Calculus Assignment:

PAGE No.
DATE / /

Q.1 $\lim_{h\to 0} \frac{2(-3+h)^2-18}{h} = \lim_{h\to 0} \frac{2[9+h^2-6h]-18}{h}$

$= \lim_{h\to 0} \frac{(\cancel{18}+2h^2-12h)-\cancel{18}}{h} = \lim_{h\to 0} \frac{2\cancel{h}[h-6]}{\cancel{h}}$

Putting the limit,

$\rightarrow 2[0-6] = 2(-6) = -12$

Q.2 a.) $f(4) = 2\times 4 = 8$

$\lim_{x\to 4^-} f(x) = 2\times 4 = 8$, $\lim_{x\to 4^+} f(x) = 2\times 4 = 8$.

As $f(4) = \lim_{x\to 4^+} f(x) = \lim_{x\to 4^-} f(x) = 8$

$\therefore f(x)$ is continous at $x=4$

b.) $f(6) = 6-1 = 5$

$\lim_{x\to 6^+} f(x) = 6-1 = 5$. $\lim_{x\to 6^-} f(x) = 2\times(6) = 12$

As $\lim_{x\to 6^-} f(x) \neq \lim_{x\to 6^+} f(x)$, $f(x)$ is discontinous at $x=6$

Q.3 $\lim_{x\to 9} \frac{x-9}{3-\sqrt{x}} \rightarrow \lim_{x\to 9} = \frac{(\sqrt{x})^2-(3)^2}{3-\sqrt{x}}$ $\left\{ \begin{array}{l} a^2-b^2 \\ (a+b)(a-b) \end{array} \right.$

$\therefore \lim_{x\to 9} \frac{(\sqrt{x}+3)\cancel{(\sqrt{x}-3)}}{-1\cancel{(\sqrt{x}-3)}} = \lim_{x\to 9} -(\sqrt{x}+3)$

Putting Value of Limit.

$= -(\sqrt{9}+3) = -(3+3)$

$\boxed{\text{Ans: } -6}$

DATE / /

Q.4 $f(x) = \frac{4x+5}{9-3x}$

a.) when $x = -1$

$f(-1) = \frac{4(-1)+5}{9-(-1)(3)} = \frac{-4+5}{9+3} = \frac{1}{12}$

$\therefore f(-1)$ exists

$\lim_{x \to -1^-} \frac{4x+5}{9-3x} = \frac{4(-1)+5}{9-3(-1)} = \frac{1}{12}$

$\lim_{x \to -1^+} \frac{4x+5}{9-3x} = \frac{4(-1)+5}{9-3(-1)} = \frac{1}{12}.$

$\therefore$ As $\lim_{x \to -1^-} f(x) = \lim_{x \to -1^+} f(x) = f(-1)$

$\therefore$ the function $f(x)$ is continous at $\underline{x=-1}$.

b.) When $x = 0$

$f(0) = \frac{4(0)+5}{9-3(0)} = \frac{5}{9}$

$\lim_{x \to 0^-} \frac{4x+5}{9-3x} \to \frac{4(0)+5}{9-3(0)} = \frac{5}{9}$

$\lim_{x \to 0^+} \frac{4x+5}{9-3x} \to \frac{4(0)+5}{9-3(0)} = \frac{5}{9}$

$\therefore$ As $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x) = f(0)$

The function is continous at $x = 0$

c] When $x = 3$

$f(3) = \frac{4(3)+5}{9-3(3)} = \frac{17}{9-9} = \frac{17}{0} \to$ Not Defined

As $f(3)$ doesnot exist the function $f(x)$ is discontinous for $x = 3$.

DATE / /

Q.5 $\lim_{x\to\infty} \frac{6e^{4x} - e^{-2x}}{8e^{4x} - e^{2x} + 3e^{-x}} \rightarrow \lim_{x\to\infty} \frac{6e^{4x} - 1/e^{2x}}{8e^{4x} - e^{2x} + \frac{3}{e^{x}}}$

$\lim_{x\to\infty} \frac{\frac{6e^{6x}-1}{e^{2x}}}{\frac{8e^{5x} - e^{3x} + 3}{e^{x}}} \rightarrow \lim_{x\to\infty} \frac{6e^{6x-1x} - \frac{1}{e^{x}}}{8e^{5x} - e^{3x} + 3}$

$\rightarrow \lim_{x\to\infty} \frac{\cancel{e^{5x}}\{6 - e^{-6x}\}}{\cancel{e^{5x}}\{8 - e^{-2x} + 3e^{-5x}\}}$

Putting limits : $\frac{6 - \frac{1}{e^{6\times\infty}}}{\left(8 - \frac{1}{e^{2\infty}} + \frac{3}{e^{5\infty}}\right)} = \frac{6}{8} = \frac{3}{4}$

Q.6 $g(y) = \begin{cases} y^2+5 & y<-2 \\ 1-3y & y\geq -2 \end{cases}$

a.) $\lim_{y\to 6} g(y) \rightarrow \lim_{y\to 6} (1-3y)$

Putting limits : $1-3(6) = 1-18 = -17$

b.) $\lim_{y\to -2} g(y) \rightarrow \lim_{y\to -2} 1-3y = 1-3(-2)$

$= 1+6 = 7$

Q.7 $f(t) = (4t^2 - t)(t^3 - 8t^2 + 12)$

D wrt 't'

$$f'(t) = \frac{d(u.v)}{dt} = u \cdot \frac{dv}{dt} + v \cdot \frac{du}{dt}$$

$$f'(t) = (4t^2 - t)[3t^2 - 16t] + (t^3 - 8t^2 + 12)[8t - 1]$$

$$= (12t^4 - 3t^3 - 64t^3 + 16t^2) + \left[\begin{array}{l} 8t^4 - 64t^3 + 96t \\ - t^3 + 8t^2 - 12 \end{array}\right]$$

$$\rightarrow (12t^4 - 67t^3 + 16t^2) + [8t^4 - 65t^3 + 8t^2 + 96t - 12]$$

$$= 20t^4 - 132t^3 + 24t^2 + 96t - 12$$

$$\therefore f'(t) = 4[5t^4 - 33t^3 + 6t^2 + 24t - 3]$$

Q.8. $R(w) = \dfrac{3w + w^4}{2w^2 + 1}$

D wrt. 'w'

$$R'(w) \rightarrow \frac{d}{dw}\left(\frac{u}{v}\right) = \frac{v \cdot \frac{du}{dw} - u \cdot \frac{dv}{dw}}{v^2}$$

$$R'(w) = \frac{(2w^2 + 1)(3 + 4w^3) - (3w + w^4)(4w)}{(2w^2 + 1)^2}$$

$$\rightarrow \frac{[6w^2 + 3 + 8w^5 + 4w^3] - [12w^2 + 4w^5]}{(2w^2 + 1)^2}$$

$$= \frac{(-6w^2 + 3 + 4w^3 - 4w^5)}{(2w^2 + 1)^2}$$

$$R'(w) = \frac{[-4w^5 + 4w^3 - 6w^2 + 3]}{(2w^2 + 1)^2}$$

DATE / /

Q.9 $f(x) = 2e^x - 8^x$

D.w.r.t. x

$f'(x) = 2e^x(1) - 8^x \times \log 8$

$\Rightarrow$ ANS: $f'(x) = 2e^x - 8^x . \log 8$

Q.10 $y = z^5 - e^z . \log z$

D.w.r.t. 'y' 'z'

$\frac{dy}{dz} = 5z^4 - \left\{ e^z . \frac{d(\log z)}{dz} + \log z . \frac{d(e^z)}{dz} \right\} = \frac{dy}{dz}$

$\frac{dy}{dz} = (5z^4) - \left[e^z \times \frac{1}{z} + \log z . e^z \right]$

$\rightarrow \frac{dy}{dz} = 5z^4 - e^z \left[\log z + \frac{1}{z} \right]$

Q.11

a.) $f(x) = [6x^2 + 7x]^4$

D.w.r.t. 'x'

$f'(x) = 4[6x^2 + 7x]^3 \times \frac{d(6x^2 + 7x)}{dx}$

$= 4[6x^2 + 7x]^3 \times (12x + 7)$

ANS: $f'(x) = 4[6x^2 + 7x]^3 \times (12x + 7)$

b.) $f(t) = 5 + e^{4t + t^7}$

D.w.r.t. 't'

$f'(t) = 0 + e^{(4t + t^7)} \times \frac{d(4t + t^7)}{dt}$

$= e^{(4t + t^7)} \times (4 + 7t^6)$

$\therefore$ ANS: $f'(t) = e^{(4t + t^7)} \times (4 + 7t^6)$

PAGE No.

DATE / /

Q.12

a. $z = \log(7-x^3)$

Dwrt. 'x'

$$\frac{dz}{dx} = \frac{1}{(7-x^3)} \times \frac{d(7-x^3)}{dx} = \boxed{\frac{0-3x^2}{7-x^3} = \frac{dz}{dx}} = \frac{-3x^2}{7-x^3}$$

Again, dwrt. 'x'

$$\frac{d^2z}{dx^2} = (7-x^3)(-3x$$

$$\frac{d^2z}{dx^2} = \frac{(7-x^3)(-6x) - (-3x^2)[0-3x^2]}{(7-x^3)^2}$$

$$\frac{d^2z}{dx^2} = \frac{(-42x+6x^4) - (9x^4)}{(7-x^3)^2} = \frac{-42x-3x^4}{(7-x^3)^2}$$

$$\frac{d^2z}{dx^2} = \frac{-3x(14+x^3)}{(7-x^3)^2}$$

b. $Q(v) = \frac{2}{(6+2v-v^2)^4} = 2[6+v-v^2]^{-4}$

Dwrt. 'v'

$$Q'(v) = 2(-4) \times [6+v-v^2]^{-4-1} \times \frac{d}{dv}(6+v-v^2)$$

$$= -8[6+v-v^2]^{-5} \times (0+1-2v)$$

$$Q'(v) = \frac{-8(1-2v)}{(6+v-v^2)^{-5}} = \frac{(16v-8)}{(6+v-v^2)^5}$$

Dwrt. 'v'

$$Q''(v) \frac{(6+v-v^2)^5 \times (16) - (16v-8)\, 5[6+v-v^2]^4[1-2v]}{([6+v-v^2]^5)^2}$$

$$= \frac{16(6+v-v^2)^5 - [80v-40](1-2v)[6+v-v^2]^4}{[6+v-v^2]^{10}}$$

Q''(v

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PAGE No.
DATE / /

$$Q''(v) = \frac{16(6+v-v^2)^5 - (-160v^2+80v+80v-40)(6+v-v^2)^4}{[6+v-v^2]^{10}}$$

$$Q''(v) = \frac{16(6+v-v^2)^5 - (6+v-v^2)^4[-160v^2+160v-40]}{(6+v-v^2)^{10}}$$

$$= \frac{(6+v-v^2)^4[16(6+v-v^2) - (-160v^2+160v-40)]}{(6+v-v^2)^{\cancel{10}6}}$$

$$\therefore \frac{(96+16v-16v^2)+160v^2-160v+40}{(6+v-v^2)^6}$$

$$Q''(v) \to \frac{144v^2-144v+136}{(6+v-v^2)^6}$$

c. $f(t) = \log(1+t^2)$

Dwrt- 't'

$$f'(t) = \frac{1}{(1+t^2)} \times \frac{d(1+t^2)}{dt} = \frac{2t}{1+t^2}$$

Dwrt- 't'

$$f''(t) = \frac{(1+t^2)2 - 2t(2t)}{(1+t^2)^2} = \frac{2+2t^2-4t^2}{(1+t^2)^2}$$

$$= \frac{2-2t^2}{(1+t^2)^2} = \frac{2[1-t^2]}{(1+t^2)^2} = f''(t)$$

DATE / /

Q.13 $f(x) = x^3 - 3x^2 - 9x + 12$

* Diwrt. 'x'

$f'(x)$ $3x^2 - 6x - 9$

Let $f'(x) = 0$

$3[x^2 - 2x - 3] = 0$

$x^2 - 3x + 1x - 3 = 0$

$(x-3)(x+1) = 0$

$\therefore x = 3$ or $x = -1$

* Diwrt. 'x'

$f''(x) = 6x - 6$

When, $x = 3$

$f''(3) = 6(3) - 6 = 12$

$\therefore$ (+ve)

When $x = -1$

$f''(-1) = 6(-1) - 6 = -12$

(−ve)

$\therefore f(x)$ is maximum at $x = -1$

$f(x)$ is minimum at $x = 3$

* Max Value, put $x = -1$

$f(-1) = (-1)^3 - 3(-1)^2 - 9(-1) + 12$

$= (-1) - 3(1) + 9 + 12$

$= -4 + 21 = 17$ → Max Value = 17

* Min Value, put $x = 3$

$f(3) = (3)^3 - 3(3)^2 - 9(3) + 12$

$= \cancel{27} - 3\cancel{(9)} - 27 + 12$

$= -27 + 12 = -15$ → Min Value = −15

PAGE No.
DATE / /

Q.14 $f(x) = 4x^3 - 18x^2 + 24x - 7$

* Diwrt. x

$f'(x)$ $12x^2 - 36x + 24$.

Let $f'(x) = 0$

$6(2x^2 - 6x + 4) = 0$

$2x^2 - 2x - 4x + 4 = 0$

$2x(x-1) - 4(x-1) = 0$

$(x-1)(2x-4) = 0$

$x = 1$ or $x = 4/2$ $\therefore$ $x = 1$ or $x = 2$

* Diwrt. x

$f''(x)$ $24x - 36$

When $x = 1$

$f''(1)$ $24(1) - 36 = -12$ (-ve)

When $x = 2$

$f''(2) = 24(2) - 36 = 12$ (+ve)

$\therefore f(x)$ is maximum at $x = 1$

$f(x)$ is minimum at $x = 2$

* Max Value, put $x = 1$

$f(1) = 4(1) - 18(1) + 24(1) - 7$

$= 28 - 25 = 3$ $\rightarrow$ Max Value $= 3$

* Min Value, put $x = 2$

$f(2)$ $4(2)^3 - 18(2)^2 + 24(2) - 7$

$= 4(8) - 18(4) + 48 - 7$

$= 32 - 72 + 48 - 7$

$= 80 - 79 = 1$ $\rightarrow$ Min Value $= 1$

Q.15 $f(x) = x^2(x+3) = x^3 + 3x^2$

$f'(x)$: Diff w.r.t x : $f'(x) = 3x^2 + 6x$

Let $f'(x) = 0$

$3x(x+2) = 0$

$\therefore x = 0$ or $x = -2$

$f''(x)$: Diff w.r.t x : $\cancel{3x^2+6x}$ $6x+6$

When $x = 0$ | When $x = -2$
$f''(0) = 6(0)+6$ | $f''(-2) = 6(-2)+6$
$= 6$ | $= -6$
(+ve) | (-ve)

$\therefore f(x)$ is Maxima at $x = -2$

$f(x)$ is Minima at $x = 0$

When $x = -2$

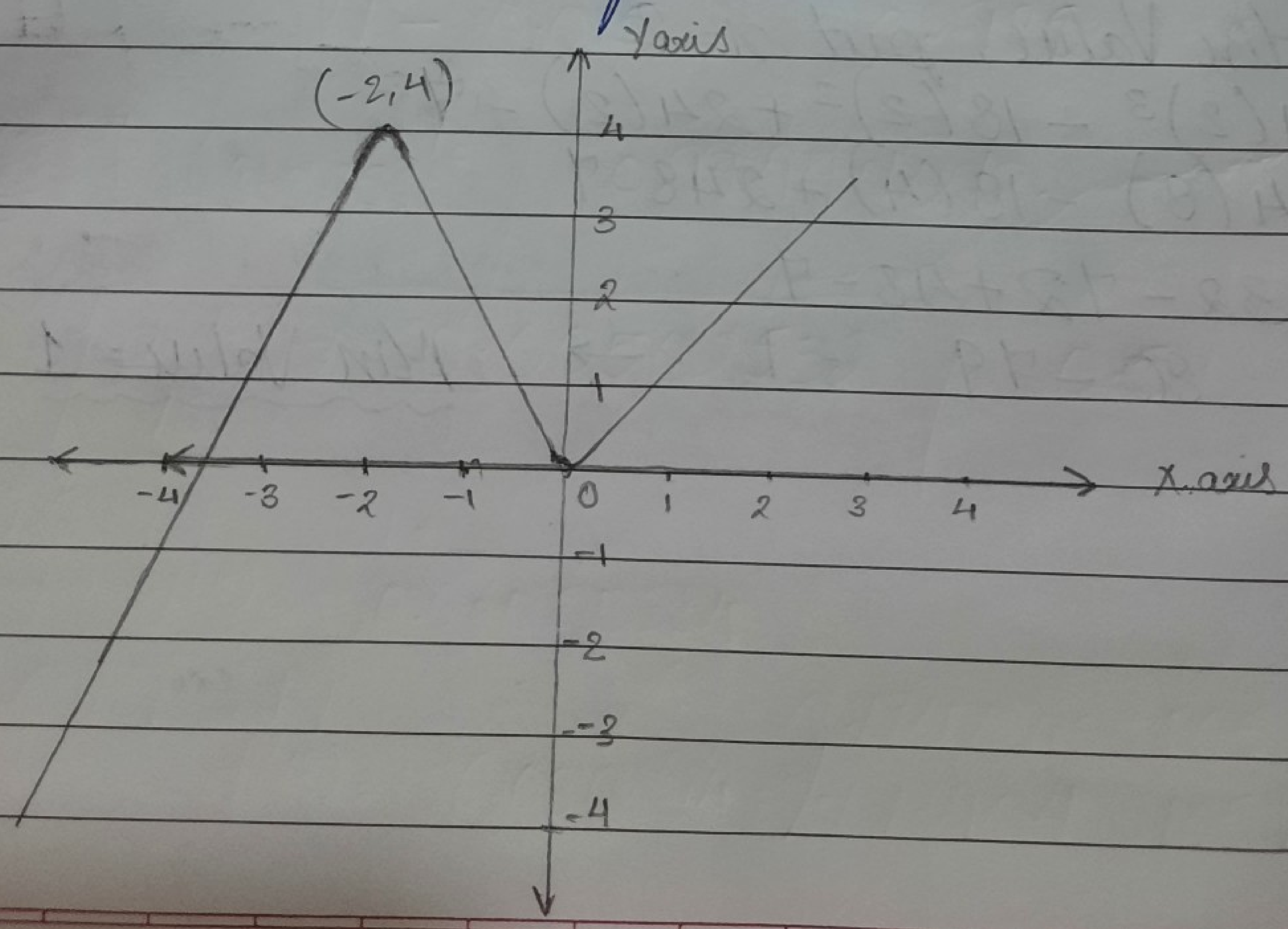
~~$f''(-2)$~~ $f(-2) = (-2)^2(-2+3) = 4(1) = 4$

When $x = 0$

$f(0) = (0)[0+3] = 0$

$f'(x) = 3x(x+2)$.

$\therefore f(x)$ is increasing for $x \in R$ except for $x = 0$ and $x = -2$



Q.16.

Let

Q.16. $f(x) = 3x^2 - 6x$

Dwrt x: $f'(x) = 6x - 6$

let $f'(x) = 0$ $\quad 6x - 6 = 0 \quad \boxed{x = 1}$

Dwrt: x.

$f''(x) = 6 \rightarrow +ve$

$\therefore f(x)$ is Minima at $x = 1$

Min Value: $f(1) = 3(1) - 6(1) = -3$

$f'(x) = 6x - 6 = 0 \quad x = 6/6 = 1$

$\therefore$ ~~f(x) is increasing~~.. $f(x)$ is increasing except for $x = 1$

$f(0) = 3(0) - 6(0) = 0$

$\therefore f(2) = 3(4) - 6(2) = 0$