FINANCIAL MATHEMATICS

Q1 ⇒

$$d^{(4)} = 8\%$$

$$\frac{d^{(4)}}{4} = 2\%$$

$$\left(1 - \frac{d^{(4)}}{4}\right)^4 = (1-d)$$

$$(1-0.02)^4 = 1-d$$

$$d = 1 - (1-0.02)^4$$

$$d = 0.07763184$$

$$i = \frac{d}{1-d} = 0.0841165785$$

ii) $d = \ln(1+i)$

$d = \ln(1.0841165785)$

$d = 0.0808108829$ $\quad d = 8.08108829\%$

iii) $i = 0.0841165785$

iv) $d^{(12)} = 12\left(1 - (1-0.07763184)^{1/12}\right)$

$= 0.0805393339$

Ques 2 →

$$\delta(t) = 0.004t + 0.0002t^2$$

$$PV = 1000 \times \frac{1}{e^{\int_0^{10} (0.04t + 0.0002t^2)\,dt}}$$

$$\int_0^{10} (0.04t + 0.0002t^2)\,dt$$

$$e^{\left[\frac{0.04t^2}{2} + \frac{0.0002t^3}{3}\right]_0^{10}}$$

$$= e^{[0.2 + \frac{1}{15}]}$$

$$= e^{[2.06666]}$$

$$PV = \frac{1000}{e^{[2.06666]}} = 765.928$$

$$AV(0,10) = 1000$$

$$AV0 = 765.928$$

$$A(0,10) = \frac{1000}{765.928}$$

$$(1+i)^{10} = 1.3056057$$

$$i = 2.7025449\%$$

Ques 3

$$d = iv$$

$$d = \frac{i}{1+i} = i \times v$$

We can write d as $\frac{i}{1+i}$ and $\frac{1}{1+i}$ is equal to discounting factor so when we can write it as i and discounting factor which is v. Hence iv.

(ii) $d^{(2)}$

a) $$\left(1 - \frac{d^{(2)}}{2}\right)^2 = \left(1 - \frac{d^{(4)}}{4}\right)^4$$

$$\left(1 - \frac{d^{(2)}}{2}\right)^2 = 1000\,(1 - 0.0225)^{\frac{2}{4}}$$

$$2\left(1 - (1 - 0.0225)^2\right) = d^{(2)}$$

$$0.0889875 = d^{(2)}$$

b) $$i^{1/2} = 5\%$$

$$\frac{i^{1/2}}{1/2} = 10\%$$

$$\left(1 - \frac{d^{(2)}}{2}\right)^2 = \left(1 - \frac{i^{1/2}}{1/2}\right)^{1/2}$$

$$\left(1-\frac{d^{(2)}}{2}\right)^2 = (1-0.1)^{1/2}$$

$$\left(1-\frac{d^{(2)}}{2}\right) = (0.9)^{1/4}$$

$$2\left(1-(0.9)^{1/4}\right) = d^{(2)}$$

$$0.051992 = d^{(2)}$$

(iii)

91 day Croatt bill

annual

$i = 5\%$

suppose bill vale is 100

$$PV = 100 \times \frac{1}{(1+0.05)^{91/365}}$$

$$PV = 98.79095608$$

$$\left(\frac{100}{98.79095608}\right)^{\frac{365}{91}} = \frac{1}{(1-d)}$$

$$1-d = 0.952380953$$

$$d = 0.047619047$$

Ques 4 (ii) $i = 0.08$

$$d = \frac{i}{1+i} = 0.07407407$$

(i) $d^{12} = 12\left(1-(1-0.07407407)^{1/12}\right)$

$= 0.076714772$

(ii) $\left(1+\frac{i^{(365)}}{365}\right)^{365} = i+1$

$$\frac{i^{(365)}}{365} = (1+i)^{1/365} - 1$$

$$i^{(365)} = 365\left((1+i)^{1/365} - 1\right)$$

$i^{(365)} = 0.076969155$

c) $\delta = \ln(1+i) = 0.076961041$

d) $i^{1/2}$

$$(i)^{1/2} = 1/2\left((1+i)^{2} - 1\right)$$

$i^{1/2} = 0.0832$

Ques 5 ⇒ (i) $d^4 = 0.06$

$$\left(1 - \frac{d^4}{4}\right)^4 = 1 - d$$

$$d = 1 - \left(1 - \frac{0.06}{4}\right)^4$$

$$d = 0.058663449$$

$$i = 0.062319315$$

a) $$\left(1 + \frac{i^{(2)}}{2}\right)^2 = 1 + i$$

$$i^{(2)} = 0.061\ 377\ 516$$

b) $$d^{(12)} = 12\left(1 - (1-d)^{1/12}\right)$$

$$= 0.060302525$$

(iii) Amount 200000

15th April 2005 ⟶ 17th June

May June

2

63 days

$$1 + i = 1.1$$

$$i = .1$$

Amt paid $= 200000 \times (1-d)^{63/365}$

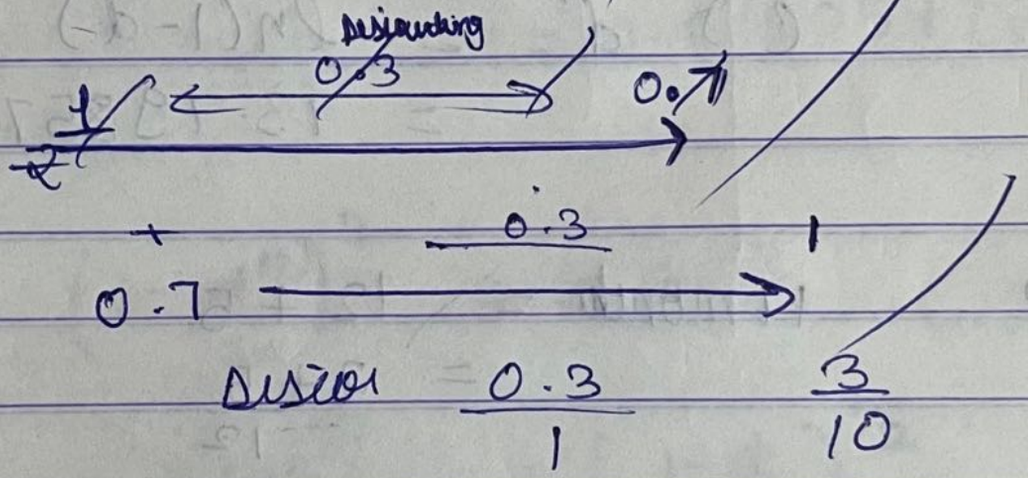
$= ₹203317.3715.$

Ques 6 ⇒

a) Cash pay = 30% below retail price

$= d = 0.3.$

b) Six mth crea = $d = 0.25$



Discount $\frac{0.3}{1}$ $\frac{3}{10}$ 30%.

Let retail price be $= x$

Retailer pays cash = 70% x

Then he has to pay

$= 0.7x$

If he use credit $= 75\%$ of x

Then he has to pay $AV = 0.75x$ in 6 months

We need to find discount on 6 mth.

$0.7x = 0.75x\,(1-d)^{1/2}$

$d = 12.88889\%$

(ii) a) $d^{(12)} = 12(1-(1-d^{(2)})^{1/12})$

$= 13.719544\%$

b) $i = \frac{d}{1-d} = 14.79591837\%$

$i^{(2)} = 2[(1+i)^{1/2} - 1]$

$= 14.285714\%$

c) $\delta = -\ln(1-d)$

$= 13.798574\%$

Ques 7 ⇒ 17 months $= 12 + 5$

$= 2 + \frac{5}{12}$

$= \frac{24+5}{12} = \frac{29}{12}$

$26000 = 20000\,(1+i)^{29/12}$

$\frac{26}{20} = (1+i)^{29/12}$

$\log \frac{26}{20} = \frac{29}{12} \log(1+i)$

$\frac{12}{29} \log \frac{26}{20} = \log(1+i)$

$0.04714897 = \log(1+i)$

$i = 0.04827816\,3$

i) $\left(1+\frac{i^{(4)}}{4}\right)^4 = 1+i$

$1+\frac{i^{(4)}}{4} = (1.048278163)^{1/4}$

$i^{(4)} = 4\left((1.048278163)^{1/4} - 1\right)$

$i^{(4)} = 0.047427947$

ii) $d = \frac{i}{1+i} = 0.046054725$

$\left(1-\frac{d^{(2)}}{2}\right)^2 = 1-d$

$1-\frac{d^{(2)}}{2} = (1-d)^{1/2}$

$2\left(1-(1-d)^{1/2}\right) = d^{(2)}$

$0.046597559 = d^{(2)}$

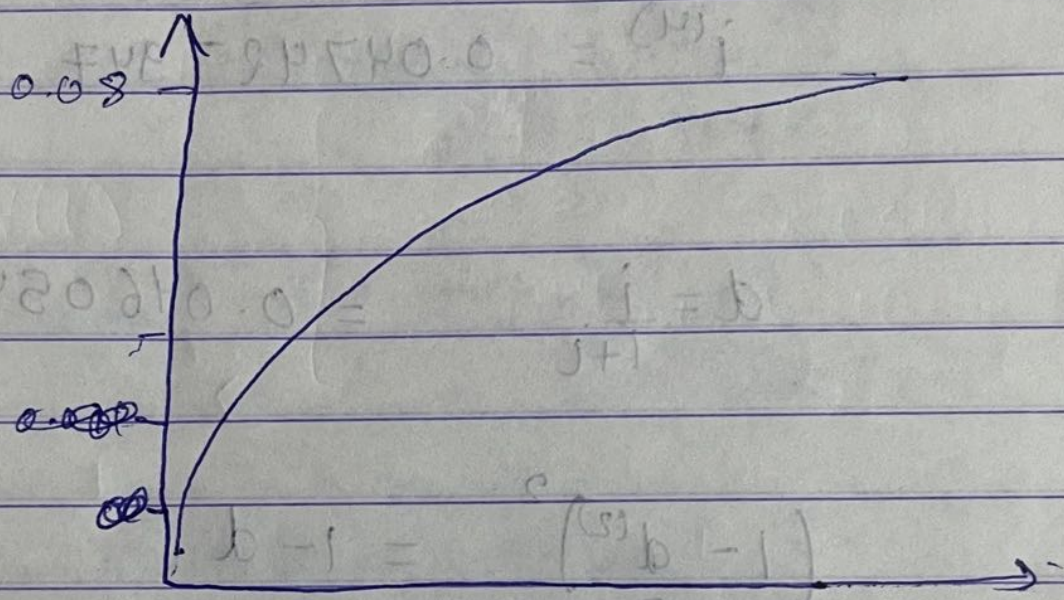
iii $26000 = 20000\left(1+\frac{29}{12}i\right)$

$\frac{26}{20} - 1 = \frac{29}{12}i$

$\frac{6}{20} \times \frac{12}{29} = i$ $\quad 0.124137931 = i$

Ques 8 $i = 0.08$

$$i > i^{(2)} > i^{(3)} > i^{(4)} \text{ ---- } i^{(n)} > \delta > \text{---}$$



Ques 9 :-

(i) Define force of int.

- Force of interest is nominal rate of interest per unit time convertible @ continuously
- → Also referred to as continously comp. force of int.

(ii) $i^{(2)} = 0.0775$

$$\left(1 + \frac{i^{(2)}}{2}\right)^2 - 1 = i$$

$$0.07900563 = i$$

$$d = \frac{i}{1+i}$$

ii $\quad d = \ln(1+i)$

$d = 0.0760361 35$

(iii) $\quad d^{(12)} = 12\left(1-(1-d)^{1/12}\right)$

$d^{(12)} = 12\left(1-(1-0.07321728 3)^{1/12}\right)$

$d^{(12)} = 0.0757957 47$

iii $\quad i^{(1/2)} = 1/2\left[(1+i)^2-1\right]$

$= 1/2\left[(1.079001563)^2-1\right]$

$= 0.0821222186$

Ques 10 ⇒

$$d(t) = \begin{cases} 0.08 & \text{for } 0 \le t \le 5 \\ 0.06 & \text{for } 5 \le t \le 10 \\ 0.04 & \text{for } t \ge 10 \end{cases}$$

For 0 to 5 $\quad = e^{[0.08t]_0^5} = e^{0.4}$

For 5 to 10 $\quad = e^{(0.06t]_5^{10}}$

$= e^{[0.6-0.3]} = e^{0.3}$

For 10 to t $\quad = e^{[0.04t]_{10}^{t}}$

$= e^{0.04t-0.4}$

PV at time 0 = Amt × Discounting Factor

$$= 1 \times \frac{1}{e^{0.4} \times e^{0.3} \times e^{0.04t} \times e^{-0.4}}$$

$$= \frac{1}{e^{0.3+0.04t}}$$

or

$$v(t) = \begin{cases} e^{-0.4} & \text{for } 0 \le t \le 5 \\ e^{-0.3} & \text{for } 5 \le t \le 10 \\ e^{0.04t-0.4} & \text{for } t \ge 10 \end{cases}$$

Ques 11 ⇒

Discount rate = 0.18

PV = Amt due × discounting factor

$$= 50000 \times (1-0.18)^{9/12}$$

$$= 43085.42623$$

b ⇒ $e^{d} = 1+i$

$e^{0.06} - 1 = i$

$0.061836547 = i$

$d = \frac{i}{1+i} = 0.058235$

$d^{(12)} = 12(1-(1-d)^{1/12})$

$= 0.059850250$

$i^{(12)}, i^{(4)}$

ii $d^{(4)} = 6\%$

$\frac{d^{(4)}}{4} = \frac{6}{4}$ 1.5%

$$\left(1 - \frac{d^{(4)}}{4}\right)^4 = (1-d)$$

$$(1 - 0.015)^4 = 1 - d$$

$$d = 0.058663449$$

$$i = \frac{d}{1-d} = 0.062319315$$

$$\left(1 + \frac{i^{(12)}}{12}\right)^{12} = 1 + i$$

$$i^{(12)} = 12\left((1+i)^{1/12} - 1\right)$$

$$= 0.060607089$$

$$i^{(4)} = 4\left((1+i)^{1/4} - 1\right)$$

$$= 0.060913705.$$

c) $d = 0.058663449 \rightarrow \delta = 0.600$

$i^{(12)} = 0.060607089$

$i^{(4)} = 0.060913705$

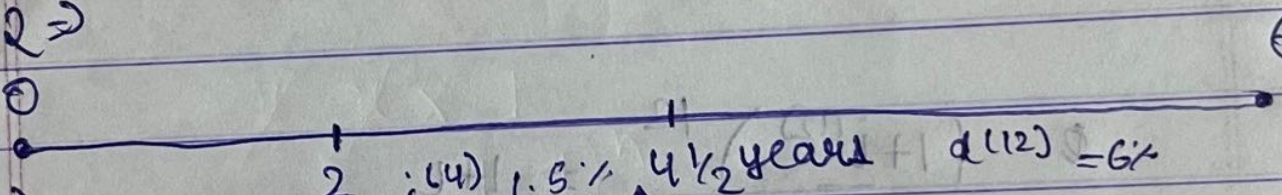
$i = 0.062319315$

Ascending order

I dont know the reason

d) = ?

Ques 12 ⇒



$$A(0,2) = \left(1 + \frac{i^{(12)}}{12}\right)^{12 \times 2}$$

$$= \left(1 + \frac{0.06}{12}\right)^{24}$$

$$= 1.127159776$$

1 year → 4
2 year → 8
2½ year ⇒ 8+2
1+nL ⇒ 10

$$A(2,4.5) = (1 + 10(0.015))$$

$$= 1 + 0.15$$

$$= 1.15$$

$$\left(1-\frac{d^{(12)}}{12}\right)^{12\times 6} = (1-d)^{1.5}$$

$$\left(1-\frac{0.06}{12}\right)^{18} = (1-d)^{1.5}$$

$$0.913724886 = (1-d)^{1.5}$$

$$\frac{1}{(1-d)^{1.5}} = 1.094421325$$

$$AV = 7049 \times (1.127159776)(1.15)(1.094421325)$$
$$= 9999.893616$$

Ques 13 $t = ?$

$$AV = PV \times A(0,t)$$
$$2x = x(1+i)^n$$
$$2 = (1+i)^n$$
$$\log 2 = n \log(1.01)$$
$$\frac{\log 2}{\log(1.1)} = n$$
$$7.2725 = n$$

(ii) $d^{(12)} = 10\% = 0.1$

$$\frac{d^{(12)}}{12} = \frac{0.1}{120}$$

$$\left(1 - \frac{d^{(12)}}{12}\right)^{12} = 1 - d$$

$$\left(1 - \frac{1}{120}\right)^{12} - 1 = -d$$

$$0.095541626 = d$$

~~$2X = X \times \frac{1}{(1-d)^n}$~~

$$(1-d)^n = 0.5$$

$$(0.904458374)^n = 0.5$$

$$n \log(1-d) = \log 0.5$$

$$n = 6.90255 \text{ years}$$