Assignment - 1.
Unit 1 & 2.

1Q. $\lim_{h \to 0} \frac{2(-3+h)^2 - 18}{h}$

$\lim_{h \to 0} \frac{2(9 - 6h + h^2) - 18}{h}$

$\lim_{h \to 0} \frac{18 - 12h + 2h^2 - 18}{h}$

$\lim_{h \to 0} \frac{\cancel{h}(2h - 12)}{\cancel{h}}$

$= -12$.

2Q. $g(x) = \begin{cases} 2x, & x < 6. \\ x - 1, & x \geq 6. \end{cases}$

a) $x = 4$.

$\lim_{x \to 4^-} g(4) = \lim_{x \to 4^+} g(4) = g(4)$

For the ~~limit~~ function to be continoous.

$8 = 8 = 8$.

$\therefore$ The function is continuos at $x = 4$.

b) $x = 6$.

$\lim_{x \to 6^-} g(6) = 2(6) = 12$.

$\lim_{x \to 6^+} g(6) = 6 - 1 = 5$.

$\therefore$ The function is discontinuos at $x = 6$.

3Q. $\lim_{x \to 9} \dfrac{x - 9}{3 - \sqrt{x}}$.

$= \lim_{x \to 9} \dfrac{(\sqrt{x})^2 - 3^2}{3 - \sqrt{x}}$.

$= \lim_{x \to 9} \dfrac{(\sqrt{x} + 3)\cancel{(\sqrt{x} - 3)}}{-\cancel{(\sqrt{x} - 3)}}$

$= \lim_{x \to 9} -(\sqrt{x} + 3)$.

$= 0$.

4Q. $f(x) = \dfrac{4x + 5}{9 - 3x}$.

a) $x = -1$.

$\lim_{x \to -1^-} f(-1) = \dfrac{4(-1) + 5}{9 - 3(-1)} = \dfrac{1}{12}$.

$\lim_{x \to -1^+} f(-1) = \dfrac{4(-1) + 5}{9 - 3(-1)} = \dfrac{1}{12}$.

$f(-1) = \dfrac{1}{12}$,

$\therefore$ $f(x)$ is continuos at $x = -1$.

b) $x = 0$.

$$\lim_{x \to 0^-} f(0) = \frac{4(0)+5}{9-3(0)} = \frac{5}{9}.$$

$$\lim_{x \to 0^+} f(0) = \frac{4(0)+5}{9-3(0)} = \frac{5}{9},$$

$$f(0) = \frac{4(0)+5}{9-3(0)} = \frac{5}{9}.$$

$\therefore$ $f(x)$ is continuos at $x=0$.

c) $x = 3$.

The function is not defined at $x=3$.
and thus is discontinuos.

5Q. $$\lim_{x \to \infty} \frac{6e^{4x} - e^{-2x}}{8e^{4x} - e^{2x} + 3e^{-x}}.$$

$$= \frac{6}{8}.$$

6Q $g(y) = \begin{cases} y^2+5, & y < -2 \\ 1-3y, & y \geq -2. \end{cases}$

a) $\lim_{y \to 6} g(y) = \lim_{y \to 6} 1-3y$

$= 1-3(6)$

$= -17.$

b) $\lim_{y \to -2} g(y).$

$\lim_{y \to -2^-} g(y) = \lim_{y \to -2^-} y^2+5$

$= (-2)^2+5$

$= 9.$

$\lim_{y \to -2^+} g(y) = \lim_{y \to -2^+} 1-3y$

$= 1-3(-2).$

$= 7.$

As LHL $\neq$ RHL, limit does not exist at $y = -2.$

7Q. $f(t) = (4t^2 - t)(t^3 - 8t^2 + 12)$

diff w.r.t t

$f'(t) = (4t^2 - t)(t^3 - 8t^2 + 12)' + (t^3 - 8t^2 + 12)(4t^2 - t)'$

$f'(t) = (4t^2 - t)(3t^2 - 16t) + (t^3 - 8t^2 + 12)(8t - 1)$

$f'(t) = 12t^4 - 64t^3 - 3t^3 + 16t^2 + 8t^4 - t^3 - 64t^3 + 8t^2 + 96t - 12$

$f'(t) = 20t^4 - 132t^3 + 24t^2 + 96t - 12$

8Q. $R(w) = \dfrac{3w + w^4}{2w^2 + 1}$

differentiating w.r.t w.

$R'(w) = \dfrac{(2w^2 + 1)(3w + w^4)' - (3w + w^4)(2w^2 + 1)'}{(2w^2 + 1)^2}$

$= \dfrac{(2w^2 + 1)(4w^3 + 3) - (3w + w^4)(4w)}{(2w^2 + 1)^2}$

$= \dfrac{8w^5 + 6w^2 + 4w^3 + 3 - (12w^2 + 4w^5)}{(2w^2 + 1)^2}$

$= \dfrac{4w^5 + 4w^3 - 6w^2 + 3}{(2w^2 + 1)^2}$

9Q. $f(x) = 2e^x - 8^x$

diff w.r.t x.

$f'(x) = 2e^x - 8^x \ln 8$

10Q. a) $y = z^5 - e^z \ln(z)$.

diff w.r.t z.

$$\frac{dy}{dz} = 5z^4 - e^z \times \frac{1}{z} + \ln(z) e^z$$

$$= 5z^4 - \frac{e^z}{z} + \ln(z) e^z$$

11Q. b) $f(t) = 5 + e^{4t + t^7}$

diff w.r.t t.

$$f'(t) = e^{(4t + t^7)} \times (4 + 7t^6).$$

$$= e^{4t+t^7} \cdot (4 + 7t^6).$$

a) $f(x) = (6x^2 + 7x)^4$

diff w.r.t x.

$$f'(x) = 4(6x^2 + 7x)^3 \times (12x + 7).$$

$$= 4(6x^2 + 7x)^3 (12x + 7).$$

12Q. a) $f(z) = \ln(7 - x^3)$.

$$f'(z) = \frac{1}{7 - x^3} \times -3x^2$$

$$= \frac{-3x^2}{7 - x^3}$$

$$= \frac{3x^2}{x^3 - 7}.$$

$$f''(z) = \frac{(x^3-7)(3x^2)' - (3x^2)(x^3-7)'}{(x^3-7)^2}$$

$$= \frac{(x^3-7)(6x) - (3x^2)(3x^2)}{(x^3-7)^2}.$$

$$= \frac{6x^4 - 42x - 9x^4}{(x^3-7)^2}$$

$$= \frac{-(3x^4 + 42x)}{(x^3-7)^2}$$

b) $$g(v) = \frac{2}{(6+2v-v^2)^4}$$

$$= 2(6+2v-v^2)^{-4}$$

diff w.r.t v.

$$g'(v) = 2 \times -4(6+2v-v^2)^3 \times (2-2v).$$

$$= -8(6+2v-v^2)^3(2-2v).$$

$$g''(v) = -8\left[(6+2v-v^2)^3(2-2v)' + (2-2v)(6+2v-v^2)'\right]$$

$$= -8\left[(6+2v-v^2)^3(-2) + (2-2v)(2-2v)\right].$$

$$= -8\left[-2(6+2v-v^2)^3 + (2-2v)^2\right].$$

c) $$f(t) = \ln(1+t^2).$$

$$f'(t) = \frac{1}{1+t^2} \times 2t.$$

$$= \frac{2t}{1+t^2}$$

$$f''(t) = \frac{(1+t^2)(2t)' - (2t)(1+t^2)'}{(1+t^2)^2}$$

$$= \frac{2(1+t^2) - (2t)(2t)}{(1+t^2)^2}$$

$$= \frac{2(1+t^2) - (2t)^2}{(1+t^2)^2}$$

$$= \frac{2[1+t^2-2t^2]}{(1+t^2)^2}$$

$$= \frac{2[1-t^2]}{(1+t^2)^2}$$

$$= \frac{2\cancel{(1+t^2)}(1-t^2)}{(1+t^2)\cancel{(1+t^2)}}$$

$$= \frac{2(1-t^2)}{1+t^2}$$

13. Q. $f(x) = x^3 - 3x^2 - 9x + 12.$

$f'(x) = 3x^2 - 6x - 9$

If $f'(x) = 0.$

$3x^2 - 6x - 9 = 0.$

$x^2 - 2x - 3 = 0.$

$x^2 + x - 3x - 3 = 0.$

$x(x+1) - 3(x+1)$

$(x-3)(x+1) = 0.$

$\therefore x = 3$ or $x = -1.$

$f''(x) = 6x - 6.$

$f''(3) = 12$ +ve $\therefore$ minima.

$f''(-1) = -12.$ -ve $\therefore$ maxima.

$\therefore f(x)$ is maxima at $x = -1$ & minima at $x = 3.$

14Q. $f(x) = 4x^3 - 18x^2 + 24x - 7$

$f'(x) = 12x^2 - 36x + 24$.

If $f'(x) = 0$.

$\therefore 12x^2 - 36x + 24 = 0$.

$x^2 - 3x + 2 = 0$.

$x^2 - x - 2x + 2 = 0$.

$x(x-1) - 2(x-1) = 0$.

$(x-2)(x-1) = 0$.

$\therefore x = +2$ or $x = 1$.

$f''(x) = 24x - 36$.

$f''(2) = 12$ +ve $\therefore$ minima. ~~maxima~~

$f''(1) = -12$ −ve $\therefore$ maxima.

$\therefore f(x)$ is maxima at $x = 1$ & minima at $x = 2$.

15Q. $f(x) = x^2(x+3)$.

$= x^3 + 3x^2$

$f'(x) = 3x^2 + 6x$.

If $f'(x) = 0$,

Then, $3x^2 + 6x = 0$.

$x^2 + 2x = 0$.

$x(x+2) = 0$.

$\therefore x = 0$ or $x = -2$

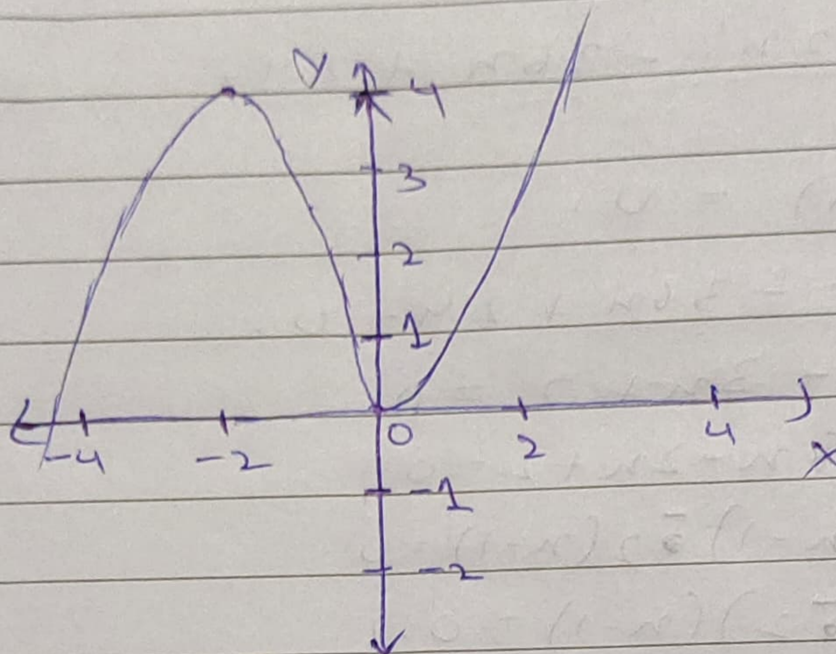
$f''(x) = 6x + 6$.

$f''(0) = 6$ +ve $\therefore$ minima.

$f''(-2) = -6$ −ve $\therefore$ maxima.

maxima minima

Increasing -2 Decreasing 0 Increasing



16Q $f(x) = 3x^2 - 6x$.

$f'(x) = 6x - 6$.

If $f'(x) = 0$.

then $6x - 6 = 0$.

$x - 1 = 0$.

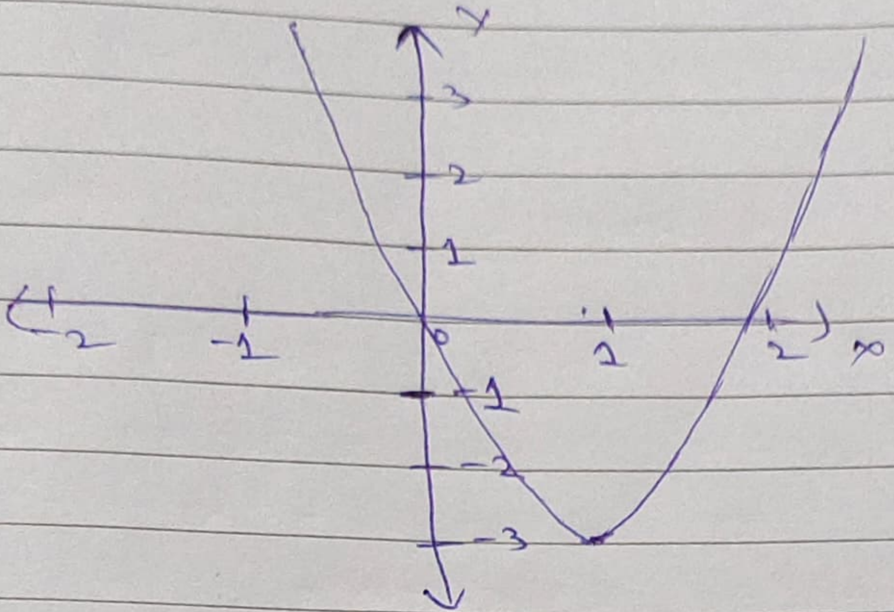
$\therefore x = 1$.

$f''(x) = 6$,

$f''(1) = 6$. +ve $\therefore$ minima.

minima

Increasing 1 Increasing



y
3
2
1
0
-1
-2
-3
-2
-1
1
2
x