Krishangi Amit Trivedi
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Calculus Assignment

1. $\lim_{h \to 0} \frac{2(-3+h)^2 - 18}{h}$

$= \lim_{h \to 0} \frac{2(9 - 6h + h^2) - 18}{h}$

$= \lim_{h \to 0} \frac{18 - 12h + 2h^2 - 18}{h}$

$= \lim_{h \to 0} \frac{2h^2 - 12h}{h}$

$= \lim_{h \to 0} \frac{2\cancel{h}(h-6)}{\cancel{h}}$

$= 2(0-6) = \underline{\underline{-12}}$

Q.2. (a) $x = 4$

$f(4) = 2(4) = 8$. hence $f(4)$ exists.

$\lim_{x \to 4^-} 2x = 2(4) = 8$ and $\lim_{x \to 4^+} 2x = 2(4) = 8$.

$\therefore \lim_{x \to 4^-} = \lim_{x \to 4^+} = \lim_{x \to 4}$

Hence the function is continuous

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(b) $x = 6$.

$f(6) = 6 - 1 = 5$. $\therefore$ $f(6)$ exists

$$\lim_{x \to 6^-} 2x = 2(6) = 12$$

$$\lim_{x \to 6^+} (x-1) = 6 - 1 = 5$$

$\lim_{x \to 6^-} \neq \lim_{x \to 6^+}$ hence the function is discontinuous

3. $\lim_{x\to 9} \dfrac{x-9}{3-\sqrt{x}}$

$$= \lim_{x\to 9} \frac{x-9}{3-\sqrt{x}} \times \frac{(3+\sqrt{x})}{(3+\sqrt{x})}$$

$$= \lim_{x\to 9} \frac{(x-9)\ (3+\sqrt{x})}{(9-x)}$$

$$= \lim_{x\to 9} \frac{-\cancel{(9-x)}\ (3+\sqrt{x})}{\cancel{(9-x)}}$$

$$= \lim_{x\to 9} -3-\sqrt{x}$$

$$= -3-\sqrt{9} = -3-3 = -6$$

4. (a) $x=-1$

$$f(x) = \frac{4(x)+5}{9-3x}.$$

$$f(-1) = \frac{4(-1)+5}{9-3(-1)}$$

$$= \frac{-4+5}{9+5} = \frac{1}{12}$$ hence $f(-1)$ exists.

$$\lim_{x\to -1} \frac{4x+5}{9-3x} = \frac{1}{12}$$ hence $\lim_{x\to -1} f(x)$ exists.

~~Ther~~ Therefore the function is <u>continous</u> at $x=(-1)$

(b) $x = 0$

$$f(x) = \frac{4x+5}{9-3x}$$

$$f(0) = \frac{4(0)+5}{9-3(0)} = \frac{5}{9}$$

hence; $f(0)$ exists.

$$\lim_{x \to 0} \frac{4(x)+5}{9-3x} = \frac{5}{9}$$ hence

$\lim_{x \to 0} f(x)$ exists.

Therefore function is continuous at $x=0$

(c) $x = 3$

$$f(x) = \frac{4x+5}{9-3x}$$

$$f(3) = \frac{4(3)+5}{9-3(3)} = \frac{17}{0}$$ hence

$f(3)$ does not exist.

Therefore function is discontinuous at $x=3$

Q.5. $$\lim_{x\to\infty} \frac{6e^{4x} - e^{-2x}}{8e^{4x} - e^{2x} + 3e^{-x}}$$

Divide by e^{-4x}

$$\lim_{x\to\infty} \frac{6 - e^{-6x}}{8 - e^{-2x} + 3e^{-5x}}$$

$$\frac{\lim_{x\to\infty} 6 - e^{-6x}}{\lim_{x\to\infty} 8 - e^{-2x} + 3e^{-5x}}$$

$$= \frac{6 - e^{-\infty}}{8 - e^{-\infty} + 3e^{-\infty}} = \frac{6-0}{8-0} = \frac{6}{8} = \frac{3}{4}$$

(6.) $g(y) = \begin{cases} y^2 + 5 & \text{if } y < -2 \\ 1 - 3y & \text{if } y \geq -2 \end{cases}$

(a) $\lim_{x \to 6} g(y)$

$= \lim_{x \to 6} 1 - 3y = 1 - 3(6)$

$= 1 - 18 = -17$

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Q6. (b) $\lim_{x \to -2} g(y)$

$\lim_{x \to -2^-} g(y) = (-2)^2 + 5 = 9$

$\lim_{x \to -2^+} g(y) = 1 - 3(-2) = 1 + 6 = 7$

$\lim_{x \to -2^-} \neq \lim_{x \to -2^+}$ hence limit does not exist.

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(7) $f(t) = (4t^2 - t)(t^3 - 8t^2 + 12)$

Let $y = (4t^2 - t)(t^3 - 8t^2 + 12)$

differentiating w.r.t 't'

$$\frac{dy}{dt} = (4t^2 - t)\frac{d}{dt}(t^3 - 8t^2 + 12) + (t^2 \cancel{\;} - 8t^2 + 12)\frac{d}{dt}(4t^2 - t)$$

$$= (4t^2 - t)(3t^2 - 16t) + (t^3 - 8t^2 + 12)(8t - 1)$$

$$= 12t^4 - 64t^3 - 3t^3 + 16t^2 + 8t^4 - 64t^3 + 96t \cancel{-12} - t^3 + 8t^2 - 12$$

$= \cancel{20t^4}$

$$= 20t^4 - 132t^3 + 24t^2 + 96t - 12$$

(8) $R(\omega) = \dfrac{3(\omega) + \omega^4}{2\omega^2 + 1}$

Let $R(\omega) = y$.

differentiation w.r.t 'ω' we

$$\frac{dy}{d\omega} = \frac{(2\omega^2 + 1)\frac{d}{d\omega}(3\omega + \omega^4) - (3\omega + \omega^4)\frac{d}{d\omega}(2\omega^2 + 1)}{(2\omega^2 + 1)^2}$$

$$= \frac{(2\omega^2 + 1)(3 + 4\omega^3) - (3\omega + \omega^4)(4\omega)}{(2\omega^2 + 1)^2}$$

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$$= \frac{(2\omega^2+1)(3+4\omega^3) - (3\omega+\omega^4)(4\omega)}{(2\omega^2+1)}$$

$$= \frac{6\omega^2 + 8\omega^5 + 3 + 4\omega^3 - 12\omega^2 \mp 4\omega^5}{(2\omega^2+1)^2}$$

$$= \frac{4\omega^5 + 4\omega^3 - 6\omega^2 + 3}{(2\omega^2+1)^2}$$

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9. $f(x) = 2e^x - 8^x$

Let $y = f(x)$

differentiating w.r.t. 'x'

$$\frac{dy}{dx} = \frac{d}{dx}(2e^x - 8^x)$$

$$= 2e^x - 8^x \log 8$$

$$= 2e^x - 8^x \log 8$$

10. $y = z^5 - e^z \ln(z)$

Differentiating w.r.t. 'z'

$$\frac{dy}{dz} = \frac{d}{dz} z^5 - e^z \ln(z)$$

$$= 5z^4 - \left[e^z \frac{d}{dz} \ln(z) + \ln(z) \frac{d}{dz} e^z \right]$$

$$= 5z^4 - \left[\frac{e^z}{z} + \ln(z)\, e^z \right]$$

$$= 5z^4 - \frac{e^z}{z} \cancel{+} - \ln(z)\, e^z$$

11. (a) $f(x) = (6x^2 + 7x)^4$

Let $f(x)$ be equal to y.

differentiating w.r.t. x

$$\frac{dy}{dx} = \frac{d}{dx} (6x^2 + 7x)^4$$

$$= 4(6x^2 + 7x)^3 \times \frac{d}{dx}(6x^2 + 7x)$$

$$= 4(6x^3 + 7x)^3 \times (12x + 7)$$

$$= 4(12x + 7)(6x^3 + 7x)^3$$

(b) $f(t) = 5 + e^{4t+t^7}$

Let $f(t) = y$

differentiating w.r.t. 't'

$$\frac{dy}{dt} = \frac{d}{dt}\left(5 + e^{4t+t^7}\right)$$

$$= 0 + e^{(4t+t^7)} \times \frac{d}{dt}\left(4t+t^7\right)$$

$$\frac{dy}{dt} = e^{(4t+t^7)} \times \left(7t^6+4\right)$$

(12) (a) $z = \ln(7-x^3)$

Differentiating w.r.t. 'x'

$$\frac{dz}{dx} = \frac{d}{dx}\left[\ln(7-x^3)\right]$$

$$= \frac{1}{(7-x^3)} \times \frac{d}{dx}(7-x^3)$$

$$= \frac{0-3x^2}{(7-x^3)}$$

$$\therefore \frac{dz}{dx} = \frac{-3x^2}{(7-x^3)} = \frac{3x^2}{(x^3-7)}$$

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Second order derivative.

$$\frac{d^2z}{dx^2} = \frac{d}{dx}\frac{(3x^2)}{(x^3-7)}$$

$$= \frac{(x^3-7)\frac{d}{dx}(3x^2) - 3x^2\frac{d}{dx}(x^3-7)}{(x^3-7)^2}$$

$$\frac{d^2z}{dx^2} = \frac{(x^3-7)(6x) - (3x^2)(3x^2)}{(x^3-7)^2}$$

$$= \frac{6x^4 - 42x - 9x^4}{(x^3-7)^2}$$

$$= \frac{-3x^4 - 42x}{(x^3-7)^2}$$

$$= \frac{-3(x^4+14x)}{(x^3-7)^2}$$

$$\cancel{\frac{d^2y}{}} = \frac{-3x(x^3+14)}{(x^3-7)^2} = \frac{d^2z}{dx^2}$$

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(b) $\phi(v) = \dfrac{2}{(6+2v-v^2)^4}$

Let $\phi(v) = y$

differtiate w.r.t. 'v'

$$\frac{dy}{dv} = \frac{d}{dv}\, 2(6+2v-v^2)^{-4}$$

$$= 2\ \cancel{2(}\ -8(6+2v-v^2)^{-5} \times (2-2v)$$

$$= \frac{-8(2-2v)}{(6+2v-v^2)^5}$$

$$= \frac{(16v-16)}{(6+2v-v^2)^5}$$

$$= \frac{16(v-1)}{(6+2v-v^2)^5}$$

$$\frac{d^2y}{dv^2} = \frac{d}{dv}\left[\frac{16(v-1)}{(-v^2+2v+6)^5}\right]$$

$$= 16\left[\frac{(-v^2+2v+6)^5 \frac{d}{dv}(v-1) - (v-1)\,5(-v^2+2v+6)^4(-2v+v)}{(-v^2+2v+6)^5}\right]$$

$$= 16\,\frac{((6+2v-v^2)^5 - 5(2-2v)(v-1)(-v^2+2v+6)^4}{(6+2v-v^2)}$$

$$= \frac{16}{(6+2v-v^2)^5} - \frac{80(2-2v)(v-1)}{(6+2v-v^2)^6}$$

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(c) $f(t) = \ln(1+t^2)$

$$f'(t) = \frac{2t}{(1+t^2)}$$

$$f''(t) = \frac{(1+t^2)\times 2 - (2t)\times(2t)}{(1+t^2)^2}$$

$$= \frac{2+2t^2 - 4t^2}{(1+t^2)^2}$$

$$= \frac{(-2t^2+2)}{(1+t^2)^2}$$

$$= \frac{-2(t^2-1)}{(1+t^2)^2}$$

Q. 13.

$f(x)\ x^3 - 3x^2 - 9x + 12$

$f'(x) : 3x^2 - 6x - 9$

Increasing / decreasing.

$f'(x) = 3x^2 - 6x - 9 = +ve / -ve$

$= 3(x^2 - 2x - 3) = +ve/-ve$

$3(x^2 - 3x + x - 3) = +ve/+e$

$3(x(x-3) + 1(x-3) = +ve / -ve$

$3(x-3)(x+1) = +ve/-ve$

If $f'(x) = 0.$

then.

$(x-3)(x+1) = 0$

hence

$(x-3) = 0$ or $(x+1) = 0$

$\therefore x = 3$ or $x = -1$

$f''(x) = 6x - 6$

$f''(3) = 6(3) - 6$

$= 18 - 6 = 12$

$f''(-1) = 6(-1) - 6$

$= -6 - 6 = -12$

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$f''(3)$ is positive hence function has minimum value at $x = 3$.

$f''(-1)$ is negative hence function has maximum value at $x = -1$

~~$f(3) = (3)^3 - 3(3)^2$~~

$$f(3) = (3)^3 - 3(3)^2 - 9(3) + 12$$
$$= 27 - 27 - 27 + 12$$
$$f(3) = \underline{-15}$$

$$f(-1) = (-1)^3 - 3(-1)^2 - 9(-1) + 12$$
$$= -1 - 3 + 9 + 12$$
$$= -4 + 21$$
$$= \underline{17}$$

Minimum value of function is -15 and Maximum value of function is 17.

Q.14. $f(x) = 4x^3 - 18x^2 + 24x - 7$.

$$f'(x) = 12x^2 - 36x + 24$$

Let $f'(x) = 0$

$$\therefore 12x^2 - 36x + 24 = 0$$
$$\therefore 12(x^2 - 3x + 2) = 0$$
$$\therefore 12(x^2 - 2x - x + 2) = 0$$
$$\therefore x(x-2) - 1(x-2) = 0$$
$$(x-2)(x-1) = 0$$

$\therefore \cancel{x=}(x-2) = 0$ or $(x-1) = 0$

$\therefore x = 2$ or $x = 1$

$f''(x) = \cancel{12}\ 24x - 36$

$f''(2) = 48 - 36$
$= 12$

$f''(1) = 24 - 36$
$= -12$

$f''(2)$ is position hence function is minimum at $x = 2$

$f''(1)$ is negetiv hence function is maximum at $x = 1$

$f(2) = 4(2)^3 - 18(2)^2 + 24(2) - 7$
$= 32 - 72 + 48 - 7$
$= 1$

$f(1) = 4 - 18 + 24 - 7$
$= 3.$

Ans: The minimum value of function is 1

The maximum value of function is 3

15. $f(x) = x^2(x+3)$

$\therefore f(x) = x^3 + 3x^2$

$\therefore f'(x) = 3x^2 + 6x$

Assuming $f'(x) = 0$

$3x^2 + 6x = 0$

$\cancel{3x(x-2)}$

$3x(x+2) = 0$

$\therefore 3x = 0$ or $(x+2) = 0$

$\therefore x = 0$ or $x = -2$

$f''(x) = 6x + 6$

$\cancel{f''(x) = 0}$

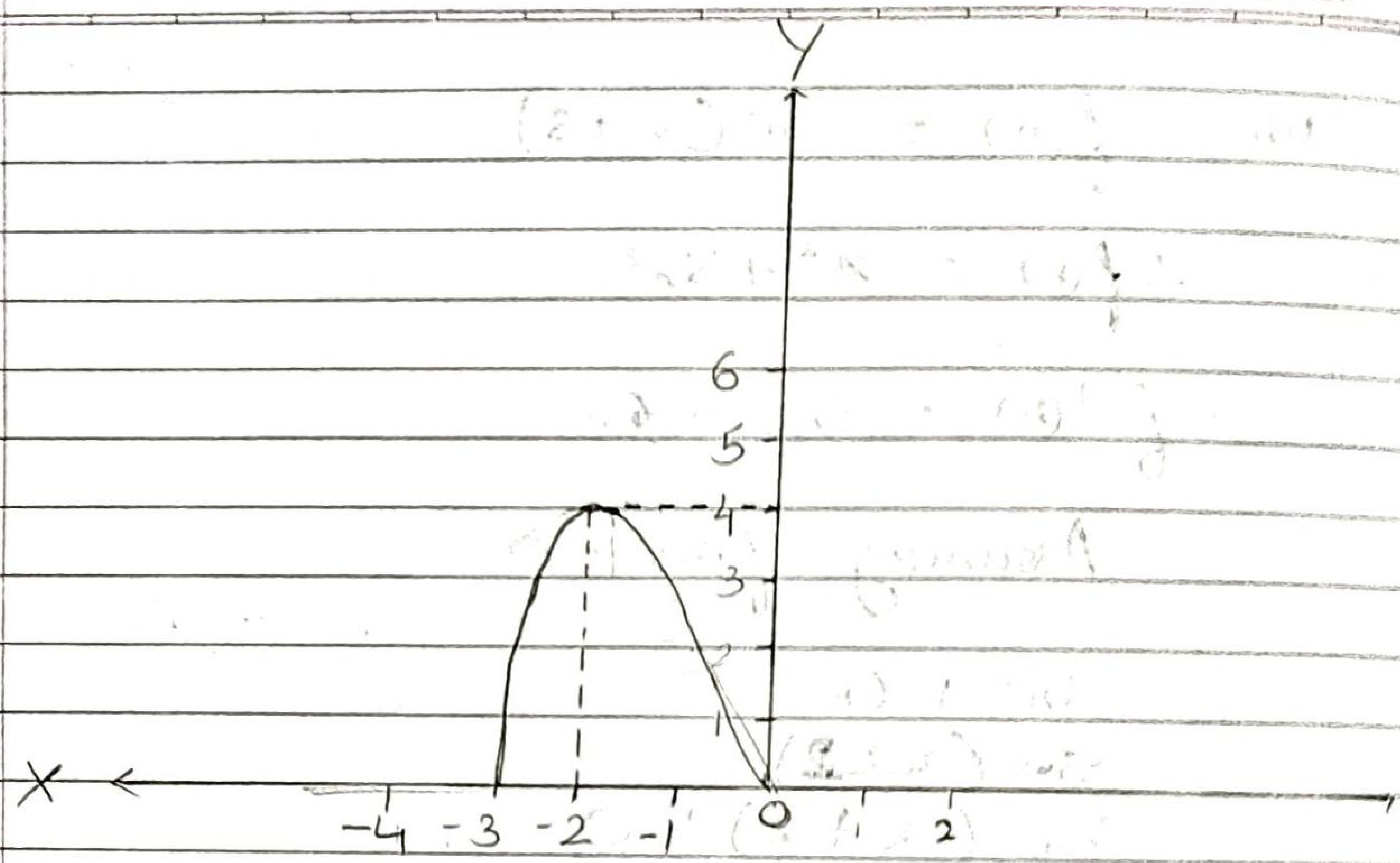
$f''(0) = 6 \quad \therefore f(0) > 0$

$f''(-2) = -12 + 6 = -6 \quad \therefore f(-2) < 0$

function is minimum at $x = 0$ and maximum at $x = -2$

$f(0) = 0(0+3) = 0$

$f(-2) = (-2)^2(-2+3)$
$= 4(1) = 4$



(16) $f(x) = 3x^2 - 6x$

$f'(x) = 6x - 6$

Assume $f'(x) = 0$

$\therefore 6(x-1) = 0$

$\therefore x = 1$

$f''(x) = 6$

$\therefore f''(1) = 6$

hence function is minimum at $x=1$

$f(1) = 3-6 = -3.$

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