Q] $\Sigma x = 850$ $\Sigma x_i^2 = 92500$
$\Sigma y = 1737$ $\Sigma y_i^2 = 372717$
$\Sigma xy = 182560$

a] Regression line: $\hat{y}_i = \hat{\alpha} + \hat{\beta} x_i$

$$\hat{\beta} = \frac{S_{xy}}{S_{xx}}$$

$$S_{xy} = \Sigma xy - \frac{\Sigma x \Sigma y}{n}$$

$$= 182560 - \frac{850 \times 1737}{10}$$

$$= 34915$$

$$S_{xx} = \Sigma x_i^2 - \frac{(\Sigma x)^2}{n}$$

$$= 92500 - \frac{850^2}{10}$$

$$= 20250$$

$$\hat{\beta} = \frac{34915}{20250} = 1.7242$$

$$\hat{\alpha} = \bar{y} - \hat{\beta}\bar{x}$$

$$= 27.143$$

$\therefore$ Regression line $\hat{y} = 27.143 + 1.7242 x_i$

b]

Q2.] $\Sigma x = 385.2$, $\Sigma x^2 = 12666.58$

$\Sigma y = 1162.5$, $\Sigma y^2 = 119026.9$

$\Sigma xy = 38191.41$

i.] ~~Q.~~ $S_{xy} = \Sigma xy - \frac{\Sigma x \Sigma y}{n}$

$= 38191.41 - \frac{385.2 \times 1162.5}{12}$

$= 875.16$

$S_{xx} = \Sigma x^2 - \frac{(\Sigma x)^2}{n}$

$= 12666.58 - \frac{385.2}{12}$

$= 12636.73$

Regression Line : $\hat{y} = \hat{\alpha} + \hat{\beta} x_i$

$\hat{\beta} = \frac{S_{xy}}{S_{xx}}$

$= 0.0693$

$\hat{\alpha} = \bar{y} - \hat{\beta}\bar{x}$

$= 94.651$

$\hat{y} = 94.651 + 0.0693 x_i$

Q3]
Q b] $\Sigma x = 39,\ \Sigma x^2 = 207,\ \Sigma y = 562,$
$\Sigma y^2 = 40508,\ \Sigma xy = 2853$

i] $r = \dfrac{S_{xy}}{\sqrt{S_{xx} S_{yy}}}$

$S_{xy} = \Sigma xy - \dfrac{\Sigma x \Sigma y}{n}$

$= 661.2$

$S_{xx} = \Sigma x^2 - \dfrac{(\Sigma x)^2}{n}$

$= 54.9$

$S_{yy} = \Sigma y^2 - \dfrac{(\Sigma y)^2}{n}$

$= 8923.6$

$r = 0.945$

ii] Regression line : $\hat{y} = \hat{\alpha} + \hat{\beta} x_i$

$\hat{\beta} = \dfrac{S_{xy}}{S_{xx}} = 12.04$

$\hat{\alpha} = \bar{y} - \hat{\beta}\bar{x}$

$= 9.244$

$\hat{y} = 9.244 + 12.04 x_i$

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ii] 95% of CI = $\hat{\beta} \pm t_{0.025, n-2} \times se(\hat{\beta})$

$$se(\hat{\beta}) = \sqrt{\frac{\hat{\sigma}^2}{S_{xx}}}$$

$$\hat{\sigma}^2 = \frac{1}{n-1}\left\{S_{yy} - \frac{S_{xy}^2}{S_{xx}}\right\}$$

$$S_{yy} = \Sigma y^2 - \frac{(\Sigma y)^2}{n}$$

$$= ~~60~~ 6409.7125$$

$$\hat{\sigma}^2 = 634.91$$

$$se(\hat{\beta}) =$$

95% of CI = $0.0693 \pm 1.812 \times 0.22415$

$$= (-0.3369, 0.475)$$

So a 95% of CI is $(-0.3369, 0.475)$

iii] Estimate of IBM share price =

$$= 94.651 + 0.0693 \times 40$$

$$\approx 97.423$$

Var of Estimate = $\sqrt{\frac{1}{12}\left\{ + \frac{(40-32.1)^2}{12636.73}\right.} \times \sqrt{634.91}$

$$= 1.7942$$

95% of CI = $97.423 \pm 1.812 \times 1.7942$

$$= (94.172, 100.67)$$

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iii] $SS_{TOT} = S_{yy}$
$= 8923.6$

$$SS_{Reg} = \frac{S^2_{xy}}{S_{xx}} = 7963.38$$

$$SS_{Res} = SS_{Tot} - SS_{Reg}$$
$$= 960.3$$

iv] $$R^2 = \frac{SS_{Reg}}{SS_{Tot}} = 0.8924$$

Q3] iii] H_0: The mean of differences is same for each city

H_1: The mean of different is not same

$SS_{Tot} = S_{yy}$
$= 1116.11$

$$SS_{Reg} = \frac{1}{7}\left(64^2 + 44^2 + 8^2 + (-13)^2\right) - \frac{103^2}{28}$$

$= 516.11$

$SS_{Res} = 600$

ANOVA Table

Source	degree of freedom	sum of squares	Mean sum of squares
Reg	3	516.11	172.04
Res	24	600	25
Tot	27	1116.11	

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Under $H_0, F = \frac{172.04}{25} = 6.88$

T.S = 6.88

$F_{3,24,5\%} = 3.009$

$\therefore$ We reject H_0

Q7] $\sum x = 174.13$, $\sum y = 197.84$

$\sum x^2 = 7545.90$, $\sum y^2 = 4747.45$

$~~\sum xy = 2~~$ $S_{xy} = 2176.84$

i] $\hat{y} = \hat{\alpha} \pm \hat{\beta} x_i$

$$\hat{\beta} = \frac{S_{xy}}{S_{xx}}$$

$$S_{xx} = \sum x^2 - \frac{(\sum x)^2}{n}$$

$$= 4789.4221$$

$$\hat{\beta} = 0.4545$$

$$\hat{\alpha} = \bar{y} - \hat{\beta}\bar{x}$$

$$= 10.78056$$

$$\hat{y} = 10.78056 + 0.45451 x_i$$

ii] $H_0 : \beta = 0$

$H_1 : \beta \neq 0$

$$\frac{\hat{\beta} - \beta}{\sqrt{\frac{\hat{\sigma}^2}{S_{xx}}}} \sim t_{n-2}$$

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$$\hat{\sigma}^2 = \frac{1}{n-2}\left\{ S_{yy} - \frac{S^2_{xy}}{S_{xx}} \right\}$$

$$= 22.20136$$

$$T.S = \frac{\hat{\beta} - 0}{\sqrt{\hat{\sigma}^2 / S_{xy}}}$$

$$= 6.67568$$

$$t_{9, 2.5\%} = 2.262$$

$\therefore$ we Reject H_0

iii) $r = \frac{S_{xy}}{\sqrt{S_{xx} S_{yy}}} = 0.91213$

iv) $x_0 = 25$

$\hat{\mu}_0 = \hat{\alpha} + \hat{\beta} x_0$

$= 22.15331$

$$se(\hat{\mu}_0) = \sqrt{\frac{1}{n} + \frac{(\bar{x} - x_0)^2}{S_{xx}} \hat{\sigma}^2}$$

$$= 1.5518$$

$$\frac{\hat{\mu}_0 - \mu_0}{se(\hat{\mu}_0)} \sim t_9$$

95% of CI $= \hat{\mu}_0 \pm t_{9, 2.5\%} \, se(\hat{\mu}_0)$

$= 22.153331 \pm 3.51018$

$= (18.64313, 25.66348)$

For individual response (y_0)

$\hat{y}_0 = \hat{\alpha} + \hat{\beta} x_0$

$= 22.15331$

$se(\hat{y}_0) = \sqrt{1 + \frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{xx}}} \, \hat{\sigma}^2$

$= 4.96079$

95% of CI $= \hat{y}_0 \pm t_{9, 2.5\%} \, se(\hat{y}_0)$

$= 22.15331 \pm 11.22131$

$= (10.932, 33.3742)$

Q8]

ii]

Principle component	Diagonal entry	PCI
PC1	0.456	65%
PC2	0.157	18.5%
PC3	0.08	11.4%
PC4	0.0165	2.4%
PC5	0.012	1.7%
	0.1015	100

Q9] $\Sigma x = 45$, $\Sigma x^2 = 277.5$, $\Sigma y = 644$

$\Sigma y^2 = 43956$, $\Sigma xy = 2602$

i] There is a negative correlation between the marks scored and hours spent per day on social media.

ii) $S_{xy} = \Sigma xy - \frac{\Sigma x \Sigma y}{n}$

$= -296$

$S_{xx} = \Sigma x^2 - \frac{(\Sigma x)^2}{n}$

$= 75$

$S_{yy} = \Sigma y^2 - \frac{(\Sigma y)^2}{n}$

$= 2482.4$

$r = \frac{S_{xy}}{\sqrt{S_{xx} S_{yy}}}$

$= -0.686002$

iii) $H_0 : \rho = 0$
$H_1 : \rho < 0$

$\tanh^{-1}(r) - \tanh^{-1}(\rho) \sim N\left(0, \frac{1}{n-3}\right)$

$\tanh^{-1}(r) - \tanh^{-1}(\rho) \sqrt{n-3} \sim N(0,1)$

T.S $\Rightarrow \tanh^{-1}(r) - \tanh^{-1}(\rho) \sqrt{n-3}$

$Z_{cal} = -2.07893$

$Z_{5\%} = 1.6449$

$|Z_{cal}| > Z_{tab}$

$\therefore$ We have sufficient evidence to reject H_0.

iv] $\hat{\beta} = \frac{S_{xy}}{S_{xx}}$

$= -3.94667$

$\hat{\alpha} = \bar{y} - \hat{\beta}\bar{x}$

$= 82.16$

$\hat{y} = 82.16 - 3.94667 x_i$

v.] $R^2 = \frac{S^2_{xy}}{S_{xx} S_{yy}}$

$= 47.05983\%$

The model is not good fit to the data

vi.] $x_0 = 1$

$\hat{\mu}_0 = \hat{\alpha} - \hat{\beta} x_0$, expected change in

$= 78.21333$

Q10]

x	0.27	0.36	0.3
y	0.05	~~0.04~~ 0.1	0.08

$\Sigma x = 0.93$ $\quad \Sigma x^2 = 0.2925$

$\Sigma y = 0.23$ $\quad \Sigma y^2 = 0.0189$

$\Sigma xy = 0.0735$

$S_{xy} = \Sigma xy - \frac{\Sigma x \Sigma y}{n}$

$= 0.0022$

$S_{xx} = \Sigma x^2 - \frac{(\Sigma x)^2}{n} = 0.0042$

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$$S_{yy} = \Sigma y^2 - \frac{(\Sigma y)^2}{n} = 0.001267$$

$$SS_{Tot} = S_{yy} = 0.001267$$

$$SS_{Reg} = \frac{S^2_{xy}}{S_{xx}} = 0.001152$$

$$SS_{Res} = SS_{Tot} - SS_{Reg}$$

$$= 0.001267 - 0.001152$$

$$= -0.000253$$

Anova table

Source of variation	degrees of freedom	Sum of squares	Mean square sum
Reg	1	0.0015	0.0015
Res	1	0.00011	0.00011
Tot	2	0.00126	

$H_0 : \beta = 0$

$H_1 : \beta \neq 0$

T.S $\Rightarrow \frac{MSS_{Reg}}{MSS_{Res}} = 10.45455$

$F_{1,1,5\%} = 161.4$

$F_{cal} < F_{tab}$

We do not reject H_0