Date : ______ Subject : ______ Roll No.: ______ Topic : ______

Calculus Assignment - 2

1. $$\int_{\frac{1}{50}}^{2} \frac{e^{\frac{2}{w}}}{w^2}$$

let $t = e^{2/w} \Rightarrow$ diff w.r.t 'x' $\frac{dt}{dx} = -2e^{2w} \cdot \frac{1}{w^2}$

$$= \int_{\frac{1}{50}}^{2} \frac{t\cancel{w^2}}{-2\cancel{w^2} \cdot e^{2w}} \, dt.$$

$$= -\frac{1}{2} \int_{1/50}^{2} dt \Rightarrow -\frac{1}{2}[x]_{1/50}^{2} \Rightarrow -\frac{1}{2}(2 - 0.02)$$

$$\Rightarrow \boxed{-0.99}$$

2. $$\int \frac{2t^3 + 1}{(t^4 + 2t)^3} \, dt.$$

Let $x = (t^4 + 2t) \Rightarrow$ ~~diff w.r.t t $\frac{dx}{dt} = 3(t^4+2t)^2(4t^3+2)$~~

$$\int \frac{\cancel{(2t^3+1)}}{x\cancel{(3)}\cancel{(t^4+2t)^2}\cancel{(4t^3+2)}} \, dx.$$ diff w.r.t $t \Rightarrow \frac{dx}{dt} = 4t^3 + 2$

$$\frac{dx}{2} = (2t^3 + 1)\, dt.$$

$$\int \frac{\cancel{(2t^3+1)}\cancel{2}}{(x)\cancel{(3)}(} = \frac{1}{2}\int \frac{dx}{x^3}$$

$$\Rightarrow \boxed{-\frac{1}{4x^2} + c}$$

3. $f'(x) = x^4 + 3x - 9$ $\qquad f(x) = f'(x)\,dx$

$f(x) = \int (x^4 + 3x - 9)\, dx$

$\Rightarrow \frac{x^5}{5} + 3.\frac{x^2}{2} - 9x + c$

4. $f(x) = \begin{cases} 6 & \text{if } x > 1 \\ 3x^2 & \text{if } x \le 1 \end{cases}$

$\int_{-2}^{3} f(x)\,dx = \int_{-2}^{1} f(x)\,dx + \int_{1}^{3} f(x)\,dx$

$= \int_{-2}^{1} (3x^2)\,dx + \int_{1}^{3} (6)\,dx$

$\Rightarrow \left[\frac{\cancel{3}x^3}{\cancel{3}}\right]_{-2}^{1} + (6x)_{1}^{3} \Rightarrow [1 - (-8)] + 6[3 - 1]$

$\Rightarrow \boxed{21}$

5. $\int 3(8y-1)\, e^{4y^2-y}\, dy$

$\int 3(8y-1)$ Let $x = e^{4y^2-y}$ $\Rightarrow$ diff w.r.t 'y' $\Rightarrow \frac{dx}{dy} = e^{4y^2-y}(8y-$

$\int \frac{3(8y-1)\, e^{4y^2-y}}{e^{4y^2-y}(8y-1)}$

$3x + c$

$\boxed{3\left[e^{4y^2-y}\right] + c}$

Grade: ____ Div.: ____ Roll No.: ____

Date : ____ Subject : ____ Topic : ____

6 $f''(x) = 15\sqrt{x} + 5x^3 + 6$ $\qquad f'(x) = \int f''(x)$

$f'(x) \Rightarrow \int 15\sqrt{x} + 5x^3 + 6\,dx$

$\Rightarrow \frac{15 \cdot x^{3/2} \cdot 2}{3} + \frac{5 \cdot x^4}{4} + 6x + c$

$f(x) \Rightarrow \int\left(\frac{\cancel{15}^{5} \cdot x^{3/2} \cdot 2}{\cancel{3}} + \frac{5x^4}{4} + 6x + c\right)dx$

$\Rightarrow \frac{\cancel{10}^{2} \cdot x^{5/2} \cdot 2}{\cancel{5}} + \frac{\cancel{5} \cdot x^5}{4(\cancel{5})} + \frac{\cancel{6}^{3} x^2}{\cancel{2}} + cx + k$

$\Rightarrow 4x^{5/2} + \frac{x^5}{4} + 3x^2 + cx + k$

$f(1) \Rightarrow -\frac{5}{4} \Rightarrow 4 + \frac{1}{4} + 3 + c + k$

$c + k \Rightarrow -8.5$ — (i)

$f(4) = 404 \Rightarrow 4(4)^{5/2} + \frac{\cancel{5x}(4)^5}{4} + 3(4)^2 + c4 + k$

$\Rightarrow 4k + c \Rightarrow 164$ — (ii)

Solving (i) and (ii)

$4k + c = 164$

$k + c = -8.5$

$(-) \quad (-) \quad (+)$

$3k = 172.5 \qquad \therefore k \Rightarrow \boxed{57.5}$ and $c = \boxed{-66}$

Q8.

8. $\frac{dr}{d\theta} = \frac{r^2}{\theta}$ $\quad r(1)=2$

$\frac{dr}{r^2} = \frac{d\theta}{\theta}$

$\int \frac{dr}{r^2} = \int \frac{d\theta}{\theta} \Rightarrow \frac{-1}{r} = \log|\theta| + c$ — (i)

At $\theta = 1$, $r = 2$.

Substituting in (i)

$\frac{-1}{2} = 0 + c \Rightarrow c = -\frac{1}{2}$

$-\frac{1}{r} = \log\theta - \frac{1}{2} \Rightarrow \boxed{\log\theta + \frac{1}{r} - \frac{1}{2}}$

9. $\frac{dv}{dt} = 9.8 - 0.196v$

$\int \frac{dv}{9.8 - 0.196v} = \int dt.$

$\log(9.8 - 0.196v) \Rightarrow \boxed{0.196t + c}$

~~10.~~

12. $f(x) = x^3 - 10x^2 + 6$ for $x = 3$.

$f(x) \Rightarrow x^3 - 10x^2 + 6$ $\qquad f(3) = -57$

$f'(x) \Rightarrow 3x^2 - 20x$ $\qquad f'(3) = -33$

$f''(x) \Rightarrow 6x - 20$ $\qquad f''(3) = -2$

$f'''(x) \Rightarrow 6$ $\qquad f'''(3) = 6.$

$f(x) \Rightarrow f(3) + (x-3)f'(x) + \frac{(x-3)^2}{2} f''(3) + \frac{(x-3)^3}{3} f'''(3)$

$\Rightarrow \cancel{-57 + (-33) + (-2) + 6}$

$\Rightarrow -57 + (x-3)(-33) + \frac{(x-3)^2}{2}(-2) + \frac{(x-3)^3}{3}(6)$

Grade: ______ Div.: ______ Roll No.: ______

Date : ______ Subject : ______ Topic : ______

10. $ty' + 2y = t^2 - t + 1$ $\qquad y(1) = \frac{1}{2}$

$\cancel{t}y' + \frac{2y}{t} = t - 1 + \frac{1}{t}$ — (i)

$y' + P(t)y = Q(t)$; $P(t) = \frac{2}{t}$; $Q(t) = t - 1 + \frac{1}{t}$.

~~IF = e~~

$IF = e^{\int P(t)dt} \Rightarrow e^{\int \frac{2}{t}dt} \Rightarrow e^{\log t^2} = \boxed{t^2}$

Multiplying (i) with IF.

$t^2y' + 2ty = t^3 - t^2 + t$

$\frac{d}{dt}(t^2y) = t^3 - t^2 + t$

$t^2y = \frac{t^4}{4} - \frac{t^3}{3} + \frac{t^2}{2} + c$ [By integrating both sides]

$y = \frac{t^2}{4} - \frac{t}{3} + \frac{1}{2} + ct^{-2}$

At $t = 1$, $y = \frac{1}{2}$.

$c = \frac{1}{2} + \left(-\frac{1}{4}\right) + \frac{1}{3} + \left(-\frac{1}{2}\right) \Rightarrow \boxed{\frac{1}{12}}$

$y \Rightarrow \boxed{\frac{t^2}{4} - \frac{t}{3} + \frac{1}{t} + \frac{1}{2}t^{-2}}$

11. $\frac{dy}{dx} = \frac{xy^3}{\sqrt{1+x^2}}$ $\quad y(0) = -1$

$\int \frac{dy}{y^3} = \int \frac{x \cdot dx}{\sqrt{1+x^2}}$

$-\frac{1}{2y^2} = \frac{1}{2}\left[2\sqrt{1+x^2}\right] + C$

$\frac{-1}{2y^2} = \sqrt{1+x^2} + c$

At $x=0$, $y=-1$

$-\frac{1}{2} = 1 + c$

$c \Rightarrow -1 - \frac{1}{2} \Rightarrow \boxed{-\frac{3}{2}}$

$\frac{1}{2y^2} = \sqrt{1+x^2} - \frac{3}{2}$

$\boxed{\sqrt{1+x^2} + \frac{1}{2y^2} = \frac{3}{2}}$

Name :

Grade: Div.: Roll No.:

Date : Subject : Topic :

13. Maclaurin series for $(1+x)^{\mu}$

$f'(x) \Rightarrow \mu(1+x)^{\mu-1}$

$f''(x) \Rightarrow \mu(\mu-1)(1+x)^{\mu-2}$

$f'''(x) \Rightarrow \mu(\mu-\cancel{2})(\mu-2)(1+x)^{\mu-3}$

$f(0)=1$

$f'(0)=\mu$

$f''(0)=\mu(\mu-1)$

$f'''(0)=\mu(\mu-1)(\mu-2)$

$f(x) \Rightarrow f(0)+\frac{x'}{1}f'(0)+\frac{x^2}{2}f''(0)+\frac{x^3}{6}f'''(0)\ldots$

$$(1+x)^{-\mu} = \boxed{1+\mu(x)+\mu(\mu-1)\left(\frac{x^2}{2}\right)+\left(\frac{x^3}{6}\right)\mu(\mu-1)(\mu-2)+\ldots}$$

14. $f(x)=\sqrt{1+x}$

$f(x)=\sqrt{1+x}$

$f'(x)=\frac{1}{2\sqrt{1+x}}$

$f''(x)=\frac{-1}{4(1+x)^{3/2}}$

$f'''(x)=\frac{+3}{8\sqrt[5]{1+x}}$

$f(0)=1$

$f'(0)=\frac{1}{2}$

$f''(0)=-\frac{1}{4}$

$f'''(0) \Rightarrow \frac{3}{8}$

$f(x) \Rightarrow 1+\frac{1}{2}(x)+\left(\frac{-1}{4}\right)\left(\frac{x^2}{2}\right)+\frac{3}{8}\left(\frac{x^3}{6}\right)$

$$\Rightarrow \boxed{1+\frac{x}{2}+\frac{x^2}{8}+\frac{x^3}{8}+\ldots}$$

15. $f(x) = e^{-6x}$ about $x=-4$

$f(x) = e^{-6x}$ $\qquad$ $f(-4) = e^{24}$

$f'(x) \Rightarrow (-6)e^{-6x}$ $\qquad$ $f'(4) = -6e^{24}$

$f''(x) = e^{-6x}(-6(-6))$ $\qquad$ $f''(4) = 36e^{24}$

$$f(x) \Rightarrow e^{24} + (-6e^{24})\left(\frac{x+4}{1}\right) + \frac{(x+4)^2}{\cancel{2}} \cdot \cancel{36}^{18}e^{24}$$

$$\boxed{\Rightarrow e^{24} + (-6e^{24})(x+4) + (x+4)^2(18e^{24}) + \ldots}$$

16. $f(x) = \ln(3+4x)$ about $x=0$

16 $\quad f(x) = \ln(3+4x)$

$f'(x) = \frac{1}{3+4x}$ $\qquad$ $f(0) = \ln(3)$

$f''(x) = \frac{-4}{(3+4x)^2}$ $\qquad$ $f'(0) = -\frac{4}{9}$

$f'''(x) \Rightarrow \frac{8}{(3+4x)^3}$ $\qquad$ $f''(0) = \frac{8}{27}$

$$f(x) = \ln(3) + \frac{x}{3} - \frac{\cancel{4}^{2}x^2}{9} \times \frac{1}{\cancel{2}} + \frac{\cancel{8}^{4}x^3}{27 \times \cancel{6}_{3}}$$

$$\Rightarrow \boxed{\ln(3) + \frac{x}{3} - \frac{2x^2}{9} + \frac{4x^3}{81}}$$

$\begin{array}{r} 27 \\ \times 3 \\ \hline 81 \end{array}$

15. $f(x) = e^{-6x}$ about $x=-4$

$f(x) = e^{-6x}$

$f'(x) \Rightarrow (-6)e^{-6x}$

$f''(x) = e^{-6x}(-6(-6))$

$f(-4) = e^{24}$

$f'(4) = -6e^{24}$

$f''(4) = 36e^{24}$

$$f(x) \Rightarrow e^{24} + (-6e^{24})\left(\frac{x+4}{1}\right) + \frac{(x+4)^2}{2} \cdot \overset{18}{\cancel{36}}e^{24}$$

$$\boxed{\Rightarrow e^{24} + (-6e^{24})(x+4) + (x+4)^2(18e^{24}) + \ldots}$$

16. $f(x) = \ln(3+4x)$ about $x=0$

$f(x) = \ln(3+4x)$

$f'(x) = \frac{1}{3+4x}$

$f(0) = \ln(3)$

$f''(x) = \frac{-4}{(3+4x)^2}$

$f'(0) = -\frac{4}{9}$

$f'''(x) \Rightarrow \frac{8}{(3+4x)^3}$

$f''(0) = \frac{8}{27}$

$$f(x) = \ln(3) + \frac{x}{3} - \frac{\overset{2}{\cancel{4}}x^2}{9} \times \frac{1}{\cancel{2}} + \frac{\overset{4}{\cancel{8}}x^3}{27 \times \cancel{6}3}$$

$$\boxed{\Rightarrow \ln(3) + \frac{x}{3} - \frac{2x^2}{9} + \frac{4x^3}{81}}$$

$27 \times 3 = 81$