FINANCIAL MATHEMATICS ASSIGNMENT

Q1) i) $APR = \dfrac{\text{average annual interest}}{1/2 \times \text{original loan}}$

$= \dfrac{2 \times \text{avg. interest}}{\text{original loan}}$

original amt = 20,000/-

$\therefore 20{,}000 = 12 \times 427.90\, a_{\overline{5}|}^{(12)} \Rightarrow 3.8499.$

$\text{flat Rate} = \dfrac{\text{total interest}}{\text{loan} \times \text{total years}}$

$= \dfrac{(5 \times 12 \times 427.90 - 20{,}000)}{20{,}000 \times 5}$

$= 5.67\%$

~~AR~~ $APR \approx 2 \times 5.67 = 11\%$.

$i = 11\%,\ a_{\overline{5}|}^{(12)} = 3.87872.$

$i = 10\%,\ a_{\overline{5}|}^{(12)} = 3.96154$

Interpolating APR, I is $\dfrac{i - 10\%}{3.8499 - 3.961} = \dfrac{11\% - 10\%}{3.8499 - 3.961}$

$i = 10.8\%$

At $i = 10.8\%$, $12 \times 427.90\, a_{\overline{5}|}^{(12)} = 20{,}000.2$.

At $i = 10.~~8~~9\%$, $12 \times 427.90\, a_{\overline{5}|}^{(12)} = 19958.2$.

$i = 8\%$ is more close.

$\therefore$ APR is ~~10%~~ 10.8%

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(ii) APR = 10.8%

After 1 year, capital left to ~~play~~ pay,

$12 \times a_{4}^{(12)} @ 10.8 \times 427.90$

$= 16775.98$

$16775.98 = 12 \times 274.49 \, a_{t}^{(12)}$

$16775.98 = \dfrac{3293.88(1 - 1.108^{-t})}{12 \times (1.108^{1/12} - 1)}$

$1.108^{-t} = 0.4754$

$t = 7.25$ years.

iii) total int $= (4 \times 12 \times 427.90) - (16775.98)$

$= 3763.22$

term $= 7.25$ years.

total int $= (7.25 \times 12 \times 274.9) - 16775.98$

$= 7104.65$

Q21) i) Discounted payback period :-

It is the time for which the present value of the payment up to time t exceeds the present value of the cost upto time 't'.

b) Payback period :-

It is the same as the discount payback period, except for the present value calculation is carried out using interest rate of 0.

Q3) a) IRR for any project is the rate of interest that satisfies the ~~project value~~ equation of value.

$$-170000 - 20{,}000v - 10{,}000v^2 + 20{,}000v + 20{,}000v^2 + 200{,}000v^2 = 0$$

$$10v^2 + 200v^3 = 170$$
$$10v^2 + 200v^3 - 170 = 0$$

$0.9 \rightarrow -16.1$
$1 \rightarrow 40$
$0.93 \rightarrow -0.479$
$0.94 \rightarrow 4.9538$
$0.9309 \rightarrow 0.0046$
$\therefore v = 0.9309.$

$$\frac{1}{1+i} = 0.9309$$

$i = 0.0742292 4055$
$i = 7.422921\% = 7.4\%$
$\therefore$ IRR of project B is exactly 74%.

b) Net Present value :-
The net present value are found by ~~joining~~ discounting payments at 6%.

Project A $= -170{,}000 + 100{,}00v^2 + 200000v^3$
$= 6823.82$

Project B $= -200{,}000 + 140000a_{\overline{6}|} + 200000v^6 = 9834.45$

c) To be profitable, Project A requires borrowing rate less than 7.42292% and project B requires a rate lower than 7%.

20%

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Q3) at 1/11/1985

Annuity 1:- $200\, v^{(1/4)} \times \ddot{a}_{22}$ at 8%

$v^{1/4} = 0.980944$

$a_{\overline{22}|} = 10.2007$

$d = 0.074074$

$= 200 \times v^{1/4} \times a_{\overline{22}|} \times \frac{i}{d}$

$= 200 \times 0.980944 \times 10.2007 \times \frac{0.08}{0.074}$

$= 2161.366301$

Annuity 2:- $320(1+i)^{1/12} + a^{(4)}_{\overline{16.25}|}$ @ 8%

$(1+i)^{1/12} = 1.006434$

$v = 0.9259261$

$i^{(4)} = 0.077706$

$\cancel{16}$ 16.26

$= 320 \times 1.00643 \times \frac{1-(0.925926)}{0.877706}$

$= 2957.870081$

Annuity 3:- $180 \times a^{(12)}_{\overline{18.75}|}$

$i^{(12)} = 0.077208$

$v = 0.925926$

$= 180 \times \frac{1-(0.925926)^{18.75}}{0.077208}$

$= 180 \times 9.892578$

$= 1780.664166$

Value of revised annuity:-

$(1+i)^{1/4} + a^{(2)}_{\overline{21.5}|}(1+i)^{1/4}$

$A = \frac{6889.90}{1.019427 \times 10.30886}$

$= 656.56.$

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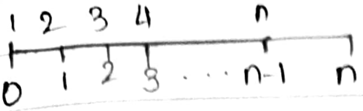
Q4) $(I\ddot{a})_{\overline{n}|}$ for $i>0$



$$(I\ddot{a})_{\overline{n}|} = 1+2v+3v^2+4v^3+\dots\dots nv^{n-1} \quad \text{---} \; (1)$$

$$(I\ddot{a})_{\overline{n}|} = (1+i)(Ia)_{\overline{n}|}$$

$$= (1+i)\frac{\ddot{a}_{\overline{n}|} - nv^n}{i}$$

$$= \frac{\ddot{a}_{\overline{n}|} - nv^n}{\frac{i}{1+i}}$$

$$\boxed{I\ddot{a}_{\overline{n}|} = \frac{\ddot{a}_{\overline{n}|} - nv^n}{d}}$$

Q5) —

Q6) Let X be the initial annual repayment.

$160{,}000 = X a_{\overline{10}|}$ @ 8%.

$$X = \frac{160{,}000}{a_{\overline{10}|}} = \frac{160{,}000}{6.7101} = 23844.65$$

ii) Loan outstanding immediately after 4^{th} payment

$23844.65 a_{\overline{6}|}$ @ 8%

$= 23844.65 \times 4.6229 = 110{,}231.442$

Let 'p' be the revised annual instaument.

$p a_{\overline{6}|} = 110231{,}442$ @ 10%

$$P = \frac{110231.442}{a_{\overline{6}|}} = \frac{110231.442}{4.3533} = 25309.72$$

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22) III) Loan outstanding immediately after 7th payment

$25309.72\, a_{\overline{3}|} @ 10\%$

$= 25309.72 \times 2.44869 = 62942.74$

Let z be the revised annual installment

$z\, a_{\overline{3}|} = 62942.74 @ 9\%$

$$z = \frac{62942.74}{a_{\overline{3}|}} = \frac{62942.74}{2.5313}$$

$= 24865.77$

Yeild :-

$160{,}000 + 1465.07\, a_{\overline{4}|} - 443.95\, a_{\overline{7}|} - 24865.77\, a_{\overline{10}|} = 0$

$160{,}000 + 1465.07\left(\frac{1-v^4}{i}\right) - 443.95\left(\frac{1-v^7}{i}\right) - 24865.77\left(\frac{1-v^{10}}{i}\right)$

Since, $v = \frac{1}{1+i}$

~~160,000 + 1465.07~~ —

Q7) The return on property is lower than that in government security. It is difficult to invest in property.

→ Less flexible because of large unit size.

Large buying and securing expenses.

Tender refers to a process wherein the government and financial institutions invite bids for large projects that must be submitted within a finite deadline.

→ Advantages :-
The government will know the price that the investor will pay at outset so it will know the cost of borrowing money.

→ Disadvantages :-
Investor may be willing to pay more for the bond than the set price and hence the money could be borrowed at a lower cost.

The securities with uncertain returns are as follows :-

→ Equity :- Rent payments may be subject to review

→ Index-linked bonds :- coupon payment and final redemption price may increase.

Q8) PV of outlay $= 2.7 + a_{\overline{3}|} @ i$ — (1)

PV of income $= 1.03v^3 + \bar{a}_{\overline{10}|}(0.25v + 0.2v^2 + 0.1v^3) @ i$ — (2)

equating eq (1) & (2),

$2.7 + a_{\overline{3}|} @ i = 1.03v^3 + \bar{a}_{\overline{10}|}(0.25v + 0.2v^2 + 0.1v^3) @ i$

(ii) $PV = 0.1v^3 + 0.85\bar{a}_{\overline{10}|}v^3$

$\Rightarrow 0.1v^3 + 0.85\bar{a}_{\overline{10}|} - 2.7 - 0.2a_{\overline{3}|} @ i = 10\%$

$\Rightarrow (0.1 \times 0.7513) + (0.85 \times 0.7513 \times 6.4469) - 2.70 - 0.2 \times 2.0966$

$\Rightarrow 4.19 - 3.19$

$\Rightarrow 1.0088$

$\because NPV > 0$, it is worth to invest.

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Q9) $(1+i)^3 = \frac{33}{30.50} \times \frac{41.05-4.5}{33+6} \times \frac{45.6}{41.05} \times \frac{47}{45.6-2.50}$

$(1+i)^3 = 1.2271326$

$(1+i) = 1.07095128$

$i = 0.070951281$

$\therefore$ TWRR $= 7.0951281\%$

ii) Linked rate of Return:-

$(30.5)(1+i) + 6(1+i)^{0.25} = 38.50$

Let $(1+i) = y$.

$30.5y + 6y^{0.25} = 38.50$

2011

$1 \rightarrow -2$

$1.1 \rightarrow 1.19468213$

$1.06 \rightarrow -0.081$

$1.07 \rightarrow 0.2373$

$1.062 \rightarrow 0.018$

$1.063 \rightarrow 0.138$

$1.0625 \rightarrow -0.00020044$

$1.0626 \rightarrow 0.00107288$

$y = 1.0626$

$1+i = 1.0626$

$i = 0.0626$ or 6.26%

Year 2012 :-

$(38.50)\ 1+i + 4.50(1+i)^{0.5} - 45$

Let $(1+i) = x$

$38.50x + 4.50x^{0.5} - 45 = 0$.

$1 \rightarrow -2$

$1.1 \rightarrow 2.0696$

$1.04 \rightarrow -0.37$

$1.05 \rightarrow 0.0361$

$1.049 \rightarrow -0.004568$

$1.0492 \rightarrow 0.003570889$

$x = 1.0492$

$1+i = 1.0492$

$i = 0.0492$ or 4.92%

Year 2013 :-

$45(1+i) - 2.5(1+i)^{0.5} = 47$

$45x - 2.5x^{0.5} = 47$

$1.1 \to -0.122$ $\quad$ $1.11 \to 0.310$ $\quad$ $1.103 \to 0.00940$

$1.2 \to 4.2013$ $\quad$ $1.102 \to -0.034404$

$1.1028 \to 0.000642288$

$x = 1.10279$

$\therefore 1+i = 1.10279$

$i = 0.10279$ or 10.279%

Linked Rate of return.

$(1+i)^3 = (1.0626)(1.0492)(1.010279)$

$(1+i)^3 = 1.229478427$

$1+i = 1.071289803$

$i = 0.071259803$ or 7.1259803%

Q10) loan $= 180{,}000\, a_{\overline{15}|} + 20000 (Ia)_{\overline{15}|}$ @ 12%

$\ddot{a}_{\overline{n}|} = a_{\overline{n}|} \cdot \frac{i}{d}$

$i = 0.12$ $\quad$ $d = 0.107143$

$a_{\overline{15}|} = 6.8109$

$v = 0.892857$

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$= 180{,}000 \times 6.8109 + 200{,}00 \left[\frac{\ddot{a}_{\overline{n}|} - 15v^{15}}{i} \right]$

$= 2040588$

total cost of house $= \frac{2040588}{0.8} = 2550735$

ii) loan at beginning of 9th year:-

$L = 180000v + 180{,}000v^2 + \ldots 180000v^7 + \{9 \times 2000 \cdot v\} + \{10 \times 20000v^2\} \ldots\ldots (15 \times 20000v^7)$

$L(9) = 1875{,}850.33$.

iii) Let the new instalment be Rs P.

$1569866.65 = P\, a_{\overline{5}|}$ @ 10%

$P = 414126.87$

extra payment ⇒ total installment − loan outstanding

$(1400000 + 420{,}000 + 440{,}000 + 460{,}000 + 480{,}000) - 15{,}69{,}366.65$

$= 630133.36$

11) i) Time weighted rate of return:-

$$(1+i)^2 = \frac{\cancel{500}}{\cancel{460}} \times \left(\frac{550+150}{500+140}\right) \times \frac{x}{\cancel{550}}$$

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$$(1+i)^2 = \frac{600x}{46 \times 540 \times 11}$$

$(1+i)^2 = 1.44$

$x = 655.776$

ii) Money weighted rate of return:-

$460(1+i)^2 + 40(1+i)^{7/4} - 50(1+i) = 655.776$

Let $1+i = x$

$460x^2 + 40x^{7/4} - 50x - 655.776 = 0$

$1.1 \rightarrow -106.925622$

$1.2 \rightarrow 1.6575$

$1.19 \rightarrow -9.6356$

$0.198 \rightarrow 0.0609$

$0.199 \rightarrow 0.5237$

$1.1985 \rightarrow -0.0427895$

$1.19854 \rightarrow 0.0025233$

$\therefore (1+i) = 1.19854$

$i = 0.19854$

$i = 19.85\%$

(iii) —.

(iv) When the sub intervals chosen for calculating LIRD coincide with the interval between the net cashflow being recieved then the linked internal rate of return will be identical to the time weighted rate of return.

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(v) 1) TWRR is a better measure for fund manager's performance.
2) MWRR is sensitive to amount & timing of net cashflows, but fund managers do not have control over these timings of cashflows.
3) TWRR is calculated using growth factors.
Hence, TWRR is better measure of fund managers performance than MWRR

Q12) The discounted payback period of an investment project is the first time when the net present value of cash flow from project is +ve.

(ii) a) $-800(1+i)^t - 50(1+i)^{t-1} + 100\, \bar{s}_{\overline{t-2}|}$ @ 7% = 0.

$$-800(1.07)^t - 50(1.07)^{t-1} + 100 \times \frac{(1.07)^{t-2} - 1}{\ln(1.07)} = 0$$

$t \to 17$	$\to -74.79$	17.76	$\to -0.757$.
18	$\to 23.4626$	17.77	$\to 0.2426$
17.7	$\to -6.741$	17.767	$\to -0.057$.
17.8	$\to 3.24632$.	17.768	$\to 0.0426$

$\therefore t = 17.767$ years.

iii) $$100\, \bar{s}_{\overline{1}|} = 100 \times \frac{(1.06)^1 - 1}{\ln(1.06)}$$

$= 100 \times 1.0297080+2$

$= 102.9708672$

the DPB is smallest integer,

$-800(1+i)^t - 50(1+i)^{t-1} + 102.97085\, s_{\overline{t-2}|}$ @ 7% > 0

$t \to 17 \to -87.1$

$18 \to 9.7866$ $\therefore t = 18$ years.

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$\therefore$ the balance $= 9.7866$

$AV(0,22) = 9.7866(1+i)^4 + 100\, s_{\overline{4}|}$ @ 6%

$12.34571169 + 450.45564$

$s_{\overline{4}|} = 4.3746 \qquad \delta = 0.058269$

$= 462.803571$

Q13) let $X/12$ = monthey repayment.

$X a_{\overline{25}|}^{(12)} = 9880$ @ 7%

$X a_{\overline{25}|} \times \frac{i}{i^{(12)}} = 9880$

$X(11.6536) \times \frac{0.07}{0.67850} = 9880 \qquad i^{(12)} = 0.067850$

$a_{\overline{25}|} = 11.6536$

$X/12 = 68.4805758$

$\therefore$ Monthey repayment $= 6848.05758$

III) loan just apter repayment on 10th september is made $X a_{83/6}$ @ 7%

loan on 10th october 2026 is made $X a_{55/4}^{(12)}$ @ 7%

capital repayment :-

$X\left[a_{83/6}^{(12)} - a_{55/4}^{(12)}\right]$ @ 7%

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$821.7669096(0.03268447611)$
$= 26.85902093.$

iv). capital repayment in the 2 years.

$$X\left[a^{(12)}_{\overline{22/3}|} - a^{(12)}_{\overline{19/3}|}\right] @ 7\%$$

$$821.7669096\left[\frac{v^{19/3} - v^{22/3}}{i^{(12)}}\right]$$

$= 516.1973861$

ii) total interest $= 821.7669096 - 516.1973861$
$= 305.5695235$ //.