Probablity and Statistics
Assignment 2

1) $X \sim N(800, 100^2)$
$Y \sim N(1200, 300^2)$

$P(y_1 + y_2 + y_3 > x_1 + x_2 + x_3 + x_4 + 800)$
$\Rightarrow P(y_1 + y_2 + y_3 - (x_1 + x_2 + x_3 + x_4) > 800$

$(y_1 + y_2 + y_3) - (x_1 + x_2 + x_3 + x_4) \sim N(3 \times 1200 - 4(800) +$
$300^2(3) + (100^2)\ )$

$P\left(Z > \frac{800 - 400}{3(10000)}\right)$

$P(Z > 0.71842) = 1 - P(0.71842)$

2) $N \sim$ Negative Binomial (k, p)
$P(N=n) = \binom{n+2}{n} 0.9^3\ 0.1^n$

$= {}^{n+2}C_n (0.9)^3 (0.1)^n$

$\therefore k = 3$ and $p = 0.9$

$M_y t = M_n(\log M_x(t))$

$X \sim$ Gamma $(6, 2)$

$M_y(t) = E(Ee^{yt} / N=n)$
$= Ee^{tx_1} \cdot E(e^{tx_2}) \dots E(e^{txn})$

$= \left(1 - \frac{t}{2}\right)^{-6n}$.

$E(E(e^{ty}) | N=n) = E\left[\left(1 - \frac{t}{2}\right)^{-6n}\right]$

$= M_n[\log(1 - t/2)^{-6}]$

$M_n(t) = \left(\frac{pe^t}{1 - qe^t}\right)^k$ $= \left[\frac{pe^{\log(1-t/2)^{-6}}}{1 - qe^{\log(1-t/2)^{-6}}}\right]$

$$= \left[\frac{(0.9)\left(1-\frac{t}{2}\right)^{-6}}{6-0.1\left(1-\frac{t}{2}\right)^{-6}}\right]$$

ii) $V(y) = E(N)\,V(X) + [E(X)]^2 \cdot V(N)$

$$= \left[\frac{3}{0.9}\times\frac{6}{4}\right] + \left(\frac{6}{2}\right)^2 \times \frac{3(0.1)}{(0.9)^2}$$

~~2~~

$= 8.333$

$S.d = 2.887$

3. $F(x) = \lambda e^{-\lambda(x-a)}$, $x \geq a$ and $\lambda, a > 0$.

$MGF = F_x(e^{tx})$

$$= \int_{\alpha}^{\infty} \cancel{e^{tx}}\ \cancel{e^{x}}\ e^{tx} \cdot \lambda e^{-\lambda(x-\alpha)}\,dx$$

$$= \lambda \int_{\alpha}^{\infty} e^{tx} \cdot e^{-\lambda x} \cdot e^{\lambda\alpha}$$

$= e^{t}$

$-$

4). $G_V(t) = \left(\frac{p}{1-qt}\right)^{K}$

~~$G_V(t) =$~~

$$P(V=4) = \frac{\sqrt{k+4}}{\sqrt{5}\,\sqrt{k}}\, p^{K} q^{K}$$

classmate
Date
Page

$\Rightarrow \frac{(k+3)}{k-1} \quad p^k (1-p)^k$

5) $f(x,y) = ax(x+y) \quad 0 < x < 5, \; 10 < y < 20$

i) $P\left(Y > \frac{1}{3}x + 10\right)$ ii) $E(Y/X)$.

i) $$a \int_{10}^{x} \int_{0}^{5} x^2 + xy \; dx \, dy = 1$$

$$\Rightarrow \int_{10}^{20} \left[\frac{x^3}{3} + \frac{x^2}{2} y \right]_0^5 dy = \frac{1}{a}$$

$$\int_{10}^{20} \frac{125}{3} + \frac{25y}{2} \Big] \, dy = \frac{1}{a}$$

$$\left[\frac{125y}{3} + \frac{25}{4} y^2 \right]_{10}^{20} = \frac{1}{a}$$

$$\Rightarrow \frac{125 \cdot 20}{3} + \frac{25(20)^2}{4} - \frac{125(10)}{3} - \frac{25(10)^2}{4} = \frac{1}{a}$$

$$833.33 + 2500 - 416.66 - 625 = \frac{1}{a}$$

$$= 2291.67 = \frac{1}{a} \Rightarrow a = 0.0004363$$

6) $X \sim Poi(\lambda)$ $\quad Y \sim Poi(\mu)$

$M_X(t) = e^{\lambda(e^t-1)}$ $\quad M_Y(t) = e^{\mu(e^t-1)}$

So $Z = x - y$

$M_z(t) = E(e^{tz}) = E(e^{t(x-y)})$

$= E(e^{tx} \cdot e^{-ty})$

$= E(e^{tx}) \cdot E(e^{-ty})$

$= \dfrac{E(e^{tx})}{(e^{ty})}$

$= \dfrac{e^{\lambda(e^t-1)}}{e^{\mu(e^t-1)}}$

$= e^{(\lambda-\mu)(e^t-1)}$

$\therefore$ MGF of $Z = e^{(\lambda-\mu)(e^t-1)} \sim Poi(\lambda-\mu)$

Let $z = x+y$.

$E(e^{tz}) = e^{(\lambda+\mu)(e^t-1)} \sim Poi(\lambda+\mu)$

Same as above.

8). $X \sim Gamma(3,2)$

$E(Y|x = x.) = 3x+1$

$E(y) = E(E\ Y|x = x)$

$= E(3x+1)$

$= 3\ \cancel{E}E(x) + 1$

$= 3 \cdot \frac{3}{2} + 1$

$= \frac{9}{2} + 1 \Rightarrow \frac{11}{2}$

$$\cancel{V(y) = E(V(X|x)) + V(E(x))\,y)}$$
$$= E(2x^2+5) + V(3x+7)$$
$$= 2E(x^2) + 5 + 9V(x)$$

$$Var(Y) = var[E(Y|x)] + E[var(Y|x)]$$

$$Var(Y) = Var(3x+1) + E[2x^2+5]$$
$$= 2E(x^2) + 5 + 9Var(x)$$

$$\cancel{Var(x) = E(x^2) - [E(x)]^2}$$

$$Var(Y) = 2(3) + 5 + 9\left(\frac{3}{4}\right)$$

$$= 11 + \frac{27}{4} \Rightarrow \frac{71}{4}$$

$E\,x = 3 \qquad var(x) = 4$

$E\,y = 4 \qquad var(y) = 1$

$Z = x+y$

$$Cov(x, x+y) = Cov(x,x) + Cov(x,y)$$
$$= Var(x) + Cov(x,y)$$
$$= 4 - 0.6$$
$$= 3.4$$

$$Var(Z) = Cov(x+y, x+y)$$
$$= Cov(x,x) + 2Cov(x,y) + Cov(y,y)$$
$$= V(x) + 2Cov(x,y) + V(y)$$
$$= 4 + 2(-0.6) + 1$$
$$= 5 - 1.2$$
$$= 3.8$$

10) $f(x,y) = kx^{-\alpha}e^{-y/\beta}$

$$k\int_1^{\infty}\int_1^{\infty} x^{-\alpha} \cdot e^{-y/\beta}\, dx\, dy = 1.$$

$$= k\int e^{-y/\beta}\left[\frac{x^{2-1}}{-2+1}\right]_1^{\infty} = 1.$$

$$= \frac{k}{\alpha-1}\int e^{-y/\beta}\, dy = 1$$

$$= \frac{k}{\alpha-1}\left[\frac{e^{-y/\beta}}{-1/\beta}\right]_1^{\infty} - \frac{k}{\alpha-1}\left(\beta e^{-y/\beta}\right) = 1$$

$$k = \frac{\alpha-1}{e^{-y/\beta}}$$