CALCULUS ASSIGNMENT-2

1. $\displaystyle\int_{1/50}^{2} \frac{e^{2/w}}{w^2}\, dw$

Let; $2/w = t$

$-\frac{2}{w^2}\, dw = dt$

$dw = -\frac{w^2}{2}\, dt$

When, $w = 2,\ t = 1$

$w = 1/50,\ t = 100$

$\therefore \displaystyle\int_{100}^{1} -\frac{1}{2} w^2 \frac{e^t}{w^2}\, dt$

$= -\frac{1}{2} \displaystyle\int_{100}^{1} e^t\, dt$

$= -\frac{1}{2} [e^t]_1^{100}$

$= -1/2\, [e^{100} - e^1]$

$= -\frac{e}{2} [e^{99} - 1] + C$

2. $\displaystyle\int \frac{2t^3+1}{(t^4+2t)^3}\, dt$

Let; $(t^4+2t) = m$

$(4t^3+2)\, dt = dm$

$dt = \frac{1}{2} \frac{dm}{(2t^3+1)}$

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$$\int \frac{2t^3+1}{(t^4+2t)^3}\,dt = \int \frac{1}{2(2t^3+1)} \frac{(2t^3+1)}{m^3}$$

$$= \frac{1}{2}\int m^{-3}$$

$$= \frac{m^{-3}}{-4}$$

$$= \frac{-1}{4m^3}$$

$$\underline{I} = -\frac{1}{4(t^4+2t)^2} + c$$

3. $f'(x) = x^4 + 3x - 9$

$$\int f'(x) = \int x^4 + 3x - 9$$

$$f(x) = \frac{4x^5}{5} + \frac{3x^2}{2} - 9x$$

4. $f(x) = \begin{cases} 6 & ; x > 1 \\ 3x^2 & ; x \leq 1 \end{cases}$

$$\int_{-2}^{3} f(x) = \int_{-2}^{1} f(x) \cdot \int_{2}^{3} f(x)$$

$$= \int_{-2}^{1} 3x^2 \cdot \int_{2}^{3} 6$$

$$= \left[\frac{3x^3}{3}\right]_{-2}^{1} \cdot \left[6x\right]_{2}^{3}$$

$$= [\cancel{x}1 + 8] \cdot [18 - 12]$$

$= 9 \times 6$

$= \underline{54}$

5. $\int 3(8y-1)e^{4y^2-y}\,dy$

Let $4y^2 - y = t$

$(8y-1)\,dy = dt$

$\Rightarrow 3\int e^t\,dt$

$\Rightarrow 3e^t + C$

Ans] $= 3e^{4y^2-y} + C$

6. $f''(x) = 15\sqrt{x} + 5x^3 + 6$

$f(1) = -5/4$

$f(4) = 404$

$\int f''(x) = \int 15\sqrt{x} + 5x^3 + 6$

$\int f''(x) = \int 15x^{1/2} + 5x^3 + 6$

$f'(x) = \dfrac{15x^{3/2}}{3/2} + \dfrac{5x^4}{4} + 6x + C$

$f'(x) = 10x^{3/2} + \dfrac{5x^4}{4} + 6x + C$

Integrating $f'(x)$

$\int f'(x) = \int 10x^{3/2} + \dfrac{5x^4}{4} + 6x + C$

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$$f(x) = \frac{10x^{5/2}}{5/2} + \frac{5x^5}{5\times4} + \frac{6x^2}{2} + cx + K$$

$$f(x) = 4x^{5/2} + \frac{x^5}{4} + 3x^2 + Cx + K$$

$f(1) = -5/4$ [given]

$$-5/4 = 4(1)^{5/2} + \frac{1^5}{4} + 3(1)^2 + C(1) + K$$

$$-\frac{5}{4} = 4 + \frac{1}{4} + 3 + C + K$$

$$-\frac{5}{4} = \frac{29}{4} + C + K$$

$$C + K = \frac{-5-29}{4} = \frac{-17}{4} \quad \text{--- I}$$

$f(4) = 404$

$$404 = 4(4)^{5/2} + \frac{4^5}{4} + 3(4)^2 + 4C + K$$

$$404 = 432 + 4C + K$$

$$4C + K = -28 \quad \text{--- II}$$

Equading I and II

$$C + K = -17/4$$
$$-(4C + K = -28)$$
$$-3C = 23.75$$
$$\boxed{C = -7.9167}$$

Subsdituding $C = -7.9167$ in I

$$\boxed{\therefore K = 3.6667}$$

8. $\frac{dr}{d\theta} = \frac{r^2}{\theta}$, $r(1) = 2$

$\frac{dr}{r^2} = \frac{d\theta}{\theta}$

$\int \frac{dr}{r^2} = \int \frac{d\theta}{\theta}$

$-\frac{1}{r} = \cancel{\frac{1}{\theta}} \log|\theta| + c$

$\theta = \log|\theta| + \frac{1}{r} + c$

7. $f(x+iy) + g(x-iy)$

Partial differentiating w.r.t x

$a = f'(x+iy) + g'(x-iy)$

Partial differentiating w.r.t y

$b = f'(x+iy)(i) + g'(x-iy)(-i)$

Partial differentiating 'a' w.r.t x

$c = f''(x+iy) + g''(x-iy)$

Partial differentiating 'b' w.r.t y

$D = f''(x+iy)(i^2) + g''(x-iy)(-i)^2$

$D = f''(x+iy)(i^2) + g''(x-iy) -i^2$

$\therefore D = i^2 [f''(x+iy) + g''(x+iy)]$

$D = i^2 [c]$ $\qquad [i = \sqrt{-1} \therefore i^2 = -1]$

$D = -c$

9. $\frac{dv}{dt} = 9.8 - 0.196v$

$\frac{1}{dt} = \frac{9.8 - 0.196v}{dv}$

$\frac{1}{dt} = \frac{9.8}{dv} - \frac{0.196v}{dv}$

reciprocating the equation

$dt = \frac{1}{9.8} dv - \frac{1}{0.196v} dv$

$\int dt = \frac{1}{9.8} \int dv - \int \frac{1}{0.196} \int \frac{1}{v} dv$

$t = \frac{v}{9.8} - \frac{1}{0.196} \log|v| + c$

10] $t \frac{dy}{dt} + 2y = t^2 - t + 1$

$\frac{dy}{dt} + \frac{2y}{t} = t - 1 + \frac{1}{t}$

$\frac{dy}{dt} + Py = Q$

$IF = e^{\int P dt}$

$= e^{\int \frac{2}{t} dt}$

$= e^{2\log|t|} = e^{\log t^2}$

$= t^2$

$$y \times IF = \int Q \times IF \, dt$$
$$t^2 y = \int t^2 (t - 1 + \frac{1}{t}) \, dt$$

$$yt^2 = \int (t^3 - t^2 + t) \, dt$$
$$yt^2 = \frac{t^4}{4} - \frac{t^3}{3} + \frac{t^2}{2} + c$$

$t = 1, y = 1/2$
$$1/2 = 1/4 - 1/3 + 1/2 + c$$
$$1/12 = c$$
$$yt^2 = \frac{t^4}{4} - \frac{t^3}{3} + \frac{t^2}{2} + c$$

$$yt^2 = \frac{t^4}{4} - \frac{t^3}{3} + \frac{t^2}{2} + \frac{1}{12}$$

$$12yt^2 = 3t^4 - 4t^3 + 6t^2 + 1$$

11 $$\frac{dy}{dx} = \frac{xy^3}{\sqrt{1+x^2}}$$
$$\frac{dy}{y^3} = dx \cdot \frac{x}{\sqrt{1+x^2}}$$
$$\int y^{-3} dy = \int \frac{x}{\sqrt{1+x^2}} dx$$
$$\frac{-y^{-2}}{2} = 2\sqrt{1+x^2} + c$$

When $x = 0, y = -1$
$$-1 = 2 + c$$
$$c = -3$$
$$2y^2\sqrt{1+x^2} - 3y^2 + 1 = 0$$

$\sqrt{1+x^2}$ is continuous at all points
Thus, the interval of validity.

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12] $f(x) = x^3 - 10x^2 + 6$ for $x = 3$

$f(3) = 27 - 90 + 6$

$= -57$

$f'(x) = 3x^2 - 20x$

$f'(3) = 3(9) - 60 = -33$

$f''(x) = 6x - 20$

$f''(3) = 18 - 20 = -2$

$f'''(x) = 6$

Taylor Series:

$f(x) = f(3) + (x-3)f'(3) + \frac{(x-3)^2}{2!}f''(3) + \frac{(x-3)^3}{3!}f'''(3$

$f(x) = -57 + (x-3)(-33) + \frac{(x-3)^2(-2)}{2} + \frac{(x-3)^3}{6} \times 6 + 0$

$f(x) = -57 - 33x + 99 + x^2 + 9 - 6x + (x-3)^3$

13] $f(0) = 1$

$f(x) = (1+x)^u$

$f'(x) = u(1+x)^{u-1}$

$f'(0) = u(1+0)^{u-1} = u$

$f''(x) = u(u-1)(1+x)^{u-2}$

$f''(0) = u(u-1)(1+0)^{u-2} = u(u-1)$

$f'''(x) = u(u-1)(u-2)(1+x)^{u-3}$

$f'''(0) = u(u-1)(u-2)$

$f''''(x) = u(u-1)(u-2)(u-3)(1+x)^{u-4}$

$f''''(0) = u(u-1)(u-2)(u-3)$

MACLAURIN SERIES:

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2} f''(0) + \frac{x^3}{6} f'''(0) + \frac{x^4}{4!} f''''(0) + \dots$$

$$f(x) = u + xu + \frac{x^2}{2} u(u-1) + \frac{x^3}{6} u(u-1)(u-2) + \frac{x^4}{4!} u(u-1)(u-2)(u-3)$$

14] $f(x) = \sqrt{1+x}$

$f(0) = \sqrt{1+0} = 1$

$f'(x) = \frac{1}{2\sqrt{1+x}}$

$f'(0) = 1/2$

$f''(x) = -1/4\,(1+x)^{3/2}$

$f''(0) = -1/4$

$f'''(x) = 3/8\,(1+x)^{5/2}$

$f'''(0) = 3/8$

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \frac{x^4}{4!} f''''(0) + \dots$$

$$= 1 + \frac{x}{2} + \frac{x^2}{2}\left(-\frac{1}{4}\right) + \frac{x^3}{6}\left(\frac{3}{8}\right) + \dots$$

$$f(x) = 1 + x/2 - x^2/8 + x^3/16 + \dots$$

15] $f(x) = e^{-6x}$ $\quad f'(x) = -6e^{-6x}$

$f(-4) = e^{24}$ $\quad f'(-4) = -6e^{24}$

$f''(x) = 36e^{-6x}$ $\quad f'''(x) = 36e^{-6x}(-6)$

$f''(-4) = 36e^{24}$ $\quad f'''(-4) = -216e^{24}$

$$f(x) = f(-4) + (x+4)f'(x) + \frac{(x+4)^2}{2} f''(x) + \frac{(x+4)^3}{3!} f'''(x) + \dots$$

$$= e^{24} + (x+4)(-6e^{24}) + \frac{(x+4)^2}{2}(36e^{24}) + \frac{(x+4)^3}{6}(-216e^{24})$$

10] $f(x) = \ln(4x+3)$

$f(0) = \ln(3)$

$f'(x) = \frac{1}{4x+3}$

$f'(0) = \frac{1}{3}$

$f''(x) = \frac{-4}{(4x+3)^2}$

$f''(0) = -\frac{4}{9}$

$f'''(x) = \frac{-4x-2(4)}{(4x+3)^3} = \frac{32}{(4x+3)^3}$

$f'''(0) = \frac{32}{9}$

$f(x) = f(0) + xf'(0) + \frac{x^2}{2} f''(0) + \frac{x^3}{3} f'''(0) + \dots$

$= \ln(3) + \frac{x}{3} + \frac{x^2}{3}\left(-\frac{4}{9}\right) + \frac{x^3}{3}\left(\frac{32}{9}\right) + \dots$

$f(x) = \ln(3) + \frac{x}{3} - \frac{2}{9}x^2 + \frac{32}{27}x^3 + \dots$