Q1.] $d^{(4)} = 8\%$

i.] $d = 1 - \left(1 - \frac{d^{(4)}}{4}\right)^4$

$= 1 - \left(1 - \frac{0.08}{4}\right)^4$

$= 0.07763184$ / 7.763184%

$\delta = -\ln(1-d)$

$= -\ln(1 - 0.07763184)$

$= 0.080811$ / 8.0811%

ii.] $i = ?$

$i = \frac{d}{1-d} = \frac{0.07763184}{1 - 0.07763184} = 0.0841668.41$

iii.] $d^{(12)} = 12\left[1 - (1-d)^{1/12}\right]$

$= 12\left[1 - (1 - 0.07763184)^{1/12}\right]$

$= 0.080539$ / 8.0539

Q2.] $\delta(t) = 0.004t + 0.0002t^2$ — (Given)

i.] $PV = 1000\left[e^{-\int_0^5 (0.004t + 0.0002t^2)\,dt}\right]$

$= 1000\left[e^{-\left[\frac{0.004t^2}{2} + \frac{0.0002t^3}{3}\right]_0^{10}}\right]$

$= 1000\left[e^{-\left(0.002 \times 100 + \frac{0.0002 \times 1000}{3}\right)}\right]$

$= 1000\left[e^{-(0.2 + 0.0667)}\right]$

$= 1000 \times e^{-(0.2667)}$

$= 765.90281$

iii.] $i = ?$

$765.90281 = 1000\,(1+i)^{10}$

$\frac{765.90281}{1000} = (1+i)^{10}$

$(0.765\,90281)^{1/10} = 1 + i$

$$i = e^{\int_0^{10} (0.004t + 0.0002t^2)\,dt} - 1$$

$$= e^{(0.002 \times 100 + 0.0002 \times \frac{1000}{3})} - 1$$

$$= e^{(0.2 + 0.0667)} - 1$$

$$= 0.305465$$

Q3] i] Let us assume that a person borrows 1 at an effective rate of discount d. Then in effect, the original principle is 1-d & the amount of discount is d. However, from the basic definition of i as the ratio of the amount of ~~int~~ discount to the principal, we obtain

$$i = \frac{d}{1-d}$$

This formula expresses i as a function of d

$$\therefore i = \frac{d}{1-d}$$

$$i - id = d$$

$$d(1+i) = i$$

$$d = \frac{i}{1+i}$$

$$\therefore \text{Discount factor } v = \frac{1}{1+i}$$

$$\therefore d = i \times \frac{1}{1+i}$$

$$d = iv$$

ii] a] $\frac{d^{(4)}}{4} = 2.25\%$

$$d = 1 - \left(1 - \frac{d^{(4)}}{4}\right)^4$$

$$= 1 - \left(1 - \frac{0.0225}{4}\right)^4$$

$$= 0.022311$$

$$d^{(2)} = 2\left[1 - (1 - 0.022311)^{1/2}\right]$$

$$= 2.244\%$$

b.] $d^{(0.5)} = 5\%$

$$d = \left[1 - 1 - \left(1 - \frac{0.05}{0.5}\right)^{0.5}\right]$$

$$= 0.051317$$

$$d^{(2)} = 2\left[1 - (1 - 0.051317)^{1/2}\right]$$

$$= 5.190 \quad 5.19\%$$

04.] Let us assume that a person borrows 1 at an effective rate of discount d. Then, in effect, the original principal is 1-d & amt of ~~int~~ discount is d. However, from the basic definition of i as the ratio of the amt of ~~int~~ discount to the principal, we obtain.

$$i = \frac{d}{1-d}$$

This formula expresses i as a function of d

$$\text{or } i = \frac{d}{1-d}$$

$$\therefore i - id = d$$

$$d(1+i) = i$$

$$d = \frac{i}{1+i}$$

$\therefore$ Discount factor $v = \frac{1}{1+i}$

$$d = i \times \frac{1}{1+i}$$

$\therefore d = iv$

ii.] $i = 0.08$

a.] $d = \frac{0.08}{1+0.08} = 0.0740741$

$d^{(12)} = 7.6715$

b.] $i^{(365)} = 365[(1+0.08)^{1/365} - 1]$

$= 7.69692\%$

c.] $\delta = \ln(1+i) = \ln(1.08) = 7.6961\%$

d.] $i^{(0.5)} = 0.5[(1+0.08)^{1/0.5} - 1]$

$= 8.32\%$

5.] i.] $d^{(6)} = 6\%$

a.] $i^{(2)} = ?$

$d = 1 - \left(1 - \frac{d^{(4)}}{4}\right)^4$

$= 1 - \left(1 - \frac{0.06}{4}\right)^4$

$= 5.8663\%$

$i = \frac{0.058663}{1-0.058663} = 6.232\%$

$i^{(2)} = 2[(1+0.06232)^{1/2} - 1]$

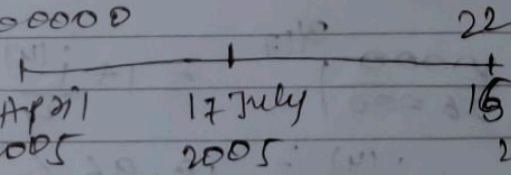
$= 6.138\%$

b.] $d = 5.8663\%$

$d^{(12)} = 12[1 - (1 - 0.058663)^{1/12}]$

$= 6.0302\%$

ii.]



Loan was paid in 93 days

So ~~Int~~ total sum paid within 93 days $= 200000 + 20000\left(\frac{93}{365}\right)$

$= 205095.89$

6b] Let us assume cash payment of 100

∴ Discounted value of 100 at time 0 = $100 \times \frac{80}{100}$

$= 70$

At six month $= 100 \times 75\% = 75$

$\therefore PV_0 = AV(1-d)^n$

$70 = 75(1-d)^{1/2}$

$\left(\frac{70}{75}\right)^2 = 1-d$

$d = 0.128889$

$= 12.8889\%$

$d^{(12)} = 12\left[1-(1-0.128889)^{1/12}\right]$

$= 13.71956\%$

$i^{(2)} = ?$

$i = \frac{0.128889}{1-0.128889} = 0.1479593$

$i^{(2)} = 2\left[(1+0.1479593)^{1/2} - 1\right]$

$= 14.2857\%$

$\delta = -\ln(1-d)$

$= 13.7986\%$

7] i] $PV_0 = 20000$

$AV_{17} = 26000$

So, $20000 = 26000\left(1+\frac{i^{(4)}}{4}\right)^{4\times\frac{17}{12}}$

$\left(\frac{20000}{26000}\right)^{3/17} = 1+\frac{i^{(4)}}{4}$

$i^{(4)} = 18.96\%$

ii] $d^{(2)} = ?$

$i = \left(1+\frac{0.1896}{4}\right)^4 - 1 = 20.34541$

$$d = \frac{0.203454}{1+0.203454} = 16.90463$$

$$d^{(2)} = 2\left[1-(1-0.1690463)^{1/2}\right]$$

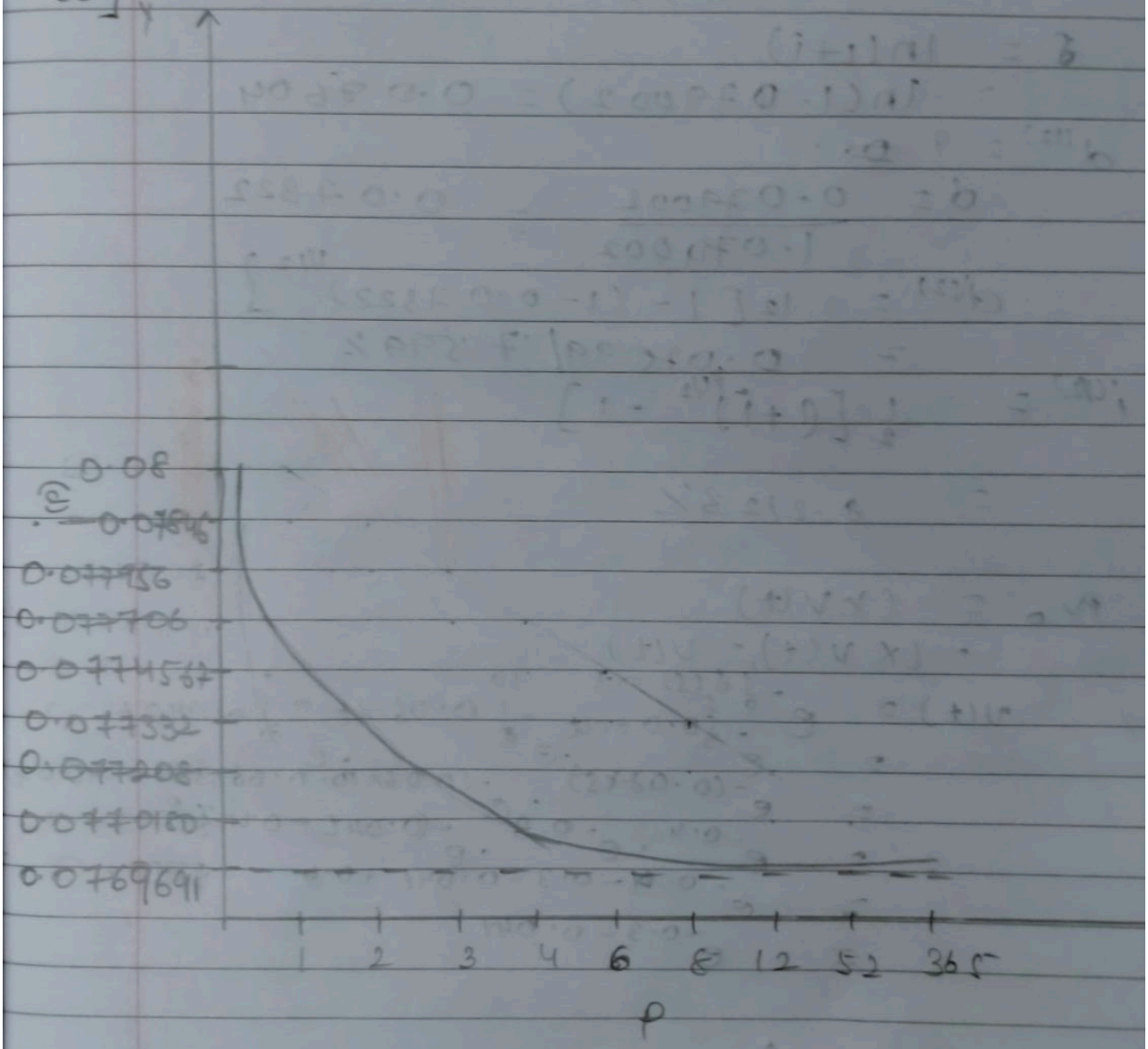
$$= 16.69\%.$$

iii.] $26000 = 20000\,(1+i)^{17/12}$

$$\left(\frac{26000}{20000}\right)^{12/17} = 1+i$$

$$i = 21.18\%.$$

Q8.]



Q 9.] i.] Force of interest refers to nominal interest rate or a discount rate compounded infinite number of times (or continuously) per time period.

ii.] $i^{(2)} = 0.0775$

$$i = \left(1 + \frac{i^{(m)}}{m}\right)^m - 1$$

$$= \left(1 + \frac{0.0775}{2}\right)^2 - 1$$

$$= 7.9002\%$$

$$\delta = \ln(1+i)$$

$$= \ln(1.079002) = 0.076004$$

$$d^{(12)} = ?$$

$$d = \frac{0.079002}{1.079002} = 0.07322$$

$$d^{(12)} = 12\left[1 - (1 - 0.07322)^{1/12}\right]$$

$$= 0.075991 \text{ or } 7.599\%$$

$$i^{(12)} = \frac{1}{2}\left[(1+i)^{1/12} - 1\right]$$

$$= 8.213\%$$

Q10.] $PV_0 = (X\,v(t))$

$$= (X\,v(t)) = v(t)$$

$$v(t) = e^{-\int_0^t \delta(s)\,ds}$$

$$= e^{-\int_0^t 0.08\,dt} \cdot e^{-\int_0^{10} 0.06\,dt - \int_0^t 0.04\,dt}$$

$$= e^{-(0.08 \times 5)} \cdot e^{-(0.06 \times 10 - 0.06 \times 5) - (0.04t - 0.4)}$$

$$= e^{-0.4} \cdot e^{-0.3} \cdot e^{-0.04t - 0.4}$$

$$= e^{-0.4 - 0.3 - 0.04t + 0.4}$$

$$= e^{-0.3 - 0.04t}$$

11.] a.] $AV_{9/12} = 50000$

$d = 18\%$

$PV = 50000\,(1-0.18)^{9/12}$

$= 43085.43$

b.] $\delta = 6\%$

$d = 1 - e^{-\delta}$

$= 5.82355\%$

$d^{(12)} = 12\left[1 - (1-0.0582355)^{1/12}\right]$

$= 5.99\%$

ii] $d^{(4)} = 6\%$

$\therefore i^{(2)} = 9$

$d = 1 - \left(1 - \frac{0.06}{4}\right)^4 = 5.866345\%$

$i = \frac{0.0586345}{1 - 0.0586345} = 6.23\%$

$i^{(12)} = 12\left[(1+0.0623)^{1/12} - 1\right]$

$= 6.0584\%$

$= 6.059\%$

c.] $d < \delta < i^{(4)} < i$

d.] Let us assume that a person borrows 1 at an effective rate of discount d. Then in effect, the original principal is $1-d$ & amt of discount is d. However, from the basic definition of i as the ratio of the amt of discount to the principal, we obtain,

$$i = \frac{d}{1-d}$$

$$\therefore d = i - id$$

~~$d+id = id$~~

$$d(1+i) = i$$

$$d = \frac{i}{1+i}$$

$$\therefore d = \frac{1}{1+i} \times i \times \frac{1}{1+i} \Rightarrow d = iv$$

Q12] $PV_0 = 7049$

$i^{(12)} = 6\%$

$i^{(4)} = 1.5\%$

$d^{(12)} = 6\%$

$d = 1 - \left(1 - \frac{d^{(12)}}{12}\right)^{12} = 0.0584$

$$AV_6 = 7049\left[\left(1+\frac{i^{(12)}}{12}\right)^{12\times 2}\cdot\left(1+\frac{i^{(4)}}{4}\right)^{4\times\frac{30}{12}}\cdot(1-d)^{\frac{18}{12}}\right]$$

$$= 7049\left[\left(1+\frac{0.06}{12}\right)^{24}\cdot\left(1+\frac{0.015}{4}\right)^{10}\cdot(1-0.0584)^{\frac{18}{12}}\right]$$

$$= 7049\left[(1.127150)(1.038141)(0.913892)\right]$$

$$= 7536.4154$$

Q13] i] ~~$i^{(2)} = \frac{1}{n}$~~

Q13] $(1+i)^n = 2$

$(1.1)^n = 2$

$n = 7.3$ years

iii] $d^{(12)} = 10\%$

$d = 1 - \left(1 - \frac{d^{(12)}}{12}\right)^{12}$

$= 9.55\%$

$2 = (1-d)^n$

$\ln(2) = n\ln(1-0.0955)$

~~$\frac{1}{2}$~~ ~~$A = 2A(1-d)^n$~~

~~$0.\frac{1}{2}$~~ $= (1-d)^n$

$\frac{\ln(0.5)}{\ln(0.9045)} = n$

$n = 6.9$ years