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Statistical & Risk Modelling

Assignment 1 - Unit 1 & 2

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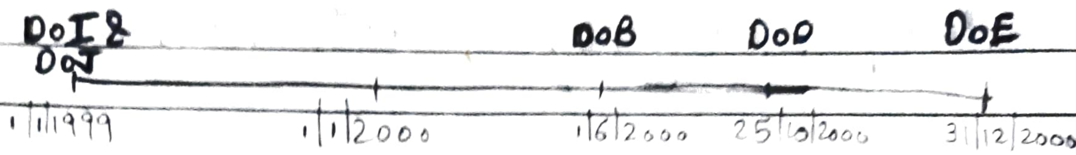
Q. 2

Timeline for the given age is:-



Herein:-

DOI = Date of Investigation
DOJ = " " Joining
DOB = " " Birth
DOD = " " Death &
DOE = " " End of Investigation.

a] Contribution of above life to E^c_{54} [Central Exposed to Risk]:-

E^c_{54}: Central exposed to risk aged 54 last birthday

~~E^c_{53} - 1/1/1999 to 31/5/2000 [approx 52 weeks + 22 weeks = 74 weeks]~~

E^c_{54} - 1/6/2000 to 24/10/2000 [approx 21 weeks]

* Assuming that the date of death isn't taken into account.

→ The above life contributes approximately ~~15~~ 21 weeks to the nearest to the Central Exposed to Risk (E^c_{54})

b] Initial Exposed to Risk for popⁿ aged 54 last b'day:-
Assuming

Q·3 Given data:-
- Policy Issue date
- Calender year of policy surrendering.
- Policy Maturity date
- Date of Exit, ~~exits~~ due to unknown reasons

Investigate:- RATE OF SURRENDER in relⁿ to DURATION of policy.

a] Left Censoring:-

This is a type of censoring wherein censoring/missing data ~~is present~~ exists even before the start of the investigation.

Classic Example would be when a person is already exposed to a co-morbidity disease* when the ~~ad~~ point of investigation too is morbidity analysis.

*even before the investigation begins.

This censoring isn't present.

b] **Right Censoring:-**

This is the most common type of censoring observed wherein the lives are censored or cut short during the observation. The reason could either be lapsation, unknown reason of ~~exit~~ exit or end of investigation.

This censoring is present for all those policies that were ~~spe~~ lapsed [Known from date of exit with reason being other than surrender]

c] **Internal Censoring:-**

As the ~~no~~ name suggests, this censoring takes place when data ~~is~~ censored or cut-short off between two points of investigation within an ongoing investigation.

This type of censoring is present since the calender year of surrender tells us the year of surrendering but not the exact point in time when it took place.

d] **Informative**

This censoring exists when the surrender gives some useful information or adds meaning to the ~~issue~~ interest of investigation. This censoring basically impacts & influences the investigation.

This ~~Exits~~ exists if some of the date of exits explicitly states that the life surrendered due to certainty about death probability / chances.

Q4 i] Mathematical formula for Complete expectation of life & what does it mean?

→ Complete Expectation of life as the name suggests, helps us find the exact ~~approximation~~ estimation of future life from age x onwards.
It is represented as $\mathring{e}_x$.

Survival is found as:-

$$_tp_x \cdot \mu_{x+t} = f(x+t) = \frac{dF_x(t)}{dt} = \frac{d\,{}_tq_x}{dt} = \frac{d(1-{}_tp_x)}{dt}$$

$$= \underbrace{0 - \frac{d\,{}_tp_x}{dt}}_{\text{we found this}}$$

∵ we want ans in terms of survival

$$\therefore \mathring{e}_x = \int_0^{\omega-x} \underbrace{t}_{u} \cdot \underbrace{{}_tp_x\,\mu_{x+t}\,dt}_{v}$$

$$\because {}_tp_x\,\mu_{x+t} = -\frac{d}{dt}({}_tp_x)$$

$$\therefore = \int_0^{\omega-x} t \cdot \left(-\frac{d}{dt}\,{}_tp_x\right) dt$$

Using $\int u \cdot v$ rule;

$$t\int_0^{\omega-x} -\frac{d}{dt}\,{}_tp_x \cdot dt - \int\left(\frac{dt}{dt}\int -\frac{d}{dt}\,{}_tp_x \cdot dt\right) dt$$

→

$$\therefore \left[t\,(-{}_tp_x)\,\cancel{dt}\right]_0^{\omega-x} - \int 1 - {}_tp_x \cdot \cancel{dt}\,.\,dt$$

[Taking Constants out]

$$\therefore -\left[t \cdot {}_tp_x\right]_0^{\omega-x} + \int_0^{\omega-x} {}_tp_x \cdot dt$$

① ②

[Since part ① would become 0, thus]

$$\mathring{e}_x = \int_0^{\omega-x} {}_tp_x \cdot dt$$

⇒ This in words represents, Complete future lifetime of age "x" is atleast for 't' years from ~~now~~ age x with death possible any time after the same. The range of t perhaps is from "0" to "ω-x" or ∞".

ii] GIVEN:- $\mu_x = 0.0325$ {Constant for all ages}

Curtate Expectation of Life - $e_x = E[K_x]$

$$\therefore E[K_x] = \sum_{k=1}^{\omega-x} {}_kp_x$$

Q.5

i] The Gompertz function perhaps described the age dynamics of human mortality rather accurately in the age range 35 years & above until 80 years.

ii] Show that

$$ {}_tp_x = (g)^{c^x(c^t-1)} $$
where $\ln g = \frac{-B}{\log c}$

We are perhaps given that,

$$ {}_tp_x = e^{-\int_0^t \mu_{x+s}\,ds} = e^{-\int_0^t Bc^{x+s}\,ds} \quad — (1) $$

Lets first understand & simplify the Exp. term.

$\therefore Bc^{x+s} = B.c^x.c^s = B.c^x e^{s\ln c}$ (taking Exp of 2nd term)

$= Bc^x e^{s \ln c}$

$$ \therefore \int_0^t Bc^{x+s}\,ds = \int_0^t Bc^x e^{s\ln c} = Bc^x\left[\frac{e^{s\ln c}}{\ln c}\right]_0^t \quad (2) $$

$$ = \frac{Bc^x}{\ln c}\left[e^{s\ln c}\right]_0^t = \frac{Bc^x}{\ln c}\left(c^s\right)_0^t $$ ......taking the ln part down

$$ = \frac{Bc^x(c^t-1)}{\ln c} \quad (A) $$

We can also make use of the given parametric:-

$$ \ln g = \frac{-B}{\log c} $$

$\therefore$ Substituting the ~~to~~ "log" part in ~~the~~ ① we get;

$$\cancel{{}_tP_x = \int_0^t Bc^x e^{s\ln c}} - \int_0^t Bc^{x+s} \cdot ds$$

$$= -\int_0^t \cancel{\log \ln c\, x^{x+s}}\ \frac{Bc^x}{\ln c}(c^t - 1) \quad \ldots \text{where } B = \log \ln c$$

$$\therefore \cancel{= \int_0^t} \log \frac{\cancel{\ln c}}{\cancel{\ln c}}\, c^x (c^t - 1)$$

$$= \log c^x (c^t - 1)$$

$$\Rightarrow {}_tP_x = e^{(\log c^x (c^t - 1))} \quad \ldots \text{taking log value down.}$$

$$\boxed{{}_tP_x = g^{c^x (c^t - 1)}}$$

Thus proved.

Q.6 Point of Study: Investigate effect of newly invented drug on Cancer suffering patients.

Given:- $y_i = 1$ if non-smoker
$ = 0$ otherwise

$z_i = 1$ if male
$ = 0$ if female

x_i = age at entry into the observation

$\rightarrow$

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a) The class of lives to which the baseline hazard fn applies is:-

$x_i = 30$ years at entry

$y_i = 0$ i.e. smoker &

$z_i = 0$ i.e female.

b) Given:-

$$h_i(t) = h_0(t) \times e^{(0.01(x-30)+0.2y-0.05z)}$$

Compare – $z_i=1$, $y_i=0$, $x=30$ with
$z_i=0$, $y_i=0$, $x=40$

$$\therefore h_{m,s}(t) = h_0(t) \times e^{(0.01(30-30)+0.2(0)-0.05)}$$

[male, smoker]

$$= h_0(t) \times e^{-0.05}$$

$$h_{f,s} = h_0(t) \times e^{(0.01(40-30)+0.2(0)-0.05(0))}$$

$$= h_0(t) \times e^{0.01(10)}$$

$$\therefore \frac{h_{m,s}}{h_{f,s}} = \frac{\cancel{h_0(t)} \times e^{-0.05}}{\cancel{h_0(t)} \times e^{0.1}}$$

$$= e^{-0.05-0.1}$$

$$= \boxed{0.86071}$$

⟹ Thus, a male smoker aged 30 has 13.93% less ~~risk~~ survival chances as compared to female smoker aged 40.

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c) As found earlier,

$$h_{m,s,30} = h_0(t) \times e^{-0.05}$$

$$h_{m,s,40} = h_0(t) \times e^{(0.01(40-30)+0.2(0)-0.05)}$$

$$= h_0(t) \times e^{0.1-0.05}$$

$$\therefore \frac{h_{m,s,30}}{h_{m,s,40}} = e^{-0.05-0.05}$$

$$= \boxed{0.9048}$$

⇒ Perhaps a male smoker aged 30 has 90.48% chance of survival as compared to male smoker aged 40.

Q.7

i] Policyholders [life] classified by :- Age Next B'day

Deaths classified as:- age next bday + curtate duration at date of death