classmate
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SRM-4 : Assignment 1 — Kareena Toted
EVT & copulas — 709
unit 1,2

Q1.

i. $\alpha = 2$

T_i = time until default of bond i, $i = 1, 2, 3$.

$$P(T_1 \le 1, T_2 \le 1, T_3 \le 1) = C[u_1, u_2, u_3]$$

$$= e^{-[(-\ln u_1)^2 + (-\ln u_2)^2 + (-\ln u_3)^2]^{1/2}}$$

$$= e^{-(3(-\ln 0.1)^2)^{1/2}}$$

$$= 0.0185$$

$$= 1.85\%$$

ii. Gumbel copula exhibits upper tail dependence, the degree of which can be varied by adjusting the single parameter. However, it exhibits no lower tail dependence.

Hence Gumbel copula is appropriate if we believe that the three investments are likely to behave similarly as the term approaches 5 years but not at early durations.

However, if one bond defaults early, it may indicate problems in the industry sector or in the economy and other investments might default early on too.

If we believe the performance of investments issued by companies within the industry are much closely associated then a copula with both lower and upper tail dependence may be more appropriate, such as a Student's-t copula

Q2

i. The three parameters of the GEV family are:
α, β, γ

ii. These parameters are:
α: location parameter
β: scale parameter
γ: shape parameter

iii.

Q3

i. 4 ways of measuring the tail weight of a distribution:
→ Existence of moments
→ Limiting density ratio
→ Hazard rate
→ Mean residual life.

ii. $\alpha_1 = 0.2$ $\quad = \lim_{x \to \infty} \frac{f_{\alpha=0.2}(x)}{f_{\alpha=0.3}(x)} = \lim_{x \to \infty} \left(\frac{0.2\lambda^{0.2}}{(\lambda+x)^{1.2}} \middle/ \frac{0.3\lambda^{0.3}}{(\lambda+x)^{1.3}} \right)$

$\alpha_2 = 0.3$

$$= \frac{2}{3} \times \frac{1}{\lambda^{0.1}} \lim_{x \to \infty} (\lambda + x)^{0.1}$$

$$= \infty$$

$\therefore$ distribution with $\alpha = 2$ has a thicker tail

Q4.

i. Gumbel copula : $\alpha = 3.5$

$_{30}P^{m}_{50} = 0.6$

$_{30}P^{f}_{50} = 0.65$

Prob of death for the same period -

$_{30}q^{m}_{50} = 0.4$

$_{30}q^{f}_{50} = 0.35$

$P(X \leq 30) = 0.4$

$P(Y \leq 30) = 0.35$

$U = 0.4$

$V = 0.35$

$$C(U,V) = e^{-\left[(-\ln U)^{\alpha} + (-\ln V)^{\alpha}\right]^{1/\alpha}}$$

$= 0.2996.$

ii. clayton with $\alpha = 3.5$

$$C[U,V] = (U^{-\alpha} + V^{-\alpha} - 1)^{(-1/\alpha)}$$

$\alpha = 3.5$

$= 0.30595.$

iii. Frank copula

$\alpha = 3.5$

$$C(u,v) = (-1/\alpha) \ln \left(1 + \frac{(e^{-\alpha u}-1)(e^{-\alpha v}-1)}{(e^{-\alpha}-1)} \right)$$

$\alpha = 3.5$

$= 0.2273$

iv. Clayton copula gives higher prob of claims because it exhibits lower tail dependence. This means that if one life does not survive for long then there is a high probability that the other life will also not survive for long.

If deaths are independent then the probability of paying the benefit is 0.19

However two lives covered are related so we would expect the prob of paying the benefit to be higher than under the assumption of independence. Hence, clayton copula is more appropriate.

5. Lifetime ~ exp($\lambda = 0.125$)

i. MRL for 2 year old phone $= \frac{\int_x^{\infty} (1 - F(y))\, dy}{1 - F(x)} = \frac{\int_x^{\infty} e^{-\lambda y}\, dy}{e^{-\lambda x}}$

$= 1/\lambda = 1/0.125$

$= 8$ years

ii. The expected lifetime of a 2 year old phone & a new phone would both be 8 years because of memoryless property of exponential distribution. Thus it's not appropriate.