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Assignment

Ans1) $\int_{1/50}^{2} \frac{e^{2/\omega}}{\omega^2} d\omega$

Put $\frac{2}{\omega} = t$

$\frac{-1}{\omega^2} d\omega = dt$

$\Rightarrow \int_{1/50}^{2} -e^t \, dt$

$= \left[\frac{-1}{2} e^{2/\omega}\right]_{1/50}^{2}$

$= \frac{-1}{2} e^1 - \left(-\frac{1}{2} e^{2/1/50}\right)$

$= \frac{-1}{2}\left(e - e^{100}\right)$

Ans2) $\int \frac{2t^3+1}{(t^4+2t)^3} dt$

Put $t^4+2t = x$

$4t^3+2 \, dt = dx$

$= \int \frac{2}{x^3} dx$

$= 2\int x^{-3} dx$

$= 2\frac{(x^{-3+1})}{(-3+1)}$

$= 2 \cdot \frac{-2}{x^2} = \frac{-4}{x^2} = \frac{-4}{(t^4+2t)^2}$

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Ans 3) $f'(x) = x^4 + 3x - 9$

$f(x) = \int f'(x)\,dx$

$\Rightarrow \int x^4 + 3x - 9$

$f(x) \Rightarrow \frac{x^5}{5} + \frac{3x^2}{2} - 9x$

or $\Rightarrow \frac{2x^5 + 15x^2 - 90x}{10}$

Ans 4) $f(x) = \begin{cases} 6 & x > 1 \\ 3x^2 & x \leq 1 \end{cases}$

$$\int_{-2}^{3} f(x)\,dx = \int_{-2}^{1} 3x^2\,dx + \int_{1}^{3} 6\,dx$$

$$= [x^3]_{-2}^{1} + [6x]_{1}^{3}$$

$$\Rightarrow [-8 - 1] + [18 - 6]$$

$$\Rightarrow -7 + 12$$

$$= 5$$

Ans 5) $\int 3(8y-1)e^{4y^2-y}\,dy$

Solving by substitution

$4y^2 - y = t$

$8y - 1\,dy = dt$

$= 3\int e^t\,dt \qquad = 3e^t$

$= 3\cancel{\left(\frac{1}{t}e^t\right)} \qquad = \underline{3(e^{4y^2-y})}$

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Ans 6) $f''(x) = 15\sqrt{x} + 5x^3 + 6$

$f'(x) = \int f''(x)\, dx$

$= \int 15\sqrt{x} + 5x^3 + 6\, dx$

$= \frac{15x^{3/2}}{3/2} + \frac{5x^4}{4} + 6x + C_1$

$= 10x^{3/2} + \frac{5x^4}{4} + 6x + C_1$

$f(x) = \int f'(x)$

$= \int 10x^{3/2} + \frac{5x^4}{4} + 6x + C_1$

$= \frac{10x^{5/2}}{5/2} + \frac{5}{4}\left(\frac{x^5}{5}\right) + \frac{6x^2}{2} + C_1x + C_2$

$= 4x^{5/2} + \frac{x^5}{4} + 3x^2 + C_1x + C_2$

$f(1) = -\frac{5}{4}$

$\Rightarrow 4(1) + \frac{(1)}{4} + 3(1) + C_1(1) + C_2 = -\frac{5}{4}$

$\Rightarrow 4 + \frac{1}{4} + 3 + C_1 + C_2 = -\frac{5}{4}$

$= \frac{16 + 12 + 1}{4} + C_1 + C_2 = -\frac{5}{4}$

$\Rightarrow \frac{29}{4} + C_1 + C_2 = -\frac{5}{4}$

$C_1 + C_2 = -\frac{34}{4}$ — (1)

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$= 128 + 256 + 42 + 4C_1 + C_2 = 404$

$4C_1 + C_2 = 404 - 426$

$4C_1 + C_2 = -22$ — ②

Subtracting ① from ②

$4C_1 + C_2 = -22$

$-C_1 - C_2 = +34/4$

$3C_1 = \frac{-54}{4}$

$C_1 = \frac{-54}{12}$

$C_1 = \cancel{-18} - \frac{9}{2} = -4.5$

$C_2 = -\cancel{25} - \frac{16}{4} = -4$

$f(x) = 4x^{5/2} + \frac{x^5}{4} + 3x^2 - 4.5x - 4$

$\frac{34}{4} - 22$

$\frac{34-88}{4} = -$

22
34
54

Ans 7) $p = f(x+iy) + g(x-iy)$

Differentiating w.r.t to x

$\frac{dp}{dx} = f'(x+iy)(1) + g'(x-iy)(1)$ — ①

Diff. w.r.t to x.

$\frac{d^2p}{dx^2} = f''(x+iy)(1) + g''(x-iy)$ — ②

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Diff. w.r.t to y

$$\frac{dp}{dy} = f'(x+iy)(i) + g'(x-iy)(-i) \quad \text{—— (3)}$$

Diff. w.r.t to y

$$\frac{d^2p}{dy^2} = f''(x+iy)(i^2) + g''(x-iy)((-i)^2)$$

$$\frac{d^2p}{dy^2} = i^2(f''(x+iy) + g''(x-iy))$$

From eq^n (2) we get

$$\frac{d^2p}{dy^2} = i^2\frac{d^2p}{dx^2} \Rightarrow \frac{d^2p}{dy^2} - i^2\frac{d^2p}{dx^2} = 0$$

Ans 8) $\frac{dr}{d\theta} = \frac{r^2}{\theta}$

$$\frac{dr}{r^2} = \frac{d\theta}{\theta}$$

$$\therefore \int\frac{dr}{r^2} = \int\frac{d\theta}{\theta}$$

$$= -\frac{1}{r} + c_1 = \log\theta + c_2 \quad \left(\text{Also } r = -\frac{1}{\log\theta}\right)$$

$$= \cancel{-\frac{1}{r} + \log}\ \log\theta + \frac{1}{r} + C = 0 \quad (C = c_2 - c_1)$$

$$r(1) = 2$$

$$\log\theta + 1 + C = 2$$

$$C = 1 - \log\theta$$

$$\Rightarrow \log\theta + \frac{1}{r} + 1 - \log\theta = 0$$

$$\Rightarrow \frac{1+r}{r} = 0$$

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Ans 9) $\frac{dv}{dt} = 9.8 - 0.196v$

$\Rightarrow \frac{dv}{9.8 - 0.196v} = dt$

$\Rightarrow \frac{dv}{dt} = 0.196(50 - v)$

$= \frac{dv}{50 - v} = 0.196\, dt$

$= \log(50 - v) + C_1 = 0.196t + C_2$

Ans 10) $ty' + 2y = t^2 - t + 1 \quad y(1) = \frac{1}{2}$

$\div$ by t on both sides.

$\frac{ty'}{t} + \frac{2y}{t} = \frac{t^2 - t + 1}{t}$

$y' + \frac{2}{t}(y) = t - 1 + \frac{1}{t}$

$P = \frac{2}{t}$ & $Q = t + \frac{1}{t} - 1$

$I.F = e^{\int \frac{2}{t} dt}$

$= e^{2\log t}$

$= t^2$

$y \times t^2 = \int t^2(t^2 - t + 1)$

$t^2 y = \int t^3 - t^2 + t$

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$$t^2y = \frac{t^4}{4} - \frac{t^3}{2} + \frac{t^2}{2} + C$$

$$y(1) = \frac{1}{2}$$

$$t^2(1) = \frac{t^4}{4} - \frac{t^3}{3} + \frac{t^2}{2} + C$$

$$\frac{1}{2} = \frac{t^2}{4} - \frac{t}{3} + \frac{1}{2} + \frac{C}{t^2}$$

$$\frac{1}{2} = \frac{1}{4} - \frac{1}{3} + \frac{1}{2} + \frac{C}{t^2}$$

$$\frac{1}{2} = \frac{5}{12} + \frac{C}{t^2}$$

$$\frac{3}{4} - \frac{1}{3}$$

$$\frac{9-4}{12}$$

$$\frac{1}{12} = \frac{C}{t^2}$$

$$\cancel{t^2 = 2C}$$

$$C = \frac{1}{12}$$

$$\Rightarrow \cancel{y}\ f(y) = \frac{t^2}{4} - \frac{t^3}{3} + \frac{t^2}{2} + \frac{1}{2}$$

Ans 11) $y' = \dfrac{xy^3}{\sqrt{1+x^2}}$ $\quad y(0) = -1$

$$\frac{dy}{dx} = \frac{xy^3}{\sqrt{1+x^2}}$$

$$\int \frac{dy}{y^3} = \int \frac{x}{\sqrt{1+x^2}}\, dx$$

Let $1+x^2 = t$

$2x\,dx = dt$

$$\int y^{-3}\, dy = \int \frac{\cancel{2}}{\sqrt{t}} \frac{dt}{2}$$

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$= \cancel{\frac{-1}{-2y^2} + C_1 = \frac{y}{\sqrt{t}} + C_2}$

$= \frac{-1}{-2y^2} + C_1 = \sqrt{t} + C_2$

$= \frac{-1}{2y^2} - \sqrt{1+x^2} + C = 0 \qquad (C = C_1 - C_2)$

We know $y(0) = -1$

$y^2 = \frac{-1}{2\sqrt{1+x^2}} + C$

$(-1)^2 = \frac{-1}{2\sqrt{1}} + C$

$1 = -\frac{1}{2} + C$

$C = \frac{3}{2}$

$\Rightarrow y^2 = \frac{-1}{2\sqrt{1+x^2}} + 3/2$

Ans 12) Taylor series

$f(x) = f(a) + \frac{(x-a)^1}{1!} f'(a) + \frac{(x-a)^2}{2!} f''(a) + \frac{(x-a)^3}{3!} f'''(a)$

$f(x) = x^3 - 10x^2 + 6 \qquad f(3) = -57$

$f'(x) = 3x^2 - 20x \qquad f'(3) = -33$

$f''(x) = 6x - 20 \qquad f''(3) = -2$

$f'''(x) = 6 \qquad f'''(3) = 6.$

$f(x) = -57 + (x-3)(-33) + \frac{(x-3)^2}{2}(-2) + \frac{(x-3)^3}{6} \times 6$

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$$f(x) = -57 - (33x + 99) - (x^2 + 9 - 6x) + (x^3 + 792 + 27 \ldots 243x)$$

Ans 13) $(1+x)^{\mu}$

$$f(x) = f(0) + \frac{x^1}{1!} f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0)$$

$f(x) = (1+x)^{\mu}$ $\quad f(0) = 1$

$f'(x) = \mu(1+x)^{\mu-1}$ $\quad f'(0) = \mu$

$f''(x) = \mu(\mu-1)(1+x)^{\mu-2}$ $\quad f''(0) = \mu^2 - \mu$

$f'''(x) = \mu(\mu-1)(\mu-2)(1+x)^{\mu-3}$ $\quad f''(0) = \mu^3 - 3\mu^2 +$

$$\cancel{f(x) = 0 + x(1) + \frac{x^2}{2}(\mu) +}$$

$$f(x) = 0 + x(\mu) + \frac{x^2}{2}(\mu^2 - \mu) + \frac{x^3}{6}(\mu^3 - 3\mu^2 +$$

$$= \mu x + \left(\frac{x^2\mu^2}{2} - \frac{x^2\mu}{2}\right) + \frac{x^3\mu^3}{6} - \frac{x^3 3\mu}{6} + \frac{2\mu}{6}$$

Ans.14) $f(x) = \sqrt{1+x}$ $\quad f(0) = 1$

$f'(x) = \frac{1}{2\sqrt{1+x}}$ $\quad f'(0) = \frac{1}{2}$

$f''(x) = \frac{1}{-4(\sqrt{1+x})^{3/2}}$ $\quad f''(0) = -\frac{1}{4}$

$f'''(x) = \frac{1}{8(\sqrt{1+x})^{5/2}}$ $\quad f'''(0) = \frac{1}{8}$

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$$f(x) = 0 + \frac{x}{1!}\left(\frac{1}{2}\right) + \frac{x^2}{2!}\left(\frac{-1}{4}\right) + \frac{x^3}{3!}\left(\frac{1}{8}\right)$$

$$= 0 + \frac{x}{2} + \frac{-x^2}{8} + \frac{x^3}{48} \dots$$

Ans 15) $f(x) = e^{-6x}$ $\qquad$ $\cancel{f(0) = 1}$

$f'(x) = -6e^{-6x}$ $\qquad$ $\cancel{f'(0) = -6}$

$f''(x) = 36e^{-6x}$ $\qquad$ $\cancel{f''(0) =}$

$f'''(x) = -216e^{-6x}$

$f(4) = e^{-24}$

$f'(4) = -6e^{-24}$

$f''(4) = 36e^{-24}$

$f'''(4) = -216e^{-24}$

$$f(x) = e^{-24} + (x-4)(-6e^{-24}) + (x-4)^2(36e^{-24}) + (x-4)^3(-216e^{-24})$$

Ans16) $f(x) = \ln(3+4x)$ $\qquad$ $f(0) = \log 3$

$f'(x) = \frac{1}{3+4x}$ $\qquad$ $f'(0) = 1/3$

$f''(x) = \frac{-4}{(3+4x)^2}$ $\qquad$ $f''(0) = \frac{-4}{9}$

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$$f(x) = \log 3 + \frac{1x}{3} + \left(\frac{x}{2} \times \frac{-4}{9}\right) \cdots$$

$$= \log 3 + \frac{1x}{3} + \left(\frac{-2x}{9}\right) \cdots$$