Assignment - I

1) $X \sim P(\theta)$

$\frac{X - \theta}{\sqrt{\theta}} \sim N(0,1)$

$\hat{\theta} = 200$, $n = 1$

95 % CI = $\bar{x} \pm Z_{\alpha/2} \sqrt{\frac{\theta}{n}}$

$= 200 \pm 1.96 \sqrt{\frac{200}{1}}$

$= (172.2814, 227.7185)$

2)

i) $\bar{x} = \frac{\sum x_i}{n}$

$E(\bar{x}) = E\left(\frac{\sum X_i}{n}\right) = \frac{1}{n} E(\sum x_i) = \frac{1}{\cancel{n}} \times \cancel{n} \times \mu = \mu$

$V(\bar{x}) = V\left(\frac{\sum x_i}{n}\right) = \frac{1}{n^2} V(\sum X_i) = \frac{n \sigma^2}{n} = \frac{\sigma^2}{n}$

Hence proved

ii) To prove : $E(S^2) = \sigma^2$

$S^2 = \frac{1}{n-1} \sum (X_i - \bar{X})^2$

$E(S^2) = E\left(\frac{1}{n-1} \sum (X_i - \bar{x})^2\right) = \frac{1}{n-1} E(\sum (X_i - \bar{x})^2$

$$= \frac{1}{n-1}\left[\sum E(x_i)^2 - n E(\bar{x}^2)\right]$$

$$= \frac{1}{n-1}\left[\sum(\mu^2 + \sigma^2) - n\left(\mu^2 + \sigma^2/n\right)\right]$$

$$\boxed{\begin{aligned} E(x_i)^2 &= V(x) + [E(x)]^2 \\ E(\bar{x})^2 &= V(\bar{x}) + [E(\bar{x})]^2 \end{aligned}}$$

$$= \frac{1}{n-1}\left(\cancel{n\mu^2} + n\sigma^2 - \cancel{n\mu^2} + \sigma^2\right)$$

$$= \sigma^2 \frac{\cancel{(n-1)}}{\cancel{(n-1)}}$$

$$= \sigma^2$$

Sol) $\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}$

$$V\left[\frac{(n-1)S^2}{\sigma^2}\right] = V(\chi^2_{n-1})$$

$$\Rightarrow \frac{(n-1)^2 V(S^2)}{\sigma^4} = 2\cancel{(n-1)}$$

$$V(S^2) = \frac{2\sigma^4}{n-1}$$

Sol) $X_i \sim N(\mu, \sigma_2)$

$\bar{X} \sim N(\mu, \sigma^2/n)$

$$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}$$

$\sigma^2 = 100, \; n = 10$

$E(S^2) = \sigma^2 = 100$

Value of S^2 can plus or minus 50% of $E(S)$

$S \pm 50$.

$\therefore P(S^2 \le 50)$

$$= P\left[\frac{(n-1)S^2}{\sigma^2} \le \frac{50 \times 9}{100}\right] = P(\chi^2_9 < 4.5)$$

$$= 0.1245$$

$$P(S^2 \le 150) = P\left[\frac{(n-1)S^2}{\sigma^2} \le \frac{150 \times 9}{100}\right] = P(\chi^2_9 \le 13.5)$$

$$= 0.8587$$

$$P(150 \le S^2 \le 50) = 0.8587 - 0.1245$$

$$= 0.7342$$

3)

No. of claims	No. of policies	$\hat{p}(\text{claim} = x_i)$ (O_i)	E_i	$(O_i - E_i)^2 / E_i$
0	86	$86/180 = 0.4778$	0.4919	0.0004041
1	75	$75/180 = 0.4167$	0.3818	0.00319
2	16	$16/180 = 0.0889$	0.1111	0.004436
3	2	$2/180 = 0.0111$	0.0144	0.00075625
4	1	$1/180 = 0.0055$	0.0006972	0.03308
				0.04187

$$L(p) = \left({}^4C_0 \cdot p^0 \cdot (1-p)^4\right)^{86} \left({}^4C_1 \cdot p^1 (1-p)^3\right)^{75} \left({}^4C_2 \cdot p^2 (1-p)^2\right)^{16}$$
$$\left({}^4C_3 \cdot p^3 \cdot (1-p)^1\right)^2 \cdot \left({}^4C_4 \cdot p^4 \cdot (1-p)^0\right)^1$$

$$= \left(1 \cdot 1 \cdot (1-p)^4\right)^{86} \cdot \left(4 \cdot p \cdot (1-p)^3\right)^{75} \cdot \left(6 \cdot p^2 \cdot (1-p)^2\right)^{16} \cdot$$
$$\left(4 \cdot p^3 \cdot (1-p)\right)^2 \left(1 \cdot p^4\right)^1$$

$$= \cancel{96 \, p^{10} \, 2(1-p)^{10}}.$$

$$= \left[(1-p)^{344}\right] \cdot \left[4^{75} \cdot p^{75} \cdot (1-p)^{225}\right] \cdot \left[6^{16} \cdot p^{32} \cdot (1-p)^{32}\right]$$

$$= \left[4^2 \cdot p^6 \cdot (1-p)^2\right] \left[(p^4)\right]$$

$= 4^{77} \cdot 6^{16} \cdot p^{117} \cdot (1-p)^{603}$

$\ln(L(p)) = c + 117 \ln(p) + 603 \ln(1-p)$

$\dfrac{d \ln(L(p))}{dp} = 0 + \dfrac{117}{p} + \dfrac{603}{1-p}$

$\dfrac{117}{p} - \dfrac{603}{(1-p)} = 0$

$117 - 117p - 603p = 0$

$720p = 117$

$p = 0.1625$

$\therefore \chi^2_{cal} = \cancel{0.04187}$

$\cancel{\chi^2_1}$

5 i) $X \sim U(a,b)$

$E(X) = \bar{X}$

$\frac{a+b}{2} = \bar{X} \Rightarrow a+b = 2\bar{X} \Rightarrow \hat{b} = 2\bar{X} - \hat{a}$

$E(X-\bar{X})^2 = S^2$

$\frac{1}{12}(b-a)^2 = S^2$

$S = \sqrt{\frac{(b-a)^2}{12}}$

$= \frac{\hat{b}-\hat{a}}{\sqrt{12}}$

$S = \frac{2\bar{X} - \hat{a} - \hat{a}}{2\sqrt{3}}$

$S = \frac{\cancel{2}(\bar{X} - \hat{a})}{\cancel{2}\sqrt{3}}$

$\hat{a} = \bar{X} - \sqrt{3}S \Rightarrow$ Hence prove.

ii) $\hat{b} = 2\bar{X} - \hat{a}$

$= 2\bar{X} - (\bar{X} - \sqrt{3}S)$

$= 2\bar{X} - \bar{X} + \sqrt{3}S$

$= \bar{X} + \sqrt{3}S$

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i) $\bar{x}_{Public} = 30$

$\bar{x}_{Private} = 35.055$

$s^2_{Public} = \frac{\Sigma(x-\bar{x})^2}{n-1} = 64.6527$

$s^2_{Private} = \frac{\Sigma(x-\bar{x})^2}{n-1} = 67.4027$

ii) Primary condition that needs to be true for testing equal means is that the sample size is same

The condition is satisfied in above example

iii) $H_0 = \bar{X}_{Private} - \bar{X}_{Public} = 0$

$H_1 = \bar{X}_{Private} - \bar{X}_{Public} > 0$

$Z_{tab} = (1.96, -1.96)$

$$Z_{cal} = \frac{(\bar{X}_1 - \bar{X}_2) - 0}{\sqrt{\frac{S^2}{n}}} = \frac{35.055 - 30}{\sqrt{\frac{67.4027 + 64.6527}{9}}}$$

$$= \frac{5.055}{3.8305}$$

$$= 1.3196$$

$\therefore Z_{tab} < Z_{cal} < Z_{tab}$

$\therefore$ Do not reject H_0

$\therefore$ mean of public hospital is not higher than private

9) i) $t_k = \dfrac{N(0,1)}{\sqrt{\left(\dfrac{\chi^2_k}{k}\right)}}$

$N(0,1)$: Standard Normal

χ^2_k : chi-square with k degrees of freedom

k : ~~deg~~ Degrees of Freedom

ii) $E(t_k) = 0$

$V(t_k) = \dfrac{k}{k-2}$

iii) $n = 10$

~~$E(x) = 50$~~

~~$V(x) = 48.667$~~

~~$x \sim N(50, 48.667)$~~

~~$\dfrac{x-50}{48.667} \sim N(0,1)$~~

$\bar{X} = 50$

$S^2 = 48.667$

$\dfrac{\bar{x} - \mu}{S/\sqrt{n}} \sim t_{n-1}$

$P\left(\dfrac{\bar{x} - \mu}{S/\sqrt{n}} \geq x\right) = 0.005$

~~$P(t_9 \leq x) = 0.995$~~

$\therefore P\left(\dfrac{\bar{x} - \mu}{S/\sqrt{n}} \geq 3.169\right) = 0.005$

$\therefore \dfrac{50 - \hat{\mu}}{6.976/\sqrt{10}} \geq 3.169$

$50 - \hat{\mu} \geq 3.169 \times 2.206$

$\hat{\mu} \leq 50 - 6.991$

$\hat{\mu} \leq 43.00899$

99% CI

$= \bar{x} \pm t_{\alpha/2, n-1} \sqrt{\frac{s^2}{n}}$

$= 50 \pm t_{0.005, 9} \sqrt{\frac{12.662}{10}}$

$= (43.00899, 56.991)$

11) $\Sigma x = 213,200$

$\Sigma x^2 = 4,919,860,000$

$\bar{x} = 17,767$

$s = 10,144$

$n = 12$

new $\Sigma x = 213,200 - 11000 - 48000 + 27500 + 31500$

$= 213,200.$

new $\bar{x} = \frac{\text{new } \Sigma x}{n} = \frac{213,200}{12} = 17,767$

new $\Sigma x^2 = 4,919,860,000 - (11,000)^2 - (48000)^2 + (27500)^2 + (31$

$= 4,243,360,000$

$\bar{x}^2 = 315666289$

new $s^2 = \frac{\Sigma x^2 - \bar{x}^2}{n-1} = \frac{4243360000 - 315666289}{11}$

$= 357063065$

new $s = \sqrt{s^2} = 18896$

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We can see that by changing the numbers standard deviation has increased while the mean remain the same this indicates the variability has increased in data

(c)
i) Type 1 error is the probablity of rejecting H_0 when in fact it is true. In above case Type 1 error (α) is Probablity of rejecting $\hat{\lambda} = 3$ but in fact it is true.

ii) Type 2 error (β) is the probablity of accepting H_0 when it is actually false.

iii) The power of a test is the probablity of rejecting H_0 when it is false.
It is given by $1-\beta$.

8)
i) unbaised estimator $\Rightarrow$ $E(\hat{\theta}) = \theta$

ii) Bais $= E(\hat{\theta}) - \theta$

iii) $MSE(\hat{\theta}) = E[(\hat{\theta} - \theta)^2]$

$= E[\{\hat{\theta} - E(\hat{\theta})\} + \{E(\hat{\theta}) - \theta\}^2]$

$= E[(\hat{\theta} - E(\hat{\theta}))^2 + 2(E(\hat{\theta}) - \theta) \cdot E(\hat{\theta} - E(\hat{\theta}))$
$+ [E(\hat{\theta}) - \theta]^2$

$= var(\hat{\theta}) + 0 + bais^2(\hat{\theta})$

$= var(\hat{\theta}) + bais^2(\hat{\theta})$