1-q)d]^2 = \sigma^2 dt$

$\therefore e^{\mu dt}(u+d) - ud - e^{2\mu dt} = \sigma^2 dt$

530

500

475

$S = 200$

$r = 10\%$

$\sigma = 35\%$

$u = 1.18063$

$d = 0.9039$

$q = 0.5161$

$p = (1-q) = 0.4839$

$T = 2$ months $= 2/12$

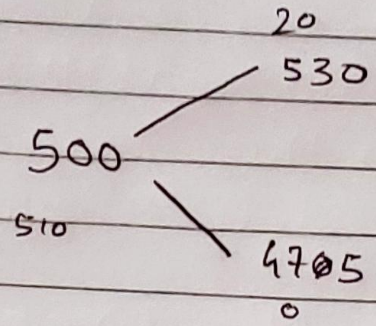
$t = 0.0833$ (1 month)

$K = 200$

$t = 0$

200

Q15]



$u = 1.06 \quad d = 0.95 \quad r = 0.05 \quad t = 0.25$

$p = \dfrac{[e^{r\cdot T} - d]}{u - d} = 0.5689$

$1 - p = 0.4311$

Value of 6-month european call $= e^{-0.5 \times 0.05} \times 51.8 \times 0.5689^2$
$= 16.351$

(ii) Value of european put $= e^{-0.05 \times 5} \times 0.4311^2 \times 58.75 + 2 \times 0.4311 \times 0.5689 \times 65$
$= 13.759$

By using put-call parity

Value of put + stock $= 500 + 13.759 = 513.759$

Value of call + discounted price $= 16.351 + 510\, e^{-0.05 \times 0.5} = 513.759$

(iii) Value at node B $= 6.5 \times e^{-0.25 \times 5\%} \times 0.4311 = 2.767$

$e^{-0.25 \times 5\%} \times 35 \times 0.4311 + 2.767 \times 0.5689 = 16.49$

(iv) Expected payoff in 3 month time is calculated by using real rate of return of 9%. $P = 0.6614$

$\therefore$ Expected payoff $= 20 \times 0.6614 = 13.228$.