

Calculus Assignment 1

$$1. \lim_{h \rightarrow 0} \frac{2(-3+h)^2 - 18}{h}$$

$$= \lim_{h \rightarrow 0} \frac{18 - 12h + 2h^2 - 18}{h}$$

$$= \lim_{h \rightarrow 0} \frac{2h^2 - 12h}{h} = 2h - 12 = \underline{\underline{-12}}$$

$$2. g(n) = \begin{cases} 2n & n < 6 \\ n-1 & n \geq 6 \end{cases}$$

L.H.L

R.H.L

$$a. \lim_{n \rightarrow 4^-} g(n)$$

$$\lim_{n \rightarrow 4^+} g(n)$$

$$g(4) = 8$$

$$= \lim_{n \rightarrow 4^-} (2n) = 8$$

$$= \lim_{n \rightarrow 4^+} (2n) = 8$$

$\therefore$ Function is continuous at $n=4$

$$b. \text{ L.H.L}$$

$$\text{R.H.L}$$

$$\lim_{n \rightarrow 6^-} g(n)$$

$$\lim_{n \rightarrow 6^+} g(n)$$

$$\text{L.H.L} \neq \text{R.H.L}$$

$$= \lim_{n \rightarrow 6^-} (2n) = 12$$

$$= \lim_{n \rightarrow 6^+} (n-1) = 5$$

$\therefore$ Function is not continuous at $n=6$.

$$3. \lim_{n \rightarrow 9} \frac{n-9}{3-\sqrt{n}} = \lim_{n \rightarrow 9} \frac{(\sqrt{n})^2 - 3^2}{(3-\sqrt{n})}$$

$$= \lim_{n \rightarrow 9} \frac{(\sqrt{n}-3)(\sqrt{n}+3)}{-(\sqrt{n}-3)} = -(\sqrt{9}+3) = -6 ; \bullet$$

4. $f(x) = \frac{4x+5}{9-3x}$

a. ~~lim~~ $\lim_{x \rightarrow -1} f(x) = \lim_{x \rightarrow -1} \frac{4x+5}{9-3x} = \frac{1}{12} \therefore \text{continuous}$

b. $\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} \frac{4x+5}{9-3x} = \frac{5}{9} \therefore \text{continuous}$

c. $\lim_{x \rightarrow 3} f(x) = \lim_{x \rightarrow 3} \frac{4x+5}{9-3x} = \frac{17}{0} \therefore \text{not defined}$

$\therefore$ infinite discontinuous

5. $\lim_{x \rightarrow \infty} \frac{6e^{4x} - e^{-2x}}{8e^{4x} - e^{2x} + 3e^{-x}}$

$= \lim_{x \rightarrow \infty} \left(\frac{6e^{4x} - e^{-2x}}{e^{4x}} \right) \left(\frac{e^{4x}}{8e^{4x} - e^{2x} + 3e^{-x}} \right)$

$= \lim_{x \rightarrow \infty} \frac{(6 - e^{-6x})}{8 - e^{-2x} + 3e^{-5x}} = \frac{6}{8} = \frac{3}{4}$

6. $g(y) = \begin{cases} y^2+5 & \text{if } y < -2 \\ 1-3y & \text{if } y \geq -2 \end{cases}$

a. $\lim_{y \rightarrow 6} g(y) = \lim_{y \rightarrow 6} (1-3y) = -17$

b. L.H.L $\lim_{y \rightarrow -2^-} g(y) = \lim_{y \rightarrow -2^-} (y^2+5) = 9$

R.H.L $\lim_{y \rightarrow -2^+} g(y) = \lim_{y \rightarrow -2^+} (1-3y) = 7$

$g(-2) = 1-3(-2) = 7$

7. $f(t) = (4t^2 - t)(t^3 - 8t^2 + 12)$

$$\frac{d}{dt} f(t) = (4t^2 - t) \frac{d}{dt} (t^3 - 8t^2 + 12) + (t^3 - 8t^2 + 12) \frac{d}{dt} (4t^2 - t)$$

$$= (4t^2 - t)(3t^2 - 16t) + (t^3 - 8t^2 + 12)(8t - 1)$$

$$= 12t^4 - 64t^3 - 3t^3 + 16t^2 + 8t^4 - 64t^3 + 96t - t^3 + 8t^2 - 12$$

$$= 20t^4 - 132t^3 + 24t^2 - 12 + 96t$$

8. $R(w) = \frac{3w + w^4}{2w^2 + 1}$

$$\frac{d}{dw} (R(w)) = \frac{(2w^2 + 1)(3 + 4w^3) - (3w + w^4)(4w)}{(2w^2 + 1)^2}$$

9. ~~10.~~ $f(x) = 2e^x - 8^x$

$$f'(x) = 2e^x - 8^x \log 8$$

10. $y = x^5 - e^x \ln(x)$

$$\frac{dy}{dx} = 5x^4 - \left[\frac{e^x}{x} + \ln(x) e^x \right]$$

$$= 5x^4 - \frac{e^x}{x} + e^x \ln(x)$$

11.

a. $f(u) = (6u^2 + 7u)^4$
 $= 4(6u^2 + 7u)^3 (12u + 7)$

$$b. f(t) = 5 + e^{4t+t^7}$$

$$f'(t) = e^{4t+t^7} (4+t^6)$$

$$a. z = \ln(7-x^3)$$

$$\frac{dz}{dx} = \frac{0x - 1}{7-x^3} (3x^2)$$

$$\frac{d^2z}{dx^2} = - \left[\frac{(7-x^3)(6x) - (3x^2)(-3x^2)}{(7-x^3)^2} \right]$$

$$\frac{d^2z}{dx^2} = - \left[\frac{42x - 6x^4 + 9x^4}{(7-x^3)^2} \right] = \left[\frac{-42x - 3x^4}{(7-x^3)^2} \right]$$

$$b. Q(v) = \frac{2}{(6+2v-v^2)^4} = y$$

$$\frac{dy}{dv} = \frac{2(-4)}{(6+2v-v^2)^5} = \frac{-16(1-v)}{(6+2v-v^2)^5}$$

$$\frac{d^2y}{dv^2} = -16 \left[\frac{(6+2v-v^2)^5(-1) - (1-v)(6+2v-v^2)^4(5)(2-2v)}{(6+2v-v^2)^{10}} \right]$$

$$= \frac{-16(6+2v-v^2)^4}{(6+2v-v^2)^4} \left[\frac{(v^2-2v-6) - (1-v)(2-2v)}{(6+2v-v^2)^6} \right]$$

$$= -16 \left[\frac{(v^2-2v-6) - 2(1-v)^2}{(6+2v-v^2)^6} \right]$$

$$= \frac{(2(1-v)^2 - (v^2) + 2v + 6)}{(6+2v-v^2)^6} 16$$

$$c. f(t) = \ln(1+t^2)$$

$$f'(t) = \frac{1}{1+t^2} (2t)$$

$$\begin{aligned} f''(t) &= \frac{(1+t^2)(2) - (2t)(2t)}{(1+t^2)^2} \\ &= \frac{2+2t^2-4t^2}{(1+t^2)^2} = \frac{2-2t^2}{(1+t^2)^2} \end{aligned}$$

$$13. f(n) = y = n^3 - 3n^2 - 9n + 12$$

$$f'(n) = 3n^2 - 6n - 9$$

$$0 = 3n^2 - 6n - 9$$

$$0 = n^2 - 2n - 3$$

$$f''(n) = 6n - 6$$

$$0 = n^2 - 3n + n - 3$$

$$0 = n(n-3) + 1(n-3)$$

$$\begin{aligned} f''(-1) &= -6 - 6 \\ &= -12 \end{aligned}$$

$$0 = (n+1)(n-3)$$

$$n = -1 ; n = 3$$

$$f''(3) = 18 - 6 = 12 \quad \text{min}$$

$$\begin{aligned} f(-1) &= -1 - 3 + 9 + 12 \\ &= 17 \quad (\text{maximum value}) \end{aligned}$$

$$\begin{aligned} f(3) &= 27 - 27 - 27 + 12 \\ &= -15 \quad (\text{minimum value}) \end{aligned}$$

$$14. f(n) = 4n^3 - 18n^2 + 24n - 7$$

$$f'(n) = 12n^2 - 36n + 24$$

$$n^2 - 3n + 2 = 0$$

$$f''(n) = 24n - 36$$

$$= n^2 - n - 2n + 2$$

$$= n(n-1) - 2(n-1)$$

$$\begin{aligned} f''(2) &= 48 - 36 \\ &= 12 \end{aligned} \quad \begin{aligned} f''(1) &= 24 - 36 \\ &= -12 \end{aligned}$$

$$= (n-2)(n-1)$$

$$n = 2 ; n = 1$$

$$f(1) = 4 - 18 + 24 - 7$$

$$= 3$$

(maximum value)

$$f(2) = 32 - 72 + 48 - 7$$

$$= 80 - 119$$

$$= -39 + 40 \text{ (minimum value)}$$

$$= 1$$

15) $f(x) = x^2(x+3) = x^3 + 3x^2$

$$f'(x) = 3x^2 + 6x = 0$$

$$x^2 + 2x = 0$$

$$x(x+2) = 0$$

$$x = 0; x = -2$$

$$f''(x) = 6x + 6$$

$$f''(0) = 6$$

$\therefore$ (At 0, $f(x)$ has minimum value)

$$f''(-2) = -6$$

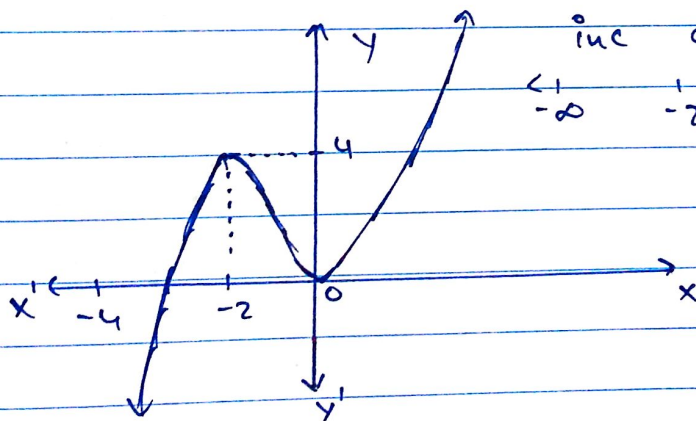
$\therefore$ (At -2, $f(x)$ has maximum value).

$$f(0) = 0 \text{ (minimum value)}$$

$$f(-2) = -8 + 12 = 4 \text{ (maximum value)}$$

$$f'(1) = 9$$

$$f'(-3) = 9$$



inc dec inc
 $< -\infty$ -2 0 ∞

16) $f(x) = 3x^2 - 6x$

$$f'(x) = 6x - 6$$

(x=1)

$$f''(x) = 6$$

$\therefore$ At $x=1$, $f(x)$ attains minimum value.

$$f(1) = -3$$

$$f'(0) = -6, f'(2) = 6$$

dec inc
 $< -\infty$ 1 ∞

