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SECTION: B
ROLL NO.: 76

# Calculus - Assignment 2

1. $\int_{1/50}^{2} \frac{e^{2/w}}{w^2} \, dw$

Let $\frac{2}{w} = t$ When $w = \frac{1}{50}$, $t = 100$

$\therefore \frac{-2}{w^2} = \frac{dt}{dw}$ When $w = 2$, $t = 1$

$\frac{1}{w^2} \, dw = \frac{-1}{2} \, dt$

$\therefore \int_{1/50}^{2} \frac{e^{2/w}}{w^2} \, dw = \int_{100}^{1} e^{t} \cdot \frac{-1}{2} \, dt$

$= \frac{-1}{2} \left[ e^{t} \right]_{100}^{1}$

$= -\frac{1}{2} \left( e^{1} - e^{100} \right)$

$= \boxed{\frac{1}{2} \left( e^{100} - e \right)}$

2. $\int \frac{2t^3+1}{(t^4+2t)^3}\,dt$

Let $t^4+2t = x$

$4t^3+2 = \frac{dx}{dt}$

$(2t^3+1)dt = \frac{dx}{2}$

$\int \frac{2t^3+1}{(t^4+2t)^3}\,dt = \int \frac{1}{x^3}\cdot\frac{1}{2}\,dx$

$= \frac{1}{2}\int x^{-3}\,dx$

$= \frac{1}{2}\frac{x^{-2}}{-2} + C = -\frac{1}{4x^2} + C$

$= \boxed{-\frac{1}{4(t^4+2t)^2} + C}$

3. $f'(x) = x^4+3x-9$

$f(x) = \int (x^4+3x-9)\,dx$

$= \int x^4\,dx + 3\int x\,dx - 9\int 1\,dx$

$= \boxed{\frac{x^5}{5} + 3\frac{x^2}{2} - 9x + C}$

4. $f(x) = \begin{cases} 6 & \text{if } x > 1 \\ 3x^2 & \text{if } x \leq 1 \end{cases}$

$$\int_{-2}^{3} f(x)\,dx = \int_{-2}^{1} f(x)\,dx + \int_{1}^{3} f(x)\,dx$$

$$= \int_{-2}^{1} 3x^2\,dx + \int_{1}^{3} 6\,dx$$

$$= \frac{3}{3}\left[x^3\right]_{-2}^{1} + 6\left[x\right]_{1}^{3}$$

$$= 9 + 12 = \boxed{21}$$

5. $\int 3(8y-1)e^{4y^2-y}\,dy$

Let $4y^2 - y = t$

$\therefore 8y - 1 = \frac{dt}{dy}$

$(8y-1)\,dy = dt$

$$\therefore \int 3(8y-1)e^{4y^2-y}\,dy = 3\int e^t\,dt$$

$$= 3.e^t + c$$

$$= \boxed{3.e^{4y^2-y} + C}$$

6. $f''(x) = 15\sqrt{x} + 5x^3 + 6$

$f'(x) = \int 15x^{1/2} + 5x^3 + 6 \; dx$

$= \cancel{15}^{5} \times \frac{2}{\cancel{3}} x^{3/2} + \frac{5}{4} x^4 + 6x + c$

$= 10x^{3/2} + \frac{5}{4}x^4 + 6x + C_1$

$f(x) = \int 10x^{3/2} + \frac{5}{4}x^4 + 6x + C_1 \; dx$

$= \cancel{10}^{2} \times \frac{2}{\cancel{5}} x^{5/2} + \frac{\cancel{5}}{4} \times \frac{1}{\cancel{5}} x^5 + \frac{\cancel{6}^{3}}{\cancel{2}} x^2 + C_1 x + C_2$

$= 4x^{5/2} + \frac{1}{4}x^5 + 3x^2 + C_1 x + C_2$

$f(1) = 4 + \frac{1}{4} + 3 + C_1 + C_2$

$-\frac{5}{4} = \frac{29}{4} + C_1 + C_2$

$C_1 + C_2 = -\frac{17}{2}$ ... (i)

$f(4) = 4(4)^{5/2} + \frac{1}{4}(4)^5 + 3(4)^2 + C_1(4) + C_2$

$404 = 32 + 256 + 48 + 4C_1 + C_2$

$4C_1 + C_2 = 68$ ... (ii)

Subtracting eqn. (i) from (ii)

$$\begin{array}{rcrcl} 4C_1 & + & \cancel{C_2} & = & 68 \\ - \quad C_1 & + & \cancel{C_2} & = & -8.5 \\ (-) & & (-) & & (+) \\ \hline & & 3C_1 & = & 76.5 \end{array}$$

$$\boxed{\therefore C_1 = 25.5}$$

Substituting $C_1$ in eqn (i)

$$C_1 + C_2 = -8.5$$
$$C_2 = -8.5 - 25.5$$
$$\boxed{\therefore C_2 = -34}$$

$$\boxed{\therefore f(x) = 4x^{5/2} + \frac{1}{4}x^5 + 3x^2 + 25.5x - 34}$$

8. $\dfrac{dr}{d\theta} = \dfrac{r^2}{\theta}$ , $r(1) = 2$

$$\int r^{-2}\,dr = \int \theta^{-1}\,d\theta$$

$$\frac{r^{-1}}{-1} = \log\theta + C \Rightarrow -\frac{1}{r} = \log\theta + C$$

Given $r = 2$, $\theta = 1$

$$-\frac{1}{2} = \log(1) + C$$

$$C = -\frac{1}{2} - \log(1)$$

$$\therefore C = -\frac{1}{2}$$

$$\boxed{\therefore -\frac{1}{r} = \log\theta - 0.5}$$

9. $\frac{dv}{dt} = 9.8 - 0.196v$

$\frac{dv}{dt} + 0.196v = 9.8$

$I.F. = e^{\int 0.196v\,dv} = e^{0.196\frac{v^2}{2}} = e^{0.098v^2}$

$\cancel{\therefore v \cdot e^{\int 0.196v\,dv} = \int 9.8 \times e^{\int}}$

$\therefore v \cdot e^{0.098v^2} = \int 9.8 \times e^{0.098v^2}\,dv$

$v \cdot \cancel{e^{0.098v^2}} = 9.8 \cdot \frac{\cancel{e^{0.098v^2}}}{0.098v}$

$\therefore v^2 = 100$

$\boxed{\therefore v = 10}$

10. $ty' + 2y = t^2 - t + 1 \qquad y(1) = \frac{1}{2}$

$t \cdot \frac{dy}{dt} + 2y = t^2 - t + 1$

$t \cdot \frac{dy}{dt} = t^2 - t + 1 - 2y$

$\cancel{t \cdot \frac{dy}{dt} = \frac{t^2}{t} - \frac{t}{t} + \frac{1}{t} - \frac{2y}{t}}$

Let $x = t^2 - t + 1 - 2y$

Diff w.r.t. t.

$$\frac{dx}{dt} = 2t - 1 - 2\frac{dy}{dt}$$

$$2\frac{dy}{dt} = 2t - 1 - \frac{dx}{dt}$$

$$\frac{dy}{dt} = t - \frac{1}{2} - \frac{1}{2}\frac{dx}{dt}$$

given $y = 1/2$ at $t = 1$

$$\therefore t\frac{dy}{dt} = t^2 - t + 1 - 2y$$

$$\therefore t\left(t - \frac{1}{2} - \frac{1}{2}\frac{dx}{dt}\right) = x$$

$$\therefore t^2 - \frac{t}{2} - \frac{t}{2}\frac{dx}{dt} = x$$

$$\therefore t^2 dt - \frac{t}{2}dt - \frac{t}{2}dx = x\,dt$$

$$\therefore \frac{t^3}{3} - \frac{t^2}{4} - \frac{tx}{2} = xt + C$$

$$\frac{t^3}{3} - \frac{t^2}{4} = \frac{3tx}{2} + C$$

$$\frac{t^3}{3} - \frac{t^2}{4} = \frac{3t}{2}(t^2 - t + 1 - 2y) + C$$

$$\frac{t^3}{3} - \frac{t^2}{4} = \frac{3t^3}{2} - \frac{3t^2}{2} + \frac{3t}{2} - 3ty + C$$

$$\therefore c = \frac{t^3}{3} - \frac{3t^3}{2} - \frac{t^2}{4} + \frac{3t^2}{2} - \frac{3t}{2} + 3ty$$

$$\therefore c = \left(\frac{2t^3 - 9t^3}{6}\right) - \left(\frac{t^2 - 6t^2}{4}\right) - \frac{3t}{2} + 3ty$$

$$\therefore c = -\frac{7t^3}{6} + \frac{5t^2}{4} - \frac{3t}{2} + 3ty$$

$$\therefore c = -\frac{7}{6} + \frac{5}{4} - \cancel{\frac{3}{2}} + \cancel{\frac{3}{2}}$$

$$\therefore c = \frac{2}{24} = \boxed{\frac{1}{12}}$$

12. $f(x) = x^3 - 10x^2 + 6$ $\qquad x = 3$

$f(3) = 27 - 90 + 6 = -57$

$\therefore f(x) = x^3 - 10x^2 + 6 \qquad f(3) = -57$

$f'(x) = 3x^2 - 20x \qquad f'(3) = -33$

$f''(x) = 6x - 20 \qquad f''(3) = -2$

$f'''(x) = 6 \qquad f'''(3) = 6$

$f^{IV}(x) = 0 \qquad f^{IV}(3) = 0$

TAYLOR SERIES

$$f(x) = f(a) + \frac{(x-a)^1}{1!} f'(a) + \frac{(x-a)^2}{2!} f''(a) + \frac{(x-a)^3}{3!} f'''(a) + \dots$$

$$f(x) = -57 + \frac{(x-3)(-33)}{1} + \frac{(x-3)^2(-\cancel{2})}{\cancel{2}\times 1} + \frac{(x-3)^3(\cancel{6})}{\cancel{3\times 2\times 1}} + \frac{(x-4)^4(0)}{4!}$$

$$\boxed{f(x) = -57 - 33(x-3) - (x-3)^2 + (x-3)^3}$$

13. $f(x) = (1+x)^{\mu}$ $\qquad f(0) = (1+0)^{\mu} = 1$

$f'(x) = \mu(1+x)^{\mu-1}$ $\qquad f'(0) = \mu(1+0)^{\mu-1} = \mu$

$f''(x) = \mu(\mu-1)(1+x)^{\mu-2}$ $\qquad f''(0) = \mu(\mu-1)(1+0)^{\mu-1} = \mu(\mu-1)$

$f'''(x) = \mu(\mu-1)(\mu-2)(1+x)^{\mu-3}$ $\qquad f'''(0) = \mu(\mu-1)(\mu-2)$

MACLAURIAN SERIES

$$f(x) = f(0) + \frac{x^1}{1!}f'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + \cdots$$

$$\cancel{(1+x)^{\mu} = 1 + \mu x + \frac{\mu^2-\mu}{2} + \mu(\mu-1)(\mu-2)}$$

$$\boxed{(1+x)^{\mu} = 1 + \mu x + \cancel{\mu(\mu} \frac{\mu(\mu-1)}{2} + \frac{\mu(\mu-1)(\mu-2)}{6} + \cdots}$$

14. $f(x) = \sqrt{1+x} = (1+x)^{1/2}$ $\qquad f(0) = \sqrt{1+0} = 1$

$f'(x) = \frac{1}{2}(1+x)^{-1/2}$ $\qquad f'(0) = \frac{1}{2}(1+0)^{-1/2} = \frac{1}{2}$

$f''(x) = -\frac{1}{4}(1+x)^{-3/2}$ $\qquad f''(0) = -\frac{1}{4}(1+0)^{-3/2} = -\frac{1}{4}$

$f'''(x) = \frac{3}{8}(1+x)^{-5/2}$ $\qquad f'''(0) = \frac{3}{8}(1+0)^{-5/2} = \frac{3}{8}$

MACLAURIAN SERIES

$$f(x) = f(0) + \frac{x^1}{1!}f'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + \cdots$$

$$\sqrt{1+x} = 1 + \frac{1}{2}x + \left(-\frac{1}{4}\right)\times\frac{1}{2}x^2 + \frac{3}{8}\times\frac{1}{3\times2}x^3 + \cdots$$

$$\boxed{\sqrt{1+x} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 + \cdots}$$

15. $f(x) = e^{-6x}$ $\qquad x = -4$

$$f(x) = e^{-6x} \qquad f(4) = e^{-6(-4)} = e^{+24}$$
$$f'(x) = e^{-6x}(-6) \qquad f'(4) = e^{-6(-4)}(-6) = e^{+24}(-6)$$
$$f''(x) = e^{-6x}(36) \qquad f''(4) = e^{-6(-4)}(36) = e^{+24}(36)$$
$$f'''(x) = e^{-6x}(-216) \qquad f'''(4) = e^{-6(-4)}(-216) = e^{+24}(-216)$$

TAYLOR SERIES

$$f(x) = f(-4) + \frac{(x+4)^1}{1!} f'(-4) + \frac{(x+4)^2}{2!} f''(-4) + \frac{(x+4)^3}{3!} f'''(-4) + \dots$$

$$f(x) = e^{24} + \frac{(x+4)}{1}(-6e^{24}) + \frac{(x+4)^2}{2}(36e^{24}) + \frac{(x+4)^3}{6}(-216e^{24})$$

$$\boxed{f(x) = e^{24} - 6e^{24}(x+4) + 18e^{24}(x+4)^2 - 36e^{24}(x+4)^3}$$

16. $f(x) = \ln(3+4x)$ $\qquad x = 0$

$$f(x) = \ln(3+4x) \qquad f(0) = \ln(3)$$
$$f'(x) = \frac{1}{3+4x}(4) \qquad f'(0) = \frac{4}{3}$$
$$f''(x) = \frac{-4}{(3+4x)^2}(4) \qquad f''(0) = \frac{-16}{3^2} = \frac{-16}{9}$$
$$f'''(x) = \frac{32}{(3+4x)^3}(4) \qquad f'''(0) = \frac{128}{3^3} = \frac{128}{27}$$

TAYLOR SERIES

$$f(x) = f(0) + \frac{(x-0)^1}{1!} f'(0) + \frac{(x-0)^2}{2!} f''(0) + \frac{(x-0)^3}{3!} f'''(0) + \dots$$

$$f(x) = \ln(3) + \frac{4}{3}x + \left(\frac{-\cancel{16}^{\,8}}{9}\right)\frac{x^2}{\cancel{2}} + \left(\frac{\cancel{128}^{\,64}}{27}\right)\frac{x^3}{3\times\cancel{2}}$$

$$\boxed{f(x) = \ln(3) + \frac{4x}{3} - \frac{8x^2}{9} + \frac{64x^3}{81}}$$