PAGE No.
DATE / /

Q1 Calculate the mean, median, mode, standard devation and interquartile range of the following data.

i) 4, 7, 9, 9, 11, 15, 18, 18, 18, 25.

Mean $(\bar{x}) = \frac{\Sigma x}{n} = \frac{4+7+9+9+11+15+18+18+18+25}{10} = \frac{134}{10} = 13.4$

Median $= \left(\frac{N+1}{2}\right)^{th}$ term $= \left(\frac{11}{2}\right)^{th}$ term $= \frac{5^{th}\text{ term} + 6^{th}\text{ term}}{2} = \frac{11+15}{2} = \frac{26}{2} = 13$

Mode = 18

Standard devation

$\Sigma x^2 = 16 + 49 + 81 + 81 + 121 + 225 + 324 + 324 + 324 + 625$

$= 2170.$

SD = σ

$\sigma^2 = \frac{1}{n}\Sigma x^2 - (\bar{x})^2$

$= 217 - (13.4)^2$

$= 217 - 179.56$

$= 36.44$

$\sigma = \sqrt{\text{variance}}$

$= \sqrt{36.44}$

$\sigma = 6.0365553309.$

Interquartile Range. = $Q_3 - Q_1$

$Q_1 = \left(\frac{N+1}{4}\right)^{th}$ term $= \frac{11}{4}^{th}$ term $= 2.75^{th}$ term

$= 2^{nd}$ term $+ 0.75\,(3-2)^{rd}$ term

$= 7 + 0.75(9-7)$

$= 8.5$

$Q_3 = 3\left(\frac{N+1}{4}\right)^{th} = \frac{33}{4}^{th}$ term $= 8.25^{th}$ term

$= 8 + 0.25(9-8)^{th}$ term

$= 18 + 0.25\ (0)$

$= 18$

Interquartile range = $18 - 8.5 = 9.5$

Number of people (x) household	Number of people f	Less than c.f.	xf	x^2f
0	5	5	0	0
1	11	16	11	11
2	26	42	52	104
3	15	57	45	135
4	3	$\boxed{60}$	12	48
			120	298
			$=\Sigma xf$	$=\Sigma x^2 f$

Mean $= \frac{\Sigma xf}{\Sigma f} = \frac{120}{60} = 2$.

Median $= \left(\frac{N+1}{2}\right)^{th}$ term $= \frac{61}{2}^{th}$ term $= 30.5^{th}$ term $= 2$

Mode $= 2$

$SD^2 = \frac{\Sigma x^2 f}{\Sigma f} - (\bar{x})^2$

$= \frac{298}{60} - (2)^2$

$= 4.966667 - 4$

$= 0.96667$

SD $(\sigma) = \sqrt{0.96667}$

$= 0.9831920803$.

Interquartile range $= Q_3 - Q_1$

$Q_3 = 3\left(\frac{N+1}{4}\right)^{th}$ term $= 3\left(\frac{61}{4}\right)^{th}$ term $= 45.75^{th}$ term $= 3$

$Q_1 = \left(\frac{N+1}{4}\right)^{th}$ term $= \left(\frac{61}{4}\right)^{th}$ term $= 15.25^{th}$ term $= 1$

Interquartile range $= 3 - 1$

$= 2$.

Q2 It is assumed that claims arising on an Industrial policy can be modelled as Possion ~~distribution~~ Process at a rate of 0.5 per year.

i) Determine the probability that no claims arise in a single year.

ii) Determine the probability that, in 3 consecutive years, there is one or more claim in one of years and no claims in each of the other two years.

iii) Suppose a claim has just occured. Determine the probability that more than 2 years will elapse before the next claim occurs.

Ans i $X \sim Possion(\lambda)$

$X \sim Possion(0.5)$

$P(X=0) = \underline{0.6053}$ where $X \sim Possion(0.5)$ (Table book pg 175).

ii Let Y = Number of claims in a year. ~~Find claims~~ Finding probability claims more than 1

$P(Y \geq 1) = 1 - P(X=0)$

$= 1 - 0.6053 = 0.39347$ (p)

$Y \sim B(3, 0.39347)$ $q = 1 - p = 1 - 0.39347 = 0.60653$.

$P(Y=1) = {}^nC_x \, p^x q^{n-1}$

$= {}^3C_1 (p)^1 (q)^2$

$= 3 \times 0.39347 \times (0.60653)^2$

$= \underline{0.4342476}$

iii Let A be time unit next claim

$A \sim Exp(0.5)$

$P(A > 2) = 1 - \int_0^2 0.5 e^{-0.5x}$

$= 1 - 0.5 \left(\dfrac{e^{-0.5x}}{-0.5}\right)_0^2$

$= 1 + (e^{-0.5x})_0^2$

$= 1 + (e^{-1} - e^0)$

$= 1 - 0.6321205588$

$= \underline{0.3678794412}$

Q3 An analyst is intrested in using gamma distribution with parameter $\alpha = 2$ and $\lambda = 1/2$. The range of x is defined as $0 < x < \infty$

i) state the mean and standard devation of the distribution

ii) Hence comment briefly on its shape.

iii) Calculate the ~~cdf~~ cumulative distribution function.

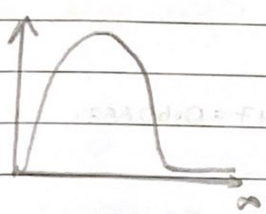
i) $X \sim \text{Gamma}(2, 1/2)$

$E(x) = \dfrac{\alpha}{\lambda} = \dfrac{2 \times 2}{1} = \underline{\underline{4}}$ $\qquad SD = \sqrt{V(x)}$

$= \sqrt{8}$

~~Var~~ $V(x) = \dfrac{\alpha}{\lambda^2} = 2 \times \left(\dfrac{1}{1/2}\right)^2 = 2 \times 4 = \underline{\underline{8}}$ $\qquad = 2.828427125.$

ii $f_x(x) = \dfrac{\lambda^{\alpha} e^{-\lambda x}}{\Gamma \alpha} x^{\alpha - 1}$



increases in the beginning & then decreases when $\alpha = 2$ and $\lambda = 1/2$.

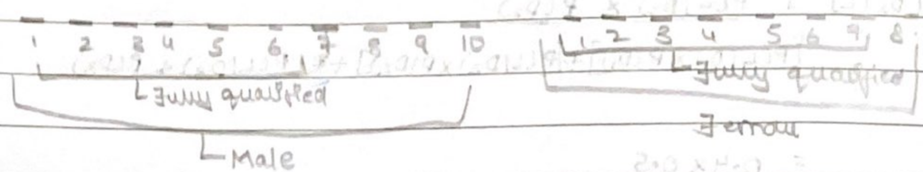
iii CDF of a Gamma distribution:

$\int_{L^L}^{x} f_x(x)\,dx = \int_0^x \lambda e^{-\lambda x}\,dx.$

$= F_x(x) = 1 - e^{\lambda x}$ (less than probability).

4 In an acturial office, there are 10 male and 8 female staff. 6 of the males and 7 of the females are fully qualified actuaries. Calculate the probability that a team of two workers randomly selected consists of 2 qualified females given that it contains at least 1 female.

actuaries



$\underline{\ }\ \underline{\ }$ $\leftarrow {}^{12}C_1$, ${}^{7}C_1$

Total No of observations possible $= {}^{18}C_2$ $\quad 18 \times 17$
$= \frac{\cancel{18}^{9} \times 17}{\cancel{2} \times 1} = 18 \times 17 = 306$
$= 9 \times 17 = 153$

P(2 qualified actuaries given at least 1 female) $= \frac{{}^{7}C_1 \times {}^{12}C_1}{\cancel{153}\ 306}$

$$\boxed{= \frac{84}{\cancel{153}\ 306} = \underline{0.2745}}$$

OR

Case I + Case II one male one female

one female ~~two females~~

one male $\quad {}^{7}C_1 \times {}^{6}C_1$

${}^{7}C_1 \times {}^{6}C_1$

$= \frac{{}^{7}C_1 \times {}^{6}C_1}{\cancel{153}\ 306} + \frac{{}^{7}C_2 \times {}^{6}C_1}{\cancel{153}\ 306}$ → 2 female

$= \frac{7 \times 6}{\cancel{153}\ 306} + \frac{7 \times \cancel{3}6}{\cancel{15}\ 306}$

$= \frac{42}{\cancel{153}\ 306} + \frac{\cancel{21}42}{\cancel{153}\ 306} = \frac{\cancel{63}84}{153} \cdot \boxed{\frac{84}{306}}$

5 The supply chain service in a country has 3 levels of service: Level 1 (next day delivery), ~~let~~ Level 2 (2-day delivery) and Level 3 (3-day delivery). 40% of packages are delivered via Level 1 & 50% are delivered via Level 2. The probability of packaging arriving late using each level is 20%, 40% and 10% respectively. Calculate the probability that a late package was send via Level 2 service.

Let L be the probability arriving late).

P(D→Level 1) = 40% $\quad$ P(D−Level 2) = 50% $\quad \therefore$ P(D−Level−3) = 10%

$P(D_1) = 40\% = 0.4 \quad P(D_2) = 0.5 \quad P(D_3) = 0.1$

PAGE No.
DATE / /

$P(D_2/L)$

$P(L|D_1) = 0.2$

$P(L|D_2) = 0.4$

$P(L|D_3) = 0.1$

Deler Delivery → Level2 and arrives late.

$$P(D_2|L) = \frac{P(L|D_2) \times P(D_2)}{[P(L|D_1) \times P(D_1)] + [P(L|D_2) \times P(D_2)] + [P(L|D_3) + P(D_3)]}$$

$$= \frac{0.4 \times 0.5}{(0.2 \times 0.4) + (0.4 \times 0.5) (0.1 \times 0.1)}$$

$$= \frac{0.2}{(0.08 + 0.2 + 0.01)}$$

$$= \frac{0.2}{0.29}$$

$$= 20/29 = 0.6896551724$$

If $X \sim U(1,2)$ and $Y = \frac{3}{6-x}$ determine PDF of X

$y = \frac{3}{6-x}$ $\qquad f_x(x) = \frac{1}{2-1} = 1$

$6 - x = \frac{3}{y}$ $\qquad f_{\frac{6y-3}{y}}\left(\frac{6y-3}{y}\right) = f_x(w^{-1}(y)) \left|\frac{dw}{dy}(w^{-1}(y))\right|$

$6 - \frac{3}{y} = x$

$F_y(y)$

$f_y(y) = \frac{6y-3}{y} = f_x\left(\frac{6y-3}{y}\right)\left(\frac{3}{y^2}\right)$

$\frac{6y-3}{y} = x$

$= 1 \times \frac{3}{y^2}$

$\frac{dx}{dy} = \frac{6y - (6y-3)(1)}{y^2}$

$= 3/y^2$

$= \frac{6y - 6y^2 + 3y}{y^2}$ $\quad \frac{6y - 6y + 3}{y^2}$

$= \frac{3y - 6y^2}{y^2} = 3/y^2$

PAGE No. DATE / /

Q7 A discrete random variable x has the following probability distribution

x	1	2	3	4	5
$P(X=x)$	0.05	0.15	0.35	0.4	0.05

i) $F(3) = P(x=1) + P(x=2) + P(x=3)$

$= 0.05 + 0.15 + 0.35$

$= 0.55$

ii) $E(x) = \sum_{\text{for all } x} x\,P(x)$

$= (0.05) + 0.15(2) + 0.35(3) + 0.4(4) + 0.05(5)$

$= 0.05 + 0.3 + 1.05 + 1.6 + 0.25$

$= 3.25$

iii) $V(x) = E(x^2) - [E(\bar{x})]^2$

for $E(x^2) = 0.05(1) + 0.3(2) + 1.05(3) + 1.6(4) + 0.25(5)$

$= 0.05 + 0.6 + 3.15 + 6.4 + 1.25$

$= 11.845$

$V(x) = 11.45 - (3.25)^2$

$= 11.45 - 10.625$

$= 0.8875$

iv) Coefficient of skewness

Skewness $= E(x^3) - E(\bar{x})^3$

for $E(x^3) = 0.05(1) + 0.6(2) + 3.15(3) + 6.4(4) + 1.25(5)$

$= 0.05 + 1.2 + 9.45 + 26.6 + 6.25$

$= 42.55$

Coefficient of skewness $= \dfrac{\text{skewness}}{[V(x)]^{1.5}}$

$= \dfrac{8.221875}{(0.8875)^{1.5}}$

$= 9.83373\overset{31}{3}07$

Skewness $= E(x^3) - [E(\bar{x})]^3$

$= 42.55 - 34.328125$

$= 8.221875$

Insurance claim amount are independent and exceed 1000 with probability 0.2. In a sample of 15 claim, calculate the probability that fewer than 3 of them exceed 1000.

X ~ Binomial distribution where $p=0.2$ & $n=15$

$X \sim (15, 0.2)$ $q = 1-0.2 = 0.8$

$P(X<3) = P(X=0)+P(X=1)+P(X=2)$ $P(X=x) = {}^nC_x\, p^x q^{n-x}$

$= {}^{15}C_0 (0.2)^0 (0.8)^{15} + {}^{15}C_1 (0.2)^1 (0.8)^{14} + {}^{15}C_2 (0.2)^2 (0.8)^{13}$

$= 0.035184372 09 + 15 \times 0.2 \times 0.04398046511 + \frac{15 \times 14}{2 \times 1} \times 0.04 \times 0.0549755 8139$

$= 0.03518437209 + 0.1319413953 + 0.2308974418$

$= 0.3980232093$

A man enters a raffle each week where the probability of winning prize is 0.01. Calculate that probability that he takes 7 weeks up to and including the week where he wins his 3rd price.

⇒ He won his 3rd price in his 7th trial.

Thus in the remaining 6 responses he won 2 times.

$p = 0.01$ ~~$p=0.03$~~ $q = 1 - 0.01$ $= 0.99$

${}^{7-1}C_{3-1}$

So $P(X=7) = {}^6C_2 (0.01)^3 (0.99)^4$ [last chance is fixed thus not taken in consideration] (Negative Binomial)

$= \frac{6 \times 5}{2 \times 1} (0.01)^3 (0.99)^4$

$= 0.0000144084015$

PAGE NO.	
DATE	/ /

10) The number of claim submitted to an insurance office averages two per working week (of 5 days). Calculate the probability that more than two claims arrive one day.

Let x be the probability of a claim per day ~~($\lambda = 0.4$ (2/5))~~ ($\lambda = 0.4$ (2/5))

$X \sim Poi(0.4)$

$P(X>2) = 1 - P(X \leq 2)$

$= 1 - 0.99207$ (from table book pg 195)

$= 0.00793$.

11) If the probability of having a male and female child is equal & woman has 5 boys & 2 girls. Calculate the probability that next 3 children are boys.

Let probability of having a ~~girl~~ female is F and probability of having a male is M

$P(F) = P(M) = 1/2 = 0.5$

Women has 5 boys and 2 girls.

P(Next 3 children being son) = $\underset{1/2}{\_} \; \underset{1/2}{\_} \; \underset{1/2}{\_}$

$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

$= 1/8 = 0.125$.

Q12 Claim follows Poisson ~~distribution~~ process with rate 10 per day. Calculate the probability that the time between two claims is less than 0.1 days.

$X \sim Poi(10)$ [$\lambda = 10$]

('Time' mentioned use exponential)

$X \sim Exp(10)$

$P(X<0.1) = \int_0^{0.1} \lambda e^{-\lambda x} dx$

$= 10 \int_0^{0.1} e^{-10x} dx$

$= 10 \left[\frac{e^{-10x}}{-10}\right]_0^{0.1}$

$= -\left[e^{-10(0.1)} - e^0\right]$

$= -(e^{-1} - 1)$

$= 1 - e^{-1}$

$= 0.6321205588$.

PAGE No.
DATE / /

If X is following uniform distribution parameters $a = 75$ and $b = 120$. Calculate the probability that X lies between 80 & 100.

$\overset{a}{75} < x < \overset{b}{120}$

Probability of $(80 < x < 100)$.

$PDF = \frac{1}{b-a} = \frac{1}{120-75} = \frac{1}{45}$

$(b-a)$

$P(80 < X < 100) = \frac{100-80}{45} = \frac{20}{45} = \frac{4}{9}$.

If $X \sim$ Gamma $(5, 0.4)$ calculate $P(X < 22.89)$

$X \sim$ Gamma $(5, 0.4)$

$\alpha = 5 \quad \lambda = 0.4$

$P(X < x) = \int_0^{22.89} \frac{\lambda^\alpha e^{-\lambda x}\, x^{\alpha-1}}{\Gamma(\alpha)}$ $\qquad \Gamma(\alpha) = (\alpha-1)!$

$P(X < 22.89) = \frac{(0.4)^5 \times e^{-0.4(22.89)} \times (22.89)^{5-1}}{4!} - \frac{(0.4)^5 \times e^{-0.4(0)} \times (0)^4}{4!}$

$= \frac{(0.4)^5 \times e^{-0.4(22.89)} \times (22.89)^4}{4 \times 3 \times 2 \times 1}$

$= 0.01236720641$

$X \sim$ Gamma $(5, 0.4)$

$\alpha = 5$. $\lambda = 0.4$ $2\lambda = 0.8$ $2\alpha = 2(5) = 10$.

$0.8X \sim \chi^2_{10}$

$P(\chi^2_{10} < 22.89 \times 0.4)$.

$P(\chi^2_{10} < 18.312)$

$18.0 \rightarrow 0.945$ $\qquad 0.312 \rightarrow \frac{0.0079}{0.5} \times 0.312 = 0.0049296$.

$18.5 \rightarrow 0.9589$

$\therefore P(\chi^2_{10} < 18.312) = 0.945 + 0.0049296$

$= 0.9499296$

$0.5 \rightarrow 0.0079$

$1 \rightarrow \frac{0.0079}{0.5}$

PAGE NO
DATE / /

Q15 A continous random variable X has the following probability density function $\frac{k}{(x+5)^4}$, $x>0$. Calculate i) k
ii) F10
iii) E(x).

i) $\int_0^\infty f_x(x)\,dx = 1$

$k\int_0^\infty (x+5)^{-4}\,dx = 1$

$\frac{k}{-3}\left[(x+5)^{-3}\right]_0^\infty = 1$

$k\left[(x+5)^{-3}\right]_0^\infty = -3$

$k\left[\frac{1}{(x+5)^{-3}}\right]_0^\infty = -3 \qquad \left(\frac{1}{\infty}=0\right)$

$k\left[\frac{1}{(\infty+5)^{-3}} - \frac{1}{5^{-3}}\right] = -3$

$k(5^{-3}) = 3$

$k = \frac{3}{5^{-3}} = 3\times5^3 = 375.$

ii) $F(10) = \int_0^{10} f_x(x)$

$= \int_0^{10} 375(x+5)^{-4}$

$= \frac{375}{-3}(x+5)^{-3}\Big|_0^{10}$

$= -125\left[\frac{1}{(x+5)^3}\right]_0^{10}$

$= \frac{-125}{(10+5)^3-(5)^3}$

$= \frac{-125}{15^3-5^3}$

$= \frac{-125}{3375-125}$

$= -125\left[\frac{1}{(15)^3} - \frac{1}{(5)^3}\right]$

$= -125\left[\frac{1}{3375} - \frac{1}{125}\right]$

$= \left[\frac{-125(125-3375)}{3375\times125}\right]$

$= \frac{125\times3250}{\cancel{3250}\,3375\times125}$

$= \frac{3250}{3375} = 0.962962963.$

iii) $E(x) = \int_0^\infty x\,f(x)\,dx$

$E(x) = \int_0^\infty x\,\frac{375}{(x+5)^4}\,dx$

$= 375\int_0^\infty x(x+5)^{-4}\,dx$

Let $x+5 = t \qquad x\to\infty,\ t\to\infty \qquad x\to0,\ t\to5.$

$\frac{dt}{dx} = 1$

$dt = dx.$

$375\int_5^\infty (t-5)\,t^{-4}\,dt$

PAGE No. DATE / /

$$= 375 \int_5^{\infty} t^{-3} - 5t^{-4}$$

$$= 375 \left[\frac{t^{-2}}{-2} - \frac{5t^{-3}}{-3} \right]_5^{\infty}$$

$$= 375 \left[- \left(\frac{5^{-2}}{-2} - \frac{5(5)^{-3}}{-3} \right) \right]$$

$$= 375 \left[- \left(\frac{1}{25(-2)} - \frac{1}{25(-3)} \right) \right]$$

$$= 375 \left[-\frac{1}{25} \left(\frac{1}{2} - \frac{1}{3} \right) \right]$$

$$= 375 \left[\frac{1}{25} \left(\frac{3-2}{6} \right) \right]$$

$$= \frac{\cancel{15}\,375}{25} \times \frac{1}{6}$$

$$= \frac{15}{6}$$

$$= 5/2$$

$E(x) = 2.5$