CMI - Assignment 1

Q1

(i) LHS is:

$(Ia)_{\overline{n}|} = v + 2v^2 + 3v^3 + \dots + nv^n \rightarrow (a)$

Multiplying b.s by $(1+i)$

$\therefore (1+i)(Ia)_{\overline{n}|} = 1 + 2v + 3v^2 + \dots + nv^{n-1} \quad (b)$

Now, (b - a),

$i(Ia)_{\overline{n}|} = 1 + v + v^2 + v^3 + \dots + v^{n-1} - nv^n$

$= \ddot{a}_{\overline{n}|} - nv^n$

$\therefore (Ia)_{\overline{n}|} = \dfrac{(\ddot{a}_{\overline{n}|} - nv^n)}{i}$

Hence proved.

(ii) PV of contributions $= 200000 \times \ddot{a}_{\overline{10}|} \times X(Ia)_{\overline{9}|}$

$i = 10\%$. $v = 0.9091$, $d = i/1+i = 9.091\%$.

$\therefore \ddot{a}_{\overline{10}|} = \dfrac{(1 - v^{10})}{d} = 6.7590$

$(Ia)_{\overline{9}|} = (\ddot{a}_{\overline{9}|} - 9v^9)/i = 6.3349$

$(Ia)_{\overline{9}|} = (6.3349 - 9 \times 0.4241)/0.1 = 25.1805$

$\therefore$ PV of contributions $= 200000 \times 6.7590 + X \times 25.1805$

$= 1351805.2 + X \times 25.1805$

PV of higher education expense $= 3000000 \times (1.06)^{10} v^{10}$ @ 10%

$= 2071350$

$\therefore 2071350 = 1351805.2 + X 25.1805$

$\therefore X = 28575$ (approximately)

Q2

$$E[\text{PV of benefits}] = 100000 \int_0^{20} v^t \cdot {}_tp_{40}^{hh} (\mu_{40+t} + \sigma_{40+t})\, dt$$

$$= 100000 \int_0^{20} e^{-\ln(1.05)t} \, {}_tp_{40}^{hh} \; 0.006 \, dt$$

$${}_tp_{40}^{hh} = {}_tp_{40}^{hh} = \exp\left(-\int_{40}^{40+t} (\mu_s + \sigma_s)\, ds\right) = e^{-0.006t}$$

$$\therefore E[\text{PV of benefits}] = 100000 \times 0.006 \int_0^{20} e^{-\ln(1.05)t} e^{-0.006t} \, dt$$

$$= 600 \int_0^{20} e^{-\ln(1.05)t} e^{-0.006t} \, dt$$

$$= 600 \int_0^{20} e^{-0.5479t} \, dt$$

$$= 600 \times \left(\frac{-e^{-0.5479t}}{0.5479} \right)_0^{20}$$

$$= 600(-6.10097 + 18.25151)$$

$$= 7290.3.$$

Q3

(i) Assuming claim is payable at end only.

$$\text{Premium} = P \ddot{a}^{(12)}_{40:\overline{20|}} = P\left[\ddot{a}_{40:\overline{20|}} - \frac{11}{24}\left(1 - \frac{P_{60}}{P_{40}}\right)\right]$$

$$= P\left[13927 - \frac{11}{24}\left(1 - \frac{882.95}{2052.96}\right)\right] = 13.6658 P.$$

$$\text{Claim Cost} = 7500000 \, A^1_{40:\overline{20|}} + 5000000 \, A^1_{[40]:\overline{20|}}$$

$$= 7500000\left[A^1_{40:20} - \frac{P_{60}}{P_{40}}\right] + 5000000\left[A^1_{[40]:\overline{20|}} - \frac{P_{60}}{P_{[40]}}\right]$$

$$= 750000\left[0.46433 - \frac{882.85}{2052.96}\right] + 5000000\left[0.46423 - \frac{882.85}{2052.54}\right]$$

$$= 427714.966$$

Commission expense $= 0.3\,P\ddot{a}^{(12)}_{40:\overline{1}|} + 0.05\,P\ddot{a}^{(12)}_{40:\overline{20}|}$

$= 0.3\,P\left[\ddot{a}_{40:1} - \frac{11}{24}\left(1 - \frac{P_{41}}{40}\right)\right] + 0.05P \times 13.6658$

$= 0.3P\left[\frac{1}{1.04}(1 - 0.000937) - \frac{11}{24} \times \left(1 - \frac{1972.15}{2052.96}\right)\right]$

$+ 0.683288\,P$

$= 0.966067\,P$

Management expense $= 0.1\,P \times \ddot{a}^{(12)}_{40:\overline{20}|} \times 1.04^{0.5}$

$= 0.1\,P \times 13.6658 \times 1.04^{0.5}$

$= 1.39364\,P$

Underwriting expense = 500

Calculation of monthly premium

$13.6658\,P = 427714.966 + 12831.4489\,P + 1.39364\,P + 500$

$\therefore 11.30607\,P = 441046.448$

$\therefore P = 39009.7$

(ii) Calculation of reserve $= 7500000\,A^{1}_{45:\overline{15}|} - 39009.7\,\ddot{a}^{(12)}_{45:\overline{15}|}$

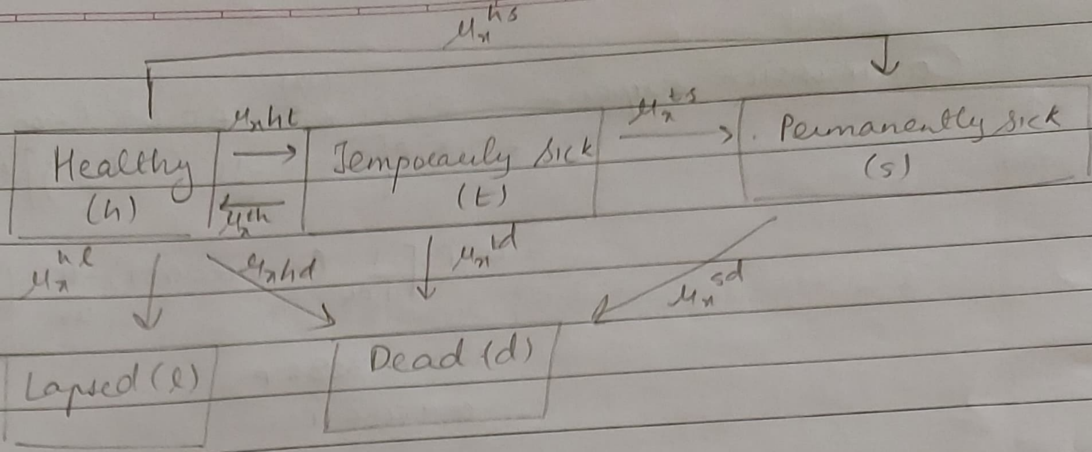
$= 7500000\left[A_{45:15} - \frac{P_{60}}{P_{45}}\right] - 39009.7\left[\ddot{a}_{45:\overline{15}|} - \frac{11}{24}\left(1 - \frac{P_{60}}{P_{45}}\right)\right]$

$= 7500000\left(0.56206 - \frac{882.85}{1677.97}\right) - 39009.7\left[11.386 - \frac{11}{24}\left(1 - \frac{882.85}{1677.97}\right)\right]$

$= 269387.1975 - 434750.6352$

$= -165363.4377$

Q4
(i)



(ii) EPV (premiums) = EPV(benefits)

EPV (temporary sickness benefit) = $10000 \int_{t=0}^{15} v^t \, {}_tp_{50}^{ht} \, dt$

EPV (permanent sickness benefit) = $20000 \int_{t=0}^{15} v^t \, {}_tp_{50}^{hs} \, dt$

Death benefit: On death at time t, benefit of 100000 would be payable regardless of which state is then occupied.

∴ EPV (death benefit) = $100000 \int_{t=0}^{15} v^t \left({}_tp_{50}^{hh} \mu_{50+t}^{hd} + {}_tp_{50}^{ht} \mu_{50+t}^{td} + {}_tp_{50}^{hs} \mu_{50+t}^{sd} \right) dt$.

EPV (premium) = $P \int_{t=0}^{15} v^t \, {}_tp_{50}^{hh} \, dt$

$$\therefore P = \frac{\left\{ 10000 \int_{t=0}^{15} v^t \, {}_tp_{50}^{ht} \, dt + 20000 \int_{0}^{15} v^t \, {}_tp_{50}^{hs} \, dt + 100000 \int_{0}^{15} v^t \left({}_tp_{50}^{hh} \mu_{50+t}^{hd} + {}_tp_{50}^{ht} \mu_{50+t}^{td} + {}_tp_{50}^{hs} \mu_{50+t}^{sd} \right) dt \right\}}{\left\{ \int_0^{15} v^t \, {}_tp_{50}^{hh} \, dt \right\}}$$

Q5

(i) EPV (premiums) = $P\ddot{a}_{[40]:25}$ = $15.887P$

EPV (expenses) $= 1200 + 500\,\bar{A}_{[40]} + 2\% \times P \times (\ddot{a}_{[40]:25} - 1)$
$= 1200 + 500\,(1.04)^{0.5} \times 0.23041 + 0.02 \times P \times (15.887 - 1)$
$= 1317.487 + 0.29774P$

EPV (benefits) $= 200000\,(v^{0.5}\,q_{[40]} + v^{1.5}(1.04)\,{}_{1|}q_{[40]} + v^{2.5}(1.04)^2\,{}_{2|}q_{[40]} + \dots)$
$= \dfrac{200000}{1.04^{0.5}}\,(q_{[40]} + (1.04)\,{}_{1|}q_{[40]} + (1.04)^2\,{}_{2|}q_{[40]} + \dots)$

$= \dfrac{200000}{(1.04)^{0.5}} = 196116.14$

$\therefore 15.887P = 196116.14 + 1317.487 + 0.29774P$
$\therefore P = 12664$

(ii) Company charges same premium rate, simple reversionary bonus rate can be calculated by equaling the EPV of benefits of 2 policy.

$\therefore$ EPV (benefits) $= 200000 \times x\%\,(I\bar{A})_{[40]} + 200000\,(1 - x\%)\,\bar{A}_{[40]}$
$= (1.04)^{0.5}[200000 \times x\%\,(IA)_{[40]} + 200000\,(1 - x\%)\,A_{[40]}]$

$\therefore (1.04)^{0.5}[200000\,x\%\,(IA)_{[40]} + 200000\,(1 - x\%)\,A_{[40]}] = 196116.14$
$\therefore x\% = 9\%$

(iii) $P\ddot{a}_{40:25} = 200000\,\bar{A}_{40}$ $\therefore P = 2960.54$

${}_{10}V = 300000\,\bar{A}_{50} - P\ddot{a}_{50:15}$
$= 300000 \times 1.04^{0.5} \times 0.32907 - 2960.54 \times 11.253$
$= 67361.1$

Q6

Let force of decrement due to death be represented as μ_x, force of decrement due to diagnosis of cancer as σ_x

Value of the benefits on death and cancer is

$$V = 100000 \int_0^{20} v^t \cdot {}_tp_{45}^{hh} (\mu_{45+t} + \sigma_{45+t})\, dt$$

$$100000 \int_0^{20} e^{-\ln(1.05)t} \cdot {}_tp_{45+t}^{hh} \; 0.008\, dt.$$

$${}_tp^{HH} = \exp\left(-\int_{45}^{45+t} (\mu_s + \sigma_s)\cdot ds\right)$$

$$= -\exp\left(-\int_{45}^{45+t} 0.008\, ds\right) = e^{-0.008t}$$

$$\therefore V = 800 \cdot \int_0^{20} \exp^{-\ln(1.05)t} \cdot e^{-0.008t}\, dt.$$

$$= 800 \int_0^{20} e^{-0.05679t}\, dt$$

$$= 800 \left[\frac{-e^{-0.05679t}}{-005679}\right]_0^{20} = 800(-5.65531 + 17.60873) = 9562.8$$

$$\text{Value of return of premium} = P \times e^{\int_{45}^{65}(0.006+0.002)\,dx} \cdot v^{20}$$

$$= P \times e^{-0.008(65-45)} \cdot v^{20}$$

$$= 0.37689 \times P \times e^{-0.16}$$

Premium = Value of benefits on death & cancer + value of return on premiums + Commission + expenses + Profit margin

$$= 9562.8 + 0.321164 \times P + 0.02 \times P + 0.1 \times P$$

$$= 17747.15$$

Q7

PV of Net CF = PV of premium − PV of benefits − PV of expenses − PV of commission

Let max. initial commission be I.

PV of premium $= 30000 \times \ddot{a}_{[45]:\overline{20|}} = 30000 \times 11.888$
$= 356640$

PV of benefits $= 1000000 \times A_{[45]:\overline{20|}} = 1000000 \times 0.32711$
$= 327110$

PV of expenses $= 5000 + 2.5\% \times 30000 \left(\ddot{a}_{[45]:\overline{20|}} - 1\right) +$
$500 \times P_{[45]} \times (1.06)^{-1} \times (1 + 1.019231\, v \times {}_1P_{[46]} +$
$1.019231^2 \cdot v^2 \cdot {}_2P_{[46]} \dots + 1.019231^{18}\, v^{18}\, {}_{18}P_{[46]})$
$= 5000 + 2.5\% \times 3000 \times (\ddot{a}_{[45]:\overline{20|}} - 1) + 500 \times P_{[45]} \times$
$(1.06)^{-1} \times \ddot{a}_{[66]:\overline{19|}}$ @ 4%
$= 5000 + 2.5\% \times 30000 \times 10.888 + 500 \left(\frac{9783.3371}{9798.0837}\right)$
$\times (1.06)^{-1} \times 13.316$
$= 19438$

PV of renewal commission $= 2\% \times 30000 \times (\ddot{a}_{[45]:\overline{20|}} - 1)$
$= 2\% \times 30000\ (10.888)$
$= 6533$

∴ PV of Net CF $= 356640 - 327110 - 19438 - 6533 - I$
$= 3559 - I$

Margin = PV of Net CF / Annual Prem $= (3559 - I) / 30000$
$= 10\%$

∴ I = Rs 559

Q8

(i) The assumption of Independence of decrements is used to derive single decrement table from a multiple decrement table.

$(a\mu)_x^j = \mu_x^j$ for all j and all x.

When we look at transition intensities (forces of decrement) we are looking at infinitesimally small time interval in which there is only time for one decrement. Thus it is reasonable to assume that the independent & dependent forces of increment are equal.

(ii) In case of term assurance, it is likely that people who lapse their policies have a lower than avg mortality.

(iii) a) $1000 \int_0^5 e^{-\delta t} \left({}_tP_{45}^{PI} \mu_{45+t} + {}_tP_{45}^{PP} \nu_{45+t} \right) dt.$

b) $250 \int_0^5 e^{-\delta t} \, {}_tP_{45}^{II} \, \nu_{45+t} \, dt.$

c) $150 \int_0^5 e^{-\delta t} \, {}_tP_{45}^{II} \, dt$

d) $250 \int_0^5 e^{-\delta t} \, {}_tP_{45}^{PP} \, g_{45+t} \, dt.$

(iv) $(aq)_{45}^c = \frac{0.05}{0.18} (1 - e^{-0.18}) = 0.045758$

$(aq)_{45}^s = \frac{0.13}{0.18} (1 - e^{-0.18}) = 0.118972$

$(ap)_{45} = 1 - 0.045758 - 0.118972 = 0.835270$

$$(aq)^{c}_{46} = \frac{0.06}{0.16}\left(1-e^{-0.16}\right) = 0.055446$$

$$(aq)^{s}_{46} = \frac{0.1}{0.16}\left(1-e^{-0.16}\right) = 0.092410$$

$$(ap)_{46} = 1-0.055446-0.092410 = 0.852144$$

∴ The prob of being in state inforce at the beginning of age 47 $= 0.835270 \times 0.852144 = 0.71770$.

Q9

(i) Under with profit policies the contract between policyholder and the company is such that whatever is the surplus arising on the underlying fund, would be shared between policyholders and shareholders in a predetermined manner. Now, as the surplus arises over future years the same is shared with policyholders in form of bonuses and hence the need of bonuses under with profit policies.

(ii) Investment surplus, expense surplus, lapse/surrender surplus, mortality surplus, reinsurance surplus.

(iii) Terminal bonus could be a strategy to build lower guarantees over the lifetime of with profit policies. For example if the proportion of RB and TB is skewed towards TB then it would mean lower guarantees to policyholders & hence the companies would have greater freedom in terms of choosing the investment assets. By doing this they can take risk of investing in real assets like equities & real estate which can lead to better final returns to policyholders which would be distributed through higher TB

Q 10

(i) Let monthly premium be 'P'

EPV (premiums) = $12 \times P \times \ddot{a}^{(12)}_{50:\overline{10|}}$ @ 6%.

$\ddot{a}^{(12)}_{50:\overline{10|}} = \ddot{a}_{50:\overline{10|}} - (11/24)(1 - {}_{10}P_{50} \times v^{10})$

$\ddot{a}_{50:\overline{10|}} = 7.694$

$\therefore \ddot{a}^{(12)}_{50:\overline{10|}} = 7.694 - (11/24) \times \left(1 - 1.06^{10}\left(\frac{9287.2164}{9712.0728}\right)\right)$

$= 7.4804$

$\therefore$ EPV (premiums) $= 12 \times P \times 7.4804$

$= 89.7648P.$

EPV (benefits) $= 1000000 \left(q_{50} v + 1.019231 v^2 \times {}_{1|}q_{50} + \dots + 1.019231^{14} \cdot v^{15} {}_{14|}q_{50} + 1.019231^{15} v^{15} \cdot {}_{15}P_{50}\right)$

$= 1000000 \times \frac{1}{1.1019231} \times q_{50} \times v \times 1.019231 + 1.019231^2 v^2 {}_{1|}q_{50} + \dots + 1.019231^{15} v^{15} {}_{14|}q_{50} + 1.019231^{15} v^{15} {}_{15}P_{50})$

Converting to 4%.

EPV (benefits) $= 1000000 (1/1.019231) \times (A_{50:\overline{15|}}$ @ 4%) + $1000000 v^{15}$ @ 6%. v ${}_{15}P_{50} \times (1.019231^{15} - 1.019231^{14})$

$A_{50:\overline{15|}} = 0.56719$

$\therefore$ EPV (benefits) $= 566004.1$

EPV (expense & commission) $= (20+35)/100 \times 12P + (2+1.5)/100 \times {}_{1}P_{50} \times 1.06^{-1} \times 12P \times \ddot{a}^{(12)}_{51:\overline{9|}} + 2500 + 400 \times {}_{1}P_{50} \times 1.06^{-1} \times \ddot{a}_{51:\overline{14|}}$ @ 6%

$= 9.3333P + 6115.06$

EPV (premiums) = EPV of benefits + EPV of expenses & commission

$\therefore 89.7648P = 566004.1 + 9.3333P + 6115.06$

$\therefore P = 7113.12$ INR

(ii) $X = 1000000 \times 1.02^{10}$

$\therefore X = 1218994$

PV of future benefits at end of yr 10 =

$$\frac{X}{1.05} \times q_{60} + \frac{X}{1.05^2} \times 1.04 \times {}_{1|}q_{60} + \frac{X}{1.05^3} \times 1.04^2 \, {}_{2|}q_{60} +$$

$$\ldots + \frac{X}{1.05^5} \times 1.04^5 \times {}_5p_{60}$$

$= 0.952948X$

$= 1160774$

PV(expenses) = $400 \, \ddot{a}_{60:\overline{5}|}$ @5%

$$= 400 \times \left(1 + 1.05^{-1} \times \frac{l_{61}}{l_{60}} + 1.05^{-2} \times \frac{l_{62}}{l_{60}} + 1.05^{-3} \times \frac{l_{63}}{l_{60}} + 1.05^{-4} \frac{l_{64}}{l_{60}}\right)$$

$= 1787.39$

Reserves req = PV of benefits + PV(expenses)

= 1162561 INR.

Q11 Let net premium be P

$P \times \ddot{a}_{30:\overline{20}|} = 400000 \times A_{30:\overline{20}|}$

$P = 400000 \times 0.315877 / 12.08705 = 10451.42$

We need to calculate the net premium prospective reserve at time 7

Total bonus till date = $7 \times 4\% \times 400000 = 112000$

$\therefore$ Guaranteed SA now $= 400000 + 112000 = 512000$

$\therefore$ Net premium at time 7 $= 512000 \times A_{37:\overline{13}|} - 10451.42 \, \ddot{a}_{37:\overline{13}|}$

$= 512000 \times 0.471638 - 10451.42 \times 9.33382$

$= 143926.89$