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Assignment - II

1.

→ $r = 12.65$ cm.

$r_{true} = 12.5$ cm.

Area $= \pi r^2 = 3.14\,(12.65)^2 = 502.47065$ cm^2.

$\text{Area}_{true} = \pi r_{true}^2 = 3.14\,(12.5)^2 = 490.625$ cm^2.

i) Absolute error $=$ ~~502.4~~ $\text{Area} - \text{Area}_{true} = 11.84565$ cm^2.

ii) Relative error $= \dfrac{\text{Absolute error}}{\text{Area}_{true}} = 0.024$

iii) Percentage error $= 2.4\%$.

2.

x	4	4.5	5.5	6.
$f(x) = \log x$	0.60206	0.6532125	0.7403627	0.7181513.

a) by lagrange's interpolation →

$$\log 5 = \frac{(5-4.5)(5-5.5)(5-6)}{(4-4.5)(4-5.5)(4-6)} \times 0.60206 \; +$$

$$\frac{(5-4)(5-5.5)(5-6)}{(4.5-4)(4.5-5.5)(4.5-6)} \times 0.6532125 \; +$$

$$\frac{(5-4)(5-4.5)(5-6)}{(5.5-4)(5.5-4.5)(5.5-6)} \times 0.7403627$$

$\Rightarrow \log 5 = -0.100343 + 0.435475 + 0.493575 - 0.129$

$\Rightarrow \log 5 = 0.699017.$

b). by $x = 4.5$ & $x = 5.5.$

$$\log 5 = \frac{(5-5.5)}{(4.5-5.5)} \times 0.6532125 + \frac{(5-4.5)}{(5.5-4.5)} \times 0.740$$

$= 0.326606 + 0.37018.$

$= 0.696786.$

3.

$\rightarrow f(x) = x^4 - x - 10.$

$a = 1.5 \qquad b = 2.$

$f(a) = -6.4375.$

$f(b) = 4.$

by bisection method $\Rightarrow$.

$x_1 = \frac{a+b}{2} = 1.75.$

$f(x_1) = (1.75)^4 - 1.75 - 10.$

$= -2.3711$

$x_2 = \frac{1.75+2}{2} = 1.875.$

$f(x_2) = (1.875)^4 - 1.875 - 10$

$= 0.4846.$

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$X_3 = \frac{1.75 + 1.875}{2} = 1.8125.$

$f(X_3) = -1.02625.$

$X_4 = \frac{1.8125 + 1.8175}{2} = 1.84375.$

$f(X_4) = -0.2877.$

$X_5 = \frac{1.84375 + 1.8175}{2} = 1.859375.$

$f(X_5) = 0.0934.$

$\therefore \boxed{X = 1.859375}$

4.

$\rightarrow f(x) = x^3 - x - 1.$

x	0	1	2
$f(x)$	-1	-1	5.

$f'(x) = 3x^2 - 1$

$X_0 = 0$
$f(X_0) = 5$
$f'(X_0) = 74.$

$X_1 = 1.93243.$
$f(X_1) = 4.28384.$
$f'(X_1) = 10.20286.$

$X_2 = 1.512563.$
$f(X_2) = 0.947952$
$f'(\cancel{X}_2) = 5.86354.$

$X_3 = 1.350894.$
$f(X_3) = 0.1143727$
$f'(X_3) = 4.47473.$

$x_4 = 1.324717$

$f(x_4) = 0.00000268$

$f'(x_4) = 4.26625$

$\therefore$ The root of this eqn $x^3 - x - 2$ is 1.324717

5

$\longrightarrow$ $A \to$ square matrix

$A^2 = A$

$7A - (I+A)^3 = 7A - (I^3 + A^3 + 3A^2 I + 3AI^2)$.

$IA = A$ & $I^3 = I$

$= 7A - (I + A^2 + 3IA + 3IA)$

$= 7A - (I + A + 6A)$

$= -I$

$\therefore$ $7A - (I+A)^3 = -I$

6

$\longrightarrow$ $\begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix}$

$|A| = 1(0-6) + 1(-4-0) + 2(4-0)$

$= -2$

$\neq 0$

$\therefore$ A^{-1} does not exist

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$$\therefore \text{Adj } A = \begin{bmatrix} -6 & 4 & 4 \\ 1 & -1 & -1 \\ -6 & 2 & 4 \end{bmatrix} = \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & -1 \\ 4 & -1 & 4 \end{bmatrix}.$$

$$\therefore A^{-1} = \frac{\text{adj} A}{|A|} = \begin{bmatrix} 3 & -1/2 & 3 \\ -2 & 1/2 & 1/2 \\ -2 & 1/2 & -2 \end{bmatrix}$$

7.

$\rightarrow$ $\vec{A} = 8\hat{i} - 9\hat{j}$

$\vec{B} = 7\hat{i} - 9\hat{j}$.

by scalar product :).

$$\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}| \cos\theta$$

$$(8\hat{i} - 9\hat{j})(7\hat{i} - 9\hat{j}) = \sqrt{8^2 + 9^2}\sqrt{7^2 + (-9)^2}\cos\theta$$

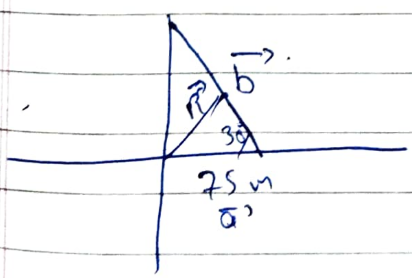
$$56 + 81 = \sqrt{145 \times 130}\cos\theta$$

$$\cos\theta = 0.997849.$$

$$\left[\theta = 3.75868\right]$$

8.

$\rightarrow$



$$\vec{R} = \vec{a} + \vec{b}.$$

$$\vec{R}^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}\cdot\vec{b}.$$

$$= 75^2 + 25^2 + 2(75)(25)\frac{\sqrt{3}}{2}$$

$$\vec{R}^2 = 9497.57.$$

$$\therefore \vec{R} = \sqrt{94757}$$

$$= 97.46 m.$$

9.

→ AP = 101, 104, 107, ..., 998

$a_n = a + (n-1)d$.

$998 = 101 + (n-1)3$

$\frac{897}{3} = n-1$

$\boxed{n = 300}$

$S_n = \frac{n}{2}\{a + a_n\}$

$= \frac{300}{2}[101 + 998]$

$= 150 \times 1099$

$= 164850$

10.

→ $a_6 = 32$. $a_8 = 128$

$ar^5 = 32$ — ① $ar^7 = 128$ — ②

②/① ⇒ $r^2 = 4$.

$\boxed{r = \pm 2}$

11.

→ Binomial Theorem $(4+3x)^5$

${}^5C_0 (3x)^5 (4)^0 + {}^5C_1 (3x)^4 (4)^1 + {}^5C_2 (3x)^3 (4)^2 + {}^5C_3 (3x)^2 (4)^3$

$+ {}^5C_4 (3x)^1 (4)^4 + {}^5C_5 (3x)^0 (4)^5$

$= 243x^5 + 1620x^4 + 4320x^3 + 5760x^2 + 3840x + 1024$

12.

$\rightarrow \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ $\quad |A - \lambda I| = 0$

13.

$\rightarrow f(x) = x^3 - 7x^2 + 8x - 3$

$x_0 = 5$

$f(x_0) = -13$

$f'(x_0) = 13$

Newton Raphson method $\rightarrow$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 5 - \frac{(-13)}{13} = 6$

$f(x_1) = 9$

$f'(x_1) = 32$

$x_2 = 6 - 9/32 = 5.71875$

14.

$\rightarrow$ To expand $= (1 - x + x^2)^4$

$(1 - x + x^2)^4 = [(1-x) + x^2]^4$

$= [1-x]^4 + 4[1-x]^3(x^2) + 6[1-x]^2(x^2)^2 + 4(1-x)(x^6) + x^8$

$= x^8 - 4x^7 + 10x^6 - 16x^5 + 19x^4 - 16x^3 + 10x^2 - 4x + 1$

15.

$\longrightarrow$ $e^{-x} = 3\log x.$

$e^{-x} - 3\log x = 0$

$f(x) = e^{-x} - 3\log x$

x	1	2
$f(x)$	0.3679	-0.7678

by bisection method $\Rightarrow$.

$x_1 = \frac{1+2}{2} = 1.5.$

$f(x_1) = -0.3051$

$x_2 = \frac{1+1.5}{2} = 1.25.$

$f(x_2) = -0.004225.$

$x_3 = 1.125.$

$f(x_3) = 0.1712$

$x_4 = 1.18275.$

$f(x_4) = 0.0811$

$\therefore \boxed{x = 1.18275}$