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Assignment

1) $r = 12.65$ cm
Actual ley $= 12.5$

(i) Absolute error $= |$Actual value $-$ Measured value$|$
$= 12.5 - 12.65$
$= ~~12.85~~ \; 0.15$

(ii) Relative Error $= \dfrac{|\text{Measured} - \text{real}|}{\text{real}}$
$= \dfrac{|12.65 - 12.5|}{12.5}$
$= 0.012$

(iii) Percentage Error $= 0.012 \times 100$
$= 1.2\%$

2)

	x	$f(x) = \log x$
a)	4	0.60206
	5	y
	6	0.778513

	x	$f(x)$
b)	4.5	0.6532125
	5	y_1
	5.5	0.7403627

a) ~~Let~~ $y = 0.60206 + \dfrac{(1)(0.778513 - 0.60206)}{2}$
$\log(5) = ~~0.6778693~~ \; 0.6902056$

b) $y_1 = 0.6532125 + \dfrac{(0.5)}{1}$
$\log(5) = 0.6967876$

3) root of equation

$$f(x) = x^4 - x - 10 = 0.$$

x	y
1.5	-6.4375
2	4

$$x_1 = \frac{1.5+2}{2} = 7/4.$$

$$y_1 = -2.3711$$

x	y
~~1.5~~	

$$x_2 = \frac{1.75+2}{2} = 1.875$$

$$y_2 = 0.48462$$

$$x_3 = \frac{1.875+1.5}{2} = 1.6875$$

$$= -3.578.$$

$$x_4 = \frac{1.6875+1.875}{2} = 1.78125$$

$$y_4 = -1.7143.$$

$$x_5 = \frac{1.78125+1.875}{2} = 1.828125$$

$$y_5 = -0.65888$$

so root is 1.828125

4) $f(x) = x^3 - x - 1$

x	$f(x)$
0	-1
1	-1
2	5

$f'(x) = 3x^2 - 1$

$x_0 = 2.$

$f(x_0) = +5.$

$f'(x_0) = 74$

$x_1 = 2 - 5/74 = \cancel{272} = 1.93243$

$f(x_1) = -\cancel{1.375}\ 4.28884$

$f'(x_1) = -\cancel{0.25}\ 10.20286$

$x_2 = 1/2 - \dfrac{1.375}{0.25} = \cancel{0.580183}\ 1.512563$

$f(x_2) = -\cancel{5}\ -\cancel{0.384886}\ 0.947952$

$f'(x_2) = \cancel{0.009662}\ 5.86354$

$x_3 = 1.512563 - \dfrac{0.947952}{5.86354} = 1.350894$

$f(x_3) = 0.1143727$

$f'(x_3) = 4.47473$

$x_4 = 1.350894 - \dfrac{0.1143727}{4.47473} = 1.31342$

$f(x_4) = -0.04767$

$f'(x_4) = 4.175216$

$x_5 = 1.31342 + \frac{0.04767}{4.17526} = 1.324838$

$f(x_5) = 0.00051474$

$f'(x_5) = 4.265587.$

$x_6 = 1.324838 - \frac{0.00051474}{4.265587} = 1.324717$

$f(x_6) = 0.00000268.$

$f'(x_6) = 4.264625$

The roots of equation $x^3 - x - 1$ is 1.324717.

5) If $A^2 = A$

$\Rightarrow 7A - (I+A)^3$

$= 7A - (I + A^3 + 3IA + 3A^2)$

$= 7A - (I + A.A^2 + 3A + 3A^2)$

$= 7A - (I + A.A + 3A + 3A)$

$= 7A - (I + A + 3A + 3A)$

$= 7A - (I + 7A)$

$= -I.$

$= -I.$

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6) $\begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix}$

$\Rightarrow$ ~~Adjoint~~ = $|A| = 1(0-6) - (-1)(-4) + 2(4)$

$= -6 - 4 + 8$

$= -2 \neq 0$

so A^{-1} exists.

Adjoint A = $\frac{1}{|A|}$

$\Rightarrow \begin{bmatrix} -6 & +4 & 4 \\ +1 & -1 & -1 \\ -6 & +2 & 4 \end{bmatrix}^T$

$\Rightarrow \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & 1 & 4 \end{bmatrix}$

$A^{-1} = \frac{\text{adj } A}{|A|}$

$= \begin{bmatrix} 3 & -1/2 & 3 \\ -2 & 1/2 & -1 \\ -2 & -1/2 & -2 \end{bmatrix}$

7)

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$$

$$\vec{a} = 8\hat{i} - 9\hat{j}$$
$$\vec{b} = 7\hat{i} - 9\hat{j}$$

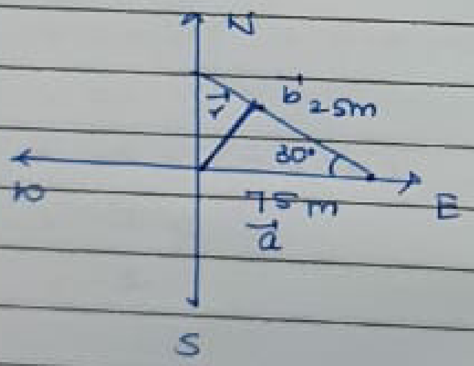
$$|\vec{a}| = \sqrt{64+72} = \sqrt{136} = 2\sqrt{34}$$
$$|\vec{b}| = \sqrt{49+81} = \sqrt{130} = \sqrt{130}.$$

$$\cos\theta = \frac{(8\hat{i}-9\hat{j}).(7\hat{i}-9\hat{j})}{2\sqrt{34}\times\sqrt{130}}$$

$$= \frac{56+81}{2\sqrt{34}\sqrt{130}} \quad \text{or} \quad \frac{137}{2\sqrt{34\times130}}$$

$$\theta = \cos^{-1}\left(\frac{137}{2\sqrt{34\times130}}\right)$$

8)



$$\vec{R} = \vec{a} + \vec{b}$$
$$R^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}.\vec{b}$$
$$= |\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|\cos 30^\circ$$
$$= 75^2 + 25^2 + 2(75)(25)\frac{\sqrt{3}}{2}.$$

$$R^2 = 9497.59$$
$$R = 97.46\ m$$

9) $a = 101$.

$d = 3$

$a_n = 998$

$a_n = a + (n-1)d$

$998 = 101 + (n-1)3$

$n - 1 = 299$

$n = 300$

$S = \frac{n}{2}(a + a_n)$

$= \frac{300}{2}(101 + 998)$

$= 150 \times 1099$

$= 164850$

10) $ar^5 = 32$

$ar^7 = 128$

$\frac{ar^7}{ar^5} = \frac{128}{32}$

$r^2 = 4$

$r = 2$.

12) Binomial theorum + $(4+3x)^5$

$\Rightarrow {}^5C_0(3x)^5 + {}^5C_1(3x)^4(4) + {}^5C_2(3x)^3(4)^2$

$+ {}^5C_3(3x)^2(4)^3 + {}^5C_4(3x)(4)^4 + {}^5C_5(4)^5$

$\Rightarrow 1(243x^5) + 5(81x^4)(4) + 10(27x^3)(16)$
$+ 10(9x^2)(64) + 5(3x)(256) + (1024)$

$\Rightarrow 243x^5 + 1620x^4 + 4320x^3 + 5760x^2 + 3840x + 1024$

12) $\begin{pmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{pmatrix}$

We know $|A - \lambda I| = 0$.

$$\begin{pmatrix} 2-\lambda & -3 & 0 \\ 2 & -5-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{pmatrix} = 0.$$

$\Rightarrow -(2-\lambda)(+5+\lambda)(3-\lambda) - (3)((2)(3-\lambda) - 0) + 0 = 0$

$\Rightarrow (-2+\lambda)(15 - \lambda^2 - 2\lambda) + 3(6 - 2\lambda)$

$\Rightarrow (-30 + 2\lambda^2 + 4\lambda + 15\lambda - \lambda^3 - 2\lambda^2) + 18 - 6\lambda$.

$\Rightarrow -30 + 18 + 19\lambda - 6\lambda - \lambda^3 = 0$

$\lambda^3 - 13\lambda + 12 = 0$.

Let $\lambda = 1$.

$(1)^3 - (13)(1) + 12 = 0$.

so $\lambda = 1$ is one of the roots.

Using synthetic division

	x^3		-13λ	$+12$
1	1	0	-13	12
		1	1	-12
	1	1	-12	0

$\lambda^2 + \lambda - 12 = 0$

$\lambda^2 + 4\lambda - 3\lambda - 12$

$\lambda(\lambda+4) - 3(\lambda+4) = 0$

$\lambda = 3$

$\lambda = -4$

Eigenvalues $= 1, 3, -4$

13) $f(x) = x^3 - 7x^2 + 8x - 3$

$f'(x) = 3x^2 - 14x + 8$.

$x_0 = 5$

$f(x_0) = -13$

$f'(x_0) = 13$.

$x_1 = 5 - \frac{(-13)}{13} \Rightarrow 6$.

$f(x_1) = 9$

$f'(x_1) = 32$.

$x_2 = 6 - \frac{9}{32} = 5.71875$

14) $1 - x = y$.

Then $(1-x+x^2)^4 = (y+x^2)^4$

$\Rightarrow {}^4C_0\, y^4 (x^2)^0 + {}^4C_1 (y^3)(x^2) + {}^4C_2\, y^2 (x^2)^2 + {}^4C_3\, y (x^2)^3 + {}^4C_4 (x^2)^4$

$\Rightarrow y^4 + 4y^3x^2 + 6y^2x^4 + 4yx^6 + x^8$

$\Rightarrow (1-x)^4 + 4x^2(1-x)^3 + 6x^4(1-x)^2 + 4x^6(1-x) + x^8$

$\Rightarrow 1 - 4x + 10x^2 - 16x^3 + 19x^4 - 16x^5 + 10x^6 - 4x^7 + x^8$

15)

$$e^{-x} = 3\log(x)$$

$$e^{-x} - 3\log(x) = 0$$

x	y
0.5	1.5096
1	0.3678
1.5	-0.305

$$x_1 = \frac{1+1.5}{2} = 1.25$$

$$y_1 = -0.004225$$

$$x_2 = \frac{1.25+1}{2} = 1.125$$

$$y_2 = 0.179$$

$$x_3 = \frac{1.125+1.25}{2} = 1.1875$$

$$y_3 = 0.08108$$

$$x_4 = 1.1875+1.25$$

$$= 1.21875$$