① Probability & Statistics I Assignment

By: Mucharla Harsha Vardhan, A, 49.

1) (i) Given,

4, 7, 9, 9, 11, 15, 18, 18, 18, 25

Here, $n = 10$

$$\text{Mean} = \frac{4+7+9+9+11+15+18+18+18+25}{10} = 13.4$$

$$\text{Median} = \left(\frac{n+1}{2}\right)^{th} \text{term} = 5.5^{th} \text{term}$$

$$= 5^{th} \text{term} + 0.5\left(6^{th}\text{term} - 5^{th}\text{term}\right)$$

$$= 11 + 0.5(15-11)$$

$$= 11 + 2$$

$$= 13$$

Mode = 18

$$\text{Standard Deviation } (\sigma) = \sqrt{\sum_{i=1}^{n} \frac{(x_i - \bar{x})^2}{n}}$$

$$= \sqrt{\frac{1}{10} \times \left[(4-13.4)^2 + (7-13.4)^2 + 2(9-13.4)^2 + (11-13.4)^2 + (15-13.4)^2 + 3(18-13.4)^2 + (25-13.4)^2\right]}$$

$$= 6.1188$$

$$Q_1 = \left(\frac{n+1}{4}\right)^{th} \text{term} = 2.75^{th} \text{ term} = 2^{nd} \text{ term} + 0.75\,(3^{rd} \text{ term} - 2^{nd} \text{ term})$$

$$= 7 + 0.75(9-7)$$

$$= 7 + 1.5$$

$$= 8.5$$

$$Q_3 = \left(\frac{3(n+1)}{4}\right)^{th} \text{term} = 8.25^{th} \text{ term} = 8^{th} \text{ term} + 0.25\,(9^{th} \text{ term} - 8^{th} \text{ term})$$

$$= 18 + 0.25\,(18-18)$$

$$= 18$$

Inter Quartile Range = $Q_3 - Q_1 = 18 - 8.5 = 9.5$

(ii)

No. of Households (x)	No. of People (f)
0	5
1	11
2	26
3	15
4	3

3

~~Mean =~~

(ii)

No of Households (x_i)	No. of People (f)	($f x_i$)	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	C.F
0	5	0	-2	4	5
1	11	11	-1	1	16
2	26	52	0	0	42
3	15	45	1	1	57
4	3	12	2	4	60
	$n = 60$	$\sum f x_i = 120$		$\sum (x_i - \bar{x})^2 = 10$	

$$\text{Mean} = \frac{\sum f x_i}{n} = \frac{120}{60} = 2 \ ;$$

$$\text{Median} = \frac{n}{2}^{th} \text{ value} = 30^{th} \text{ value} = 2 \ ;$$

$$\text{Mode} = \text{value with largest } f = 2 \ ;$$

$$\text{Standard Deviation} = \sqrt{\frac{1}{n} \sum (x_i - \bar{x})^2} = \sqrt{\frac{1}{60} \times 10} = 0.40825$$

(4)

$Q_1 = \left(\frac{60}{4}\right)^{th}$ term $= 15^{th}$ term $= 1$

$Q_3 = 3\left(\frac{60}{4}\right)^{th}$ term $= 45^{th}$ term $= 3$

Inter quartile Range $= Q_3 - Q_1 = 3 - 1 = 2$

2) ~~Given~~

Let X ~~represent~~ be a random variable representing the claims arising on an industrial policy.

Given, $X \sim Poi(0.5)$

(i) Probability that no claims arise in a single year

$= P(X=0) = \frac{0.5^0}{0!} e^{-0.5} = 0.60653066$

(ii) Required Probability $= 3\left[(1-P(X=0)) \times P(X=0) \times P(X=0)\right]$

$= 0.434247627$

(iii) Required Probability $= \cancel{[1-(P(X=0))^2]}$ $\cancel{[1-2(P(X=0)\cdot P(X=1))}$

$= \left(1-(P(X=0))^2\right)$

$= 0.63212136$

3) Given,

$X \sim \text{Gamma}(2, 0.5)$

(i) $E(X) = \dfrac{\alpha}{\lambda} = \dfrac{2}{0.5} = 4$

$Var(X) = \dfrac{\alpha}{\lambda^2} = \dfrac{2}{0.25} = 8 = \sigma^2$

$\therefore \sigma = \sqrt{8} = 2.828$

(ii) The shape of gamma distribution is a positively skewed / right skewed as gamma distribution is a continuous distribution that models right-skewed data.

(iii) $F_X(x) = \int_0^x f_X(x) = \int_0^x \dfrac{\lambda^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\lambda x} \, dx$

Integration by Parts with $U = \dfrac{\lambda^\alpha}{\Gamma(\alpha)} x^{\alpha-1}$; $V = e^{-\lambda x}$, we have,

$F_X(x) = \left[\dfrac{\lambda^\alpha}{\Gamma(\alpha)} x^{\alpha-1} \times \dfrac{-1}{\lambda} e^{-\lambda x} \right]_0^x + \int_0^x \dfrac{\lambda^\alpha}{\Gamma(\alpha)} (\alpha-1) x^{\alpha-2} \times e^{-\lambda x} \, dx$

(iii) Here, $X \sim \text{Gamma}(2, 0.5)$

$$f_X(x) = \frac{(0.5)^2}{\Gamma(2)} x^{2-1} e^{-0.5x} = \frac{0.25}{1!} x \cdot e^{-0.5x}$$

$$= 0.25\, x\, e^{-0.5x}$$

Then, $F_X(x) = \int_0^x 0.25\, x e^{-0.5x}\, dx$

By integrating by parts with $u = x$ and $v = e^{-0.5x}$, we have

$$= 0.25 \left[\left(x \times -2 e^{-0.5x} \right)_0^x - \left(\int_0^x -2 e^{-0.5x}\, dx \right) \right]$$

$$= 0.25 \left[\left(-2x e^{-0.5x} - 0 \right) + 2 \left(\frac{e^{-0.5x}}{-0.5} \right)_0^x \right]$$

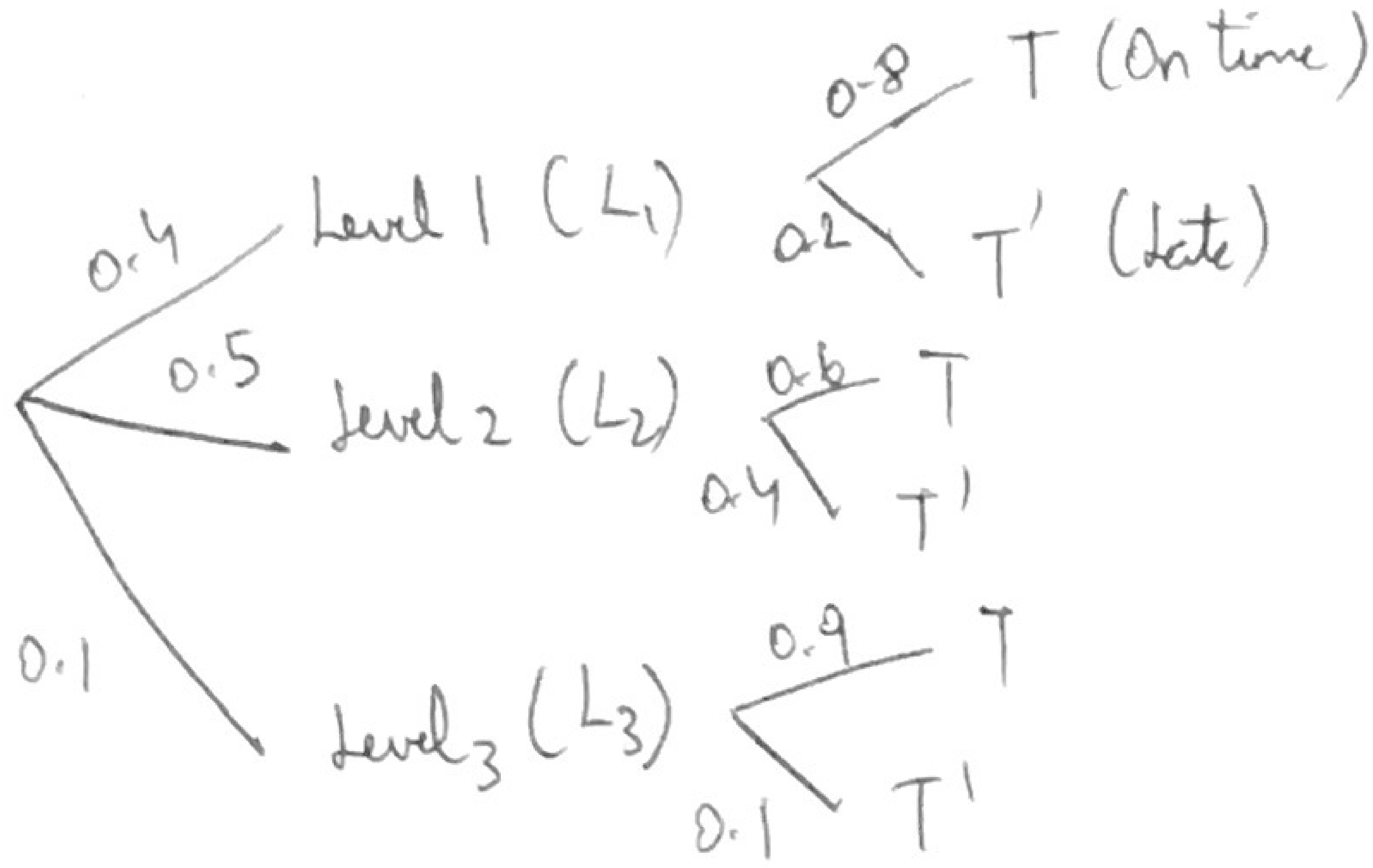
$$= 0.25 \left[-2x e^{-0.5x} - 4\left(e^{-0.5x} - 1 \right) \right]$$

$$= -\frac{x}{2} e^{-0.5x} - \left(e^{-0.5x} - 1 \right)$$

$$= e^{-0.5x} \left(-\frac{x}{2} - 1 \right) \cdot -1$$

$$F_X(x) = 1 - \left(\frac{x}{2} + 1 \right) e^{-0.5x}$$

5)



To find, $P(L_2 | T')$

Here, $P(L_2 | T') = \dfrac{P(L_2 \cap T')}{P(T')}$

$$= \frac{0.5 \times 0.4}{(0.4 \times 0.8) + (0.5 \times 0.4) + (0.1 \times 0.1)}$$

$$= \frac{0.2}{0.32 + 0.2 + 0.01}$$

$$= 0.37736$$

6) Given, $X \sim U(1,2)$

Since we need to find PDF, the distribution is a continuous uniform distribution.

So, $f_x(x) = \frac{1}{\beta - \alpha} = \frac{1}{2-1} = 1$

Also given, $Y = \frac{3}{6-X}$

Then,
$$f_y(y) = \frac{3}{6 - f_x(x)} = \frac{3}{6-1} = \frac{3}{5}$$

7) (i) $F(3) = P(X \leq 3) = 0.05 + 0.15 + 0.35 = 0.55$

(ii) $E(X) = \sum_{i=1}^{n} x \, P(X=x) = 1 \times 0.05 + 0.15 \times 2 + 3 \times 0.35 + 4 \times 0.4 + 5 \times 0.05$

$= 3.25$

(iii) $E(X^2) = 1^2 \times 0.05 + 2^2 \times 0.15 + 3^2 \times 0.35 + 4^2 \times 0.4 + 5^2 \times 0.05$

$= 11.45$

$V(X) = E(X^2) - [E(X)]^2 = 11.45 - (3.25)^2 = 0.8875$

(iv) Coefficient of Skewness $= \frac{\text{Skew}(X)}{\sigma^3}$

$$\text{Skew}(X) = E(x_i - \bar{x})^3 = \sum (x_i - \bar{x})^3 P(X=x)$$

$$= (1-3.25)^3 \times 0.05 + (2-3.25)^3 \times 0.15 + (3-3.25)^3 \times 0.35 + (4-3.25)^3 \times 0.4$$

$$+ (5-3.25)^2 \times 0.05$$

$$= -0.43125$$

$$\sigma^3 = (0.8875)^{\frac{3}{2}} = 0.836089$$

$$\text{Coefficient of Skewness} = \frac{-0.43125}{0.836089} = -0.515794$$

8) Here, the number of insurance claims exceeding 1000 are distributed binomially with parameters, $n = 15$, $p = 0.2$.

To find $P(X<3) = P(X=0) + P(X=1) + P(X=2)$

$$= {}^{15}C_0 (0.2)^0 (1-0.2)^{15} + {}^{15}C_1 (0.2)^1 (1-0.2)^{14} + {}^{15}C_2 (0.2)^2 (1-0.2)^{13}$$

$$= (0.8)^{15} + 15 \times 0.2 \times (0.8)^{14} + \frac{15 \times 14}{2} (0.2)^2 (0.8)^{13}$$

$$= 0.398023$$

9) ~~Let X be the week where he wi~~

Here, the given data follows negative binomial with parameters,

$\cancel{X} = 7$, $k = 3$, $p = 0.01$.

To Find, $P(X=7) = {}^{6}C_{2}\,(0.01)^{3}\,(0.99)^{4}$

$= \frac{6\times 5}{2} \times (0.01)^{3}\,(0.99)^{4}$

$= 0.000014409$

10) Here, the given data follows poisson distribution with parameters, $\lambda = \frac{2}{5} = 0.4$

To find, $P(X > 2) = 1 - P(X \leq 2)$

$= 1 - 0.99207$

$= 0.00793$

11) Since the given data follows geometric distribution with parameter $p = 0.5$, the children the woman has so far is not taken into consideration while calculating the probability of future children.

Hence, the required probability $= \left(\frac{1}{2}\right)^3 = \frac{1}{8} = 0.125$

12) Here, random variable 'x' follows poisson distribution with mean $\lambda = 10$. And the time between two claims T, can be modelled using exponential distribution with

$\lambda = 10$.

Hence,

$$P(T \leq 0.1) = \int_0^{0.1} 10\, e^{-10\times 0.1}\, dx = 10\, e^{-1} [x]_0^{0.1}$$

$$= 0.367879$$

13) Given,

$X \sim U(75, 120)$

$$f_X(x) = \frac{1}{120-75} = \frac{1}{45}$$

$$P(80 < X < 100) = \int_{80}^{100} \frac{1}{45} \, dx = \frac{1}{45} [x]_{80}^{100} = \frac{20}{45} = \frac{4}{9} = 0.44445$$

14) Given,

$X \sim Gamma(5, 0.4)$, we know that $0.8X \sim \chi^2_{10}$

$$P(X < 22.89) = \cancel{P(0.8X < 9.156)} \; P(0.8X < 9.156)$$

According to tables Pg - 165,

$$P(0.8X < 9) = 0.4679$$

$$P(0.8X < 9.5) = 0.5146$$

By linear interpolation, we have,

$$P(0.8X < 9.156) = 0.48247$$

15) Given,

$$(PDF) \quad f_x(x) = \frac{K}{(x+5)^4} \quad ; \; x>0$$

(i) We know that,

$$\int_0^\infty \frac{K}{(x+5)^4} dx = 1$$

Let $x+5=t$. Then $dx = dt$. When $x=0$, $t=5$ and $x=\infty$, $t=\infty$. So,

$$K \int_5^\infty \frac{K}{t^4} = 1$$

$$\Rightarrow \frac{-K}{3}\left[\frac{1}{K^3}\right]_5^\infty = 1$$

$$\Rightarrow \frac{-K}{3}\left[0 - \frac{1}{125}\right] = 1$$

$$\Rightarrow \frac{K}{375} = 1 \Rightarrow \boxed{K=375}$$

$$\therefore f_x(x) = \frac{375}{(x+5)^4}$$

(ii) Now,

$$F_x(10) = \int_0^{10} \frac{375}{(x+5)^4} dx$$

Let, $x+5 = t$. Then, $dx = dt$ and when $x=0$, $t=5$ and $x=10$, $t=15$.

$$\Rightarrow F_x(10) = 375 \int_5^{15} \frac{1}{t^4} dt$$

$$= 375 \left[\frac{-1}{3t^3}\right]_5^{15}$$

$$= 375 \left[\frac{-1}{10125} + \frac{1}{375}\right]$$

$$= 1 - \frac{375}{10125}$$

$$\Rightarrow F_x(10) = 0.963$$

(iii) Now,

$$E(X) = \int_0^\infty x\, f_X(x)\, dx$$

$$= \int_0^\infty x \frac{K}{(x+5)^4}\, dx$$

$$= 375 \int_0^\infty \frac{x}{(x+5)^4}\, dx$$

Let $x+5 = t$ then, $dx = dt$ and, at $x=0$, $t=5$ and at $x=\infty$, $t=\infty$.

$$\Rightarrow E(X) = 375 \int_5^\infty \frac{t-5}{t^4}\, dt = 375 \int_5^\infty \frac{1}{t^3} - \frac{5}{t^4}\, dt$$

$$= 375 \left[\left[\frac{-1}{2t^2}\right]_5^\infty - 5\left[\frac{-1}{3t^3}\right]_5^\infty \right]$$

$$= 375 \left[\left(0 + \frac{1}{50}\right) - 5\left(0 + \frac{1}{375}\right) \right]$$

$$E(X) = \frac{375}{50} - \frac{5 \times 375}{375} = 2.5$$

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4) Here,

Probability of selecting 2 members from a group $= {}^{18}C_2$

Probability of selecting 2 qualified Females $= \dfrac{{}^{7}C_2}{{}^{18}C_2} = P(F)$

Probability of selecting atleast 1 female $= 1 - \dfrac{{}^{10}C_2}{{}^{18}C_2}$

$= 0.705882353$

$= P(A)$

Then, $P(F \cap A) = \dfrac{{}^{7}C_2}{{}^{18}C_2} = 0.137254902$ = Probability of qualified female

Therefore $P(F|A) = \dfrac{P(F \cap A)}{P(A)} = \dfrac{0.137254902}{0.705882353}$

$= 0.19445$