CALCULUS ASSIGNMENT

LITTLE JAIN

Ques 1 $\displaystyle\int_{1/50}^{2} \frac{e^{2/w}}{w^2}\,dw$

= Let,

$2/w = t$

$-2/w^2 = \frac{dt}{dw}$

$\frac{1}{w^2}\,dw = -\frac{dt}{2}$

$dw = -\frac{w^2}{2}\,dt$

$2 = wt$ $\qquad$ $2 = wt$

$\cancel{2} = \cancel{2}t$ $\qquad$ $2 = \frac{1}{50}t$

$1 = t$ $\qquad$ $100 = t$

$$I = \int_{100}^{1} \frac{e^t}{\cancel{w^2}} \times \frac{-\cancel{w^2}}{2}\,dt$$

$$I = \frac{1}{2}\int_{1}^{100} e^t\,dt$$

$$I = \frac{1}{2}\left[e^{100} - e^{1}\right] = \frac{e}{2}\left[e^{99} - 1\right] + c$$

Ques 2 $I = \int \frac{2t^3+1}{(t^4+2t)^3} dt$

Let,

$(t^4+2t) = u$

$4t^3+2 = \frac{du}{dt}$

$2(2t^3+1) = \frac{du}{dt}$

$(2t^3+1)dt = \frac{du}{2}$

$I = \frac{1}{2} \int \frac{du}{u^3}$

$I = 1/2 \int u^{-3} du$

$I = 1/2 \times \frac{u^{-2}}{-2} + C$

$I = \frac{-1}{4(t^4+2t)^2} + C$

Ques 3 $f'(x) = x^4 + 3x - 9$

$$\boxed{\int f'(x)\,dx = f(x)}$$

$$f(x) = \int (x^4 + 3x - 9)\,dx$$

$$f(x) = \frac{x^5}{5} + \frac{3x^2}{2} - 9x + C$$

Ques 4 ⇒

$$f(x) = \begin{cases} 6 & \text{if } x > 1 \\ 3x^2 & \text{if } x \le 1 \end{cases}$$

Evaluate

$$\int_{-2}^{3} f(x)\,dx = 3\int_{-2}^{1} x^2\,dx + \int_{1}^{3} 6\,dx$$

$$= \frac{3}{3}\left[x^3\right]_{-2}^{1} + \left[6x\right]_1^3$$

$$= 1 - (-2)^3 + 18 - 6$$

$$= 1 - (-8) + 12$$

$$= 1 + 8 + 12$$

$$= 21$$

Ques 5 ⇒

$$I = \int 3(8y-1)\, e^{(4y^2-y)}\, dy$$

$$4y^2 - y = t$$

$$(8y-1)\,dy = dt$$

$$I = \int 3e^t \, dt$$

$$I = 3e^t + C$$

$$I = 3e^{4y^2-y} + C$$

Ques 6

$$f''(x) = 15\sqrt{x} + 5x^3 + 6$$

$$\int f''(x)\, dx = f'(x)$$

$$f'(x) = \int (15\, x^{1/2} + 5x^3 + 6)\, dx$$

$$f'(x) = 2 \times \cancel{15}^{5} \frac{x^{3/2}}{\cancel{3}} + \frac{5x^4}{4} + 6x + C$$

$$f'(x) = 10x^{3/2} + \frac{5}{4}x^4 + 6x + c$$

$$\int f'(x)\,dx = f(x)$$

$$f(x) = \int \left(10x^{3/2} + \frac{5}{4}x^4 + 6x + c\right)dx$$

$$= \frac{2 \times \cancel{10}^{2}\, x^{5/2}}{\cancel{5}} + \frac{\cancel{5}}{4}\frac{x^5}{\cancel{5}} + \frac{\cancel{6}x^2}{\cancel{2}} + cx + K$$

$$f(x) = 4x^{5/2} + \frac{x^5}{4} + 3x^2 + cx + K$$

$$f(1) = -5/4$$

$$f(1) = 4 + \frac{1}{4} + 3 + c + K$$

$$-5/4 = 9/4 + 3 + c + K$$

$$-5/4 = \frac{21}{4} + c + K$$

$$-\cancel{26}/4 = c + K$$

$$-13/2 = c + K$$

$$f(4) = 404$$

$$404 = 4 \times (2)^{5/2 \times 2} + \frac{(2)^{10}}{(2)^2} + 3(2)^4 + 4c + K$$

$$-28 = 4C + K$$

$$-13/2 = C + K$$

$(+)$ $(-)$

$$13/2 - 28 = 3C$$

~~13 = 28~~

$$C = -13/2$$

$$K = -2$$

$$f(x) = 4x^{5/2} + \frac{x^5}{4} + 3x^2 - \frac{13x}{2} - 2$$

Ques 7

$$f(x+iy) + g(x-iy)$$

$\Rightarrow$ Partially diffrentiation w.r.t x
let it be denote by a

$$a = f'(x+iy) + g'(x-iy) \quad -(1)$$

Partially diffrentiating w.r.t y. Let it denote by b

$$b = f'(x+iy) \times i + g'(x-iy)(-i) \quad -(2)$$

Partiall diffientiating ① wrt x
let it denote by

$$c = f''(x+iy) + g''(x-iy)$$

Partially diffrenting ② wrt y. let it be denote by d

$$d = f''(x+iy)(i)^2 + g''(x-iy)(-i)^2$$

$$d = f''(x+iy)\,i^2 + g''(x-iy)\,i^2$$

$$d = i^2(f''(x+iy) + g''(x-iy))$$

$$d = (\sqrt{-1})^2 c$$

$$\boxed{d = -c}$$

Ques 8

$$\frac{dr}{d\theta} = \frac{r^2}{\theta} \qquad r(1) = 2$$

$r(1) = 2$

$\theta = 1,\ r = 2$

$$\int r^{-2}\,dr = \int \frac{d\theta}{\theta}$$

$0 = \log(1) + \frac{1}{2} +$

$C = -\frac{1}{2}$

$$\frac{r^{-1}}{-1} = \log|\theta| + c$$

$$-\frac{1}{r} = \log|\theta| + c$$

$$\boxed{0 = \log|\theta| + \frac{1}{r} + c}$$

$$\log|\theta| + \frac{1}{x} = -\frac{1}{2}$$

Ques 9 $$\frac{dv}{dt} = 9.8 - 0.196V$$

$$\int \frac{dv}{9.8-0.196V} = \int dt$$

$$\frac{1}{9.8}\int \frac{dv}{1-0.02V} = t+C$$

$$1-0.02V = A$$

$$-0.02 = \frac{dA}{dv}$$

$$dv = \frac{dA}{-0.02}$$

$$\frac{1}{9.8(-0.02)}\int \frac{dA}{A} = t+C$$

$$-\frac{10}{98} \times \frac{\cancel{100}^{50}}{\cancel{2}} \log|A| = t+C$$

$$\frac{-500}{98}\log|1-0.02V| = t+c$$

Ques10 $\Rightarrow$ $t\frac{dy}{dt} + 2y = t^2 - t + 1$

divide whole equation by t

$$\frac{dy}{dt} + \frac{2}{t}y = t - 1 + \frac{1}{t}$$

$$\frac{dy}{dt} + Py = Q$$

$$IF = e^{\int P dt}$$
$$= e^{\int \frac{2}{t} dt}$$
$$= e^{2\int \frac{1}{t} dt}$$
$$= e^{2\log|t|}$$
$$= e^{\log t^2}$$
$$= t^2$$

$$y \times IF = \int Q \times IF \, dt$$
$$t^2 y = \int t^2 \left(t - 1 + \frac{1}{t}\right) dt$$

$$yt^2 = \int (t^3 - t^2 + t) \, dt$$

$$yt^2 = \frac{t^4}{4} - \frac{t^3}{3} + \frac{t^2}{2} + c$$

$t = 1,\ y = 1/2$

$$1/2 = 1/4 - 1/3 + 1/2 + c$$
$$1/2 = 3/4 - 1/3 + c$$

$1/2 + 1/3$

$\frac{3+2}{6}$

$5/6$

$$\frac{5}{6} - \frac{3}{4} = c$$

$$\frac{20-18}{24} = c$$

$$\frac{2}{24} = c$$

$$\frac{1}{12} = c$$

$$yt^2 = \frac{t^4}{4} - \frac{t^3}{3} + \frac{t^2}{2} + c$$

let $c = \frac{1}{12}$

$$yt^2 = \frac{t^4}{4} - \frac{t^3}{3} + \frac{t^2}{2} + \frac{1}{12}$$

$$\boxed{12yt^2 = 3t^4 - 4t^3 + 6t^2 + 1}$$

Ques 11 $$\frac{dy}{dx} = \frac{xy^3}{\sqrt{1+x^2}}$$

$$\frac{dy}{y^3} = dx \frac{x}{\sqrt{1+x^2}}$$

$$\int y^{-3}\, dy = \int \frac{x}{\sqrt{1+x^2}}\, dx$$

$$\frac{-y^{-2}}{-2} = 2\sqrt{1+x^2} + C$$

$x = 0,\ y = -1$

$$-1 = 2 + C$$

$$C = -3$$

$$2y^2\sqrt{1+x^2} - 3y^2 + 1 = 0.$$

$\sqrt{1+x^2}$ is continious at all points

Thus, the intertal of validtly $(-\infty, \infty)$

Ques 12 ⇒

$f(x) = x^3 - 10x^2 + 6$ for $x = 3$

$f(3) = 27 - 90 + 6$

$f(3) = -57$

$f'(x) = 3x^2 - 10x \times 2$

$f'(3) = 3(9) - 60$

$f'(3) = 27 - 60$

$f'(3) = -33$

$$f''(x) = 6x - 20$$

$$f''(3) = 18 - 20$$

$$= -2$$

$$f'''(x) = 6$$

Taylor series

$$f(x) = f(3) + (x-3)f'(3) + \frac{(x-3)^2}{2!}f''(3) + \frac{(x-3)^3}{3!}f'''(3)$$

$$+ \frac{(x-3)^4}{4!}f''''(3)$$

$$f(x) = -57 + (x-3)(-33) + \frac{(x-3)^2}{\cancel{2}}(-\cancel{2}) + \frac{(x-3)^3}{\cancel{6}} \times \cancel{6} + 0$$

$$= -57 - 33x + 99 + x^2 + 9 - 6x + (x-3)^3$$

Ques 13 $f(0) = 1$

$f(x) = (1+x)^u$

$f'(x) = u(1+x)^{u-1}$

$f'(0) = u(1+0)^{u-1}$

$f'(0) = u$

$f''(x) = u(u-1)(1+x)^{u-2}$

$f''(0) = u(u-1)(1+0)^{u-2}$

$f''(0) = u(u-1)$

$f'''(x) = u(u-1)(u-2)(1+x)^{u-3}$

$f'''(0) = u(u-1)(u-2)$

$f''''(x) = u(u-1)(u-2)(u-3)(1+x)^{u-4}$

$f''''(0) = u(u-1)(u-2)(u-3)$

→ MACLAURIN SERIES

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2} f''(0) + \frac{x^3}{6} f'''(0) + \frac{x^4}{4!} f''''(0) + \cdots$$

$$= u + xu + \frac{x^2}{2} u(u-1) + \frac{x^3}{6} u(u-1)(u-2) + \frac{x^4}{4!} u(u-1)(u-2)(u-3) \cdots$$

$(1+x)^{1/2}$

Ques 14

$$f(x) = \sqrt{1+x}$$

$$f(0) = \sqrt{1+0}$$

$$f(0) = \underline{1}$$

$$f'(x) = \frac{1}{2\sqrt{1+x}}$$

$$f'(0) = \frac{1}{2}$$

$$f''(x) = -\frac{1}{4(1+x)^{3/2}}$$

$$f''(0) = -\frac{1}{4}$$

$$f'''(x) = \frac{3}{8(1+x)^{5/2}}$$

$$f'''(0) = \frac{3}{8}$$

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \cdots$$

$$= 1 + \frac{x}{2} + \frac{x^2}{2}\left(-\frac{1}{4}\right) + \frac{x^3}{6_2}\left(\frac{3}{8}\right) + \cdots$$

$$= 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} + \cdots$$

Ques 15 $f(x) = e^{-6x}$

$f(-4) = e^{24}$

$f'(x) = e^{-6x} \times (-6)$

$f'(-4) = -6e^{24}$

$f''(x) = 36e^{-6x}$

$f''(-4) = 36e^{24}$

$f'''(x) = 36e^{-6x} \times (-6)$

$f'''(-4) = -216e^{24}$

$$f(x) = f(-4) + (x+4) f'(x) + \frac{(x+4)^2}{2} f''(x)$$

$$+ \frac{(x+4)^3}{3!} f'''(x) + \cdots$$

$$= e^{24} + (x+4)(-6e^{24}) + \frac{(x+4)^2}{2}(36e^{24}) - (-216e^{24})\frac{(x+4)^3}{3!}$$

Ques16 $\Rightarrow$ $f(x) = \ln(4x+3)$

$f(0) = \ln(3)$

$$f'(x) = \frac{1}{4x+3}$$

$$f'(0) = 1/3$$

$$f''(x) = \frac{-4}{(4x+3)^2}$$

$$f''(0) = \frac{-4}{9}$$

$$f'''(x) = \frac{-4 \times -2 \times 4}{(4x+3)^3}$$

$$f'''(x) = \frac{32}{(4x+3)^3}$$

$$f'''(0) = \frac{32}{9}$$

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2} f''(0) + \frac{x^3}{\cancel{3}\,3} f'''(0) + \cdots$$

$$f(x) = \ln(3) + \frac{x}{3} + \frac{x^2}{\cancel{2}}\left(\frac{\cancel{-4}^{\,-2}}{9}\right) + \frac{x^3}{3}\,\frac{32}{9} + \cdots$$

$$f(x) = \ln(3) + \frac{x}{3} - \frac{2}{9}x^2 + \frac{32}{27}x^3 + \cdots$$