Calculus Assignment-2

Q.1) $\int_{1/50}^{2} \frac{e^{2/w}}{w^2}\, dw$

let $t = e^{2/w}$

diff w.r.t 'w'.

$\frac{dt}{dw} = -2e^{2/w}\frac{1}{w^2}$

$= \int_{1/50}^{2} \frac{tw^2}{-2w^2e^{2/w}}\, dt$

$\frac{-1}{2}\int_{1/50}^{2} dt = \frac{-1}{2}[x]_{1/50}^{2}$

$= \frac{-1}{2}(2-0.002)$

$= -0.99$

Q.2) $\int^{8} \frac{2t^3+1}{(t^4+2t)^3}\, dt$

let x be $=(t^4+2t)^3$

diff w.r.t t

$\frac{dx}{dt} = 3(t^4+2t)^2(4t^3+2)$

$\int \frac{2t^3+1}{x(3)(t^4+2t)^2(4t^3+2)}$

~~Multiply and~~ Multiply and divide Nr and Dr by 2

$\int \frac{(4t^3\cancel{+}2)}{(x)(6)((\cancel{4}t^3+2)\cdot(x)^{2/3}}$

$= \frac{1}{6}\int x^{-5/3}\, dx$

$= \frac{1}{6}\left(\frac{x^{-5/3+1}}{-5/3+1}\right) + C = \frac{-1}{4} x^{-2/3} + C$

Q.3) $f'(x) = x^4 + 3x - 9$ $\quad f(x) = \int f'(x)\, dx$

$f(x) = \int x^4 + 3x - 9 \; dx$

$= \frac{x^5}{5} + \frac{3x^2}{2} - 9x + C$

Q.4) $f(x) = \begin{cases} 6 & \text{if } x > 1 \\ 3x^2 & \text{if } x \le 1 \end{cases}$

$\int_{-2}^{3} f(x)\, dx = \int_{-2}^{1} f(x)\, dx + \int_{1}^{3} f(x)\, dx$

$\int_{-2}^{1} f(x)\, dx + \int_{1}^{3} f(x)\, dx$

$x^3 \quad \int_{-2}^{1} 3x^2\, dx + \int_{1}^{3} 6\, dx$

$$\left[x^3\right]_{-2}^{1} + \left[6x\right]_{1}^{3}$$

$$(1+8) + (18-6) = 4.$$

S.54 $\int 3(8y-1)\, e^{4y^2-y}\, dy$.

$x = e^{4y^2-y}$.

diff. w.r.t

$$\frac{dx}{dy} = (e^{4y^2-y})(8y-1)$$

$$\int \frac{3\cancel{(8y-1)}\,\cancel{e^{4y^2-y}}}{\cancel{e^{(4y^2-y)}}\cancel{(8y-1)}}\, dx.$$

$$\int 3\, dx$$

$$\int 3x + C$$

Q.6) $f''(x) = 15\sqrt{x} + 5x^3 + 6$.

$f'(x) = \int f''(x)\,dx$

$f'(x) = \int 15\sqrt{x} + 5x^3 + 6\,dx$.

$= 15 \cdot \frac{2}{3} x^{3/2} + \frac{5x^4}{4} + 6x + C$.

$f(x) = \int f'(x)\,dx$.

$= \int 10x^{3/2} + \frac{5x^4}{4} + 6x + C_1\,dx$

$= \int \frac{10\,x^{5/2}}{5/2} + \frac{5x^5}{20} + \frac{6x^2}{2} + C_1x + C_2$

$= 2 \cdot 2x^{5/2} + \frac{x^5}{4} + 3x^2 + C_1x + C_2$

$= 4x^{5/2} + \frac{x^5}{4} + 3x^2 + C_1x + C_2$.

$f(1) = \frac{-5}{4} = 4 + \frac{1}{4} + 3 + C_1 + C_2 \quad C_1 + C_2 = -8.5$

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$$f(4) = 464 = 4(4)^{5/2} + \frac{(4)^4}{4} + 3(4)^2 + 4K + C$$

$$= 4\cancel{K}C_1 + C_2 = 164$$

$$4\cancel{K}C_1 + C_2 = 164$$

$$+ \cancel{K}C_1 + C_2 = \cancel{-}8.5$$

$$C_1 = 57.5$$

$$C_2 = -66.$$

$$f(x) = 4x^{5/2} + \frac{x^4}{4} + 3x^2 + 57.5x - 66$$

Q.9) $\frac{dv}{dt} = 9.8 - 0.196v.$

$$\int \frac{dv}{9.8 - 0.196v} = \int dt$$

$$\log(9.8 - 0.196v) = -0.196t + C$$

~~12) Taylor Series~~

10) $ty' + 2y = t^2 - t + 1$

~~dividing by t~~

$$t\frac{dy}{dt} + 2y = t^2 - t + 1$$

dividing by t

$$\frac{dy}{dt} + \frac{2}{t}y = t - 1 + \frac{1}{t}$$

$$I_f = e^{\int \frac{2}{t} dt}$$

$$I_f = e^{2\ln t}$$

$$I_f = t^2$$

eqn -

$$yt^2 = \frac{t^4}{4} - \frac{t^3}{3} + \frac{t^2}{2} + \frac{1}{12}$$

11) $y' = \frac{xy^3}{\sqrt{1+x^2}}$, $y(0) = -1$

$\frac{dy}{dx} = \frac{xy^3}{\sqrt{1+x^2}}$

$\int \frac{1}{y^3} dy = \int \frac{x}{\sqrt{1+x^2}} dx.$

$\frac{y^{-2}}{-2} = \frac{1}{2} \int \frac{2x}{\sqrt{1+x^2}} dx$

$\frac{-1}{2y^2} = \frac{1}{2}\left(2\sqrt{1+x^2}\right) + C$

when $x = 0$, $y = -1$.

$\frac{-1}{2(-1)^2} = \sqrt{1} + C$

eqn $= \frac{-1}{2y^2} = \sqrt{x^2+1} + 1/2$ or $-3/2$

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Q.12Y $f(x) = x^3 - 10x^2 + 6 \qquad x = 3$

Taylor Series

$x^3 - 10x^2 + 6 = f(3) + 1 \cdot f'(3)(x-3) + \frac{1}{2} f''(3)$
$(x-3)^2 + \frac{1}{6} f'''(3)(x-3)^3 + \frac{1}{24} f^{iv}(3)(x-3)^4.$

$x^3 - 10x^2 + 6 = -57 - 33(x-3) - (x-3)^2 + (x-3)^3.$

14Y $f(x) = \sqrt{1+x}, \quad f(0) = 1$

$f'(x) = \frac{1}{2\sqrt{1+x}}, \quad f'(0) = \frac{1}{2}$

$f''(x) = -1, \quad f''(0) = \frac{1}{4}$

$f'''(x) = \quad f'''(0) = \frac{1}{8}$

Maclaurian Series

$\sqrt{1+x} = 1 + \frac{1}{2}x + \frac{1}{2^2 \cdot 2!} + \frac{1 \cdot 3 \cdot x^3}{2^3 3!} + \frac{1 \cdot 3 \cdot 5 \cdot x^4}{2^4 \cdot 4!} + \cdots$

Q.14) $f(x) = (1+x)^{\mu}$

Maclaurian Series

$$(1+x)^{\mu} = 1 + \mu x + \frac{\mu(\mu-1)x^2}{2!} + \frac{\mu(\mu-1)(\mu-3)x^3}{3!}$$

$$+ \frac{\mu(\mu-1)(\mu-2)(\mu-3)\cdot x^4}{4!}$$

Q.15) $f(x) = e^{-6x}$, $x = -4$

Taylor Series

$$f(x) = f(4) + f'(4)\cancel{(x+4)}(x+4) + \frac{f''(4)(x+4)^2}{2!} + \ldots\ldots$$

$$e^{-6x} = e^{24} - 6e^{24}(x+4) + 18e^{24}(x+4)^2 - 36e^{24}(x+4)^3$$
$$+ 54e^{24}(x+4)^4 - \frac{324}{5}e^{24}(x+4)^5$$

Q.164 $f(x) = \ln(3+4x)$, $x = 0$

Taylor Series.

$\ln(3\cancel{x}+4x) = \ln(3) + \frac{4\cancel{x}}{3} - \frac{8}{9}x^2 + \frac{64}{81}x^3 -$

$\frac{64}{81}x^4 + \frac{1024}{1215}x^5 + \ldots\ldots$