# Assignment

1. $X \sim N(800, 100^2)$ $\quad$ $Y \sim N(1200, 300^2)$

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2. $P(N=n) = {}^{n+2}C_n \, 0.9^3 \, 0.1^n = {}^{x-1}C_{k-1} \, p^k \, (1-p)^{x-k}$

$n+2 = x-1 \qquad k=3$

$n = k-1 \qquad p = 0.9$

$n = 2$

$n+2 = x-1$

$x = 5$

$N \sim \text{negBi}(0.9, 3)$

$M_y(t) = E(e^{ty}) = E(e^{tx_1} \, e^{tx_1}) = [M_x(t)]^n = [1 - t/2]^{-6n}$

4. $V(S) = V(E(S/N)) + E(V(S/N))$

$= V(n\,E(x)) + E(V(x) \cdot n)$

$= V(x)E(N) + E(x)^2 V(N)$

$= \frac{6}{4} \cdot \frac{2 \times 0.1}{0.9 \times 0.9} + \frac{6}{2} \times \frac{6}{2} \times \frac{2}{6}$

$= 20 + \frac{10}{27} = \frac{550}{27}$

STD. DEV $= \sqrt{\frac{550}{27}} = 4.51335$

3. $f(x) = \lambda e^{-\lambda(x-\alpha)}$

MGF: $M_x(t) = E(e^{tx}) = \int_0^\infty e^{tx} \lambda e^{-\lambda(x-\alpha)} dx$

$= \int_0^\infty e^{tx - \lambda x + \lambda\alpha} \lambda dx$

$= \int_0^\infty e^{x(t-\lambda) + \lambda\alpha} \lambda dx$

$= \dfrac{e^{x(t-\lambda)+\lambda\alpha}}{t-\lambda} \lambda$

ii $E(x) = M_x(t)'$ $\qquad V(x) = ?$

$= e^{t(x)} \lambda e^{-\lambda(x-\alpha)}$

$t=0$ $\quad = e^{0(x)} \lambda e^{-\lambda(x-\alpha)}$

$= \lambda e^{-\lambda(x-\alpha)}$

$E(x^2) = M_x''(t)$

$= e^{tx} \lambda e^{-\lambda(x-\alpha)\,'} = e^{tx + \cancel{\lambda\alpha} - \cancel{\lambda x}} \lambda (t-\lambda)$

$t=0$ $\quad = e^{\lambda\alpha - \lambda x} \lambda(-\lambda) = e^{\lambda(\alpha - x)} - \lambda^2$

$V(x) = E(x^2) - [E(x)]^2 = e^{\lambda(\alpha-x)} - \lambda^2 - [\lambda e^{-\lambda(x-\alpha)}]^2$

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4. $G_v(t) = \left(\frac{p}{1-qt}\right)^k$ $\quad p+q=1$

$G_v(t) = p^k(1-qt)^{-k}$

$P(V=4) = ?$

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5. $f(x,y) = ax(x+y)$ $\qquad 0 < x < 5$

$10 < y < 20$

$P\left(Y > \frac{1}{3}x + 10\right)$ , $E(Y/X)$

$$\int_{10}^{20}\int_{0}^{5} ax^2 + axy = a\int_{10}^{20}\int_{0}^{5} x^2 + xy \, dx \, dy$$

$$= a\int_{10}^{20} \left[\frac{x^3}{3} + \frac{x^2 y}{2}\right]_0^5 dy$$

$$= a\int_{10}^{20} \frac{125}{3} + \frac{25}{2}y$$

$$= a\left[\frac{125y}{3} + \frac{25y^2}{4}\right]_{10}^{20}$$

$$= a\left[\left(\frac{2500}{3} + \frac{10000}{4}\right) - \left(\frac{1250}{3} + \frac{2500}{4}\right)\right]$$

$$= a\left[\frac{1250}{3} + \frac{7500}{4}\right] = 1$$

$$= a\left[\frac{5000 + 22500}{12}\right] = a\left[\frac{27500}{12}\right] = 1$$

$$a = \frac{12}{27500}$$

6. $X \sim Poi(\lambda)$ $\quad Y \sim Poi(\mu)$ $\quad X - Y \sim Poi(\lambda - \mu)$

$X \sim Poi(\lambda)$ $\quad Y \sim Poi(\mu)$ $\quad X + Y \sim Poi(\lambda + \mu)$

$$M_x(t) = E(e^{tx}) = E(e^{t(x-y)}) = E(e^{tx-ty}) = E\left(\frac{e^{tx}}{e^{ty}}\right)$$

$M_x(t) = e^{\lambda(e^t - 1)}$ for Poisson

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?. $V(X/Y=2) = E(X^2/Y=2) - \left(E(X/Y=2)\right)^2$

$E\left(X^2/Y=2\right) = 4\times0.3 + 9\times0.1 + 16\times0.2$
$= 1.2 + 0.9 + 3.2 = 5.3$

$E(X/Y=2) = 0.6 + 0.3 + 0.8 = 1.7$

$V(X/Y=2) = 5.3 - 1.7 = 3.6$

ii $f(u,v) = \frac{48}{67}(2uv - u^2) \qquad 0 < u < 1 \qquad \frac{u}{2} < v <$

$E(U/V=v) = ?$

$\frac{48}{67}\int_{u/2}^{2}\int_{0}^{1} 2uv - u^2 \, du \, dv$

$\frac{48}{67}\int_{u/2}^{2}\left[u^2 v - \frac{u^3}{3}\right]_0^1 dv = \frac{48}{67}\int_{u/2}^{2} v - \frac{1}{3} \, dv$

$\frac{48}{67}\left[\frac{v^2}{2}\right]_{u/2}^{2} = \frac{48}{67}\left[\frac{4 - u^2}{2}\right]$

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8. $X \sim \text{Gamma}(3, 2)$

$E(Y/X=x) = 3x+1 \qquad V(Y/X=x) = 2x^2+5$

$E(Y) = ? \qquad \sqrt{V(Y)} = ?$

$E(Y) = E(E(Y/X=x)) = E(3x+1)$

$V(Y) = E(V(Y/X=x)) + V(E(Y/X=x))$

$= E(2x^2+5) + V(3x+1)$

9. $E(x) = 3 \qquad V(x) = 4$

$E(Y) = 4 \qquad V(Y) = 1$

$r = -0.3$

$Z = X + Y$

i $cov(x, Z) = E(xZ) \cdot E(x) E(Z)$

$E(Z) = E(x) E(Y) = 12 \qquad cov(x, z) = \frac{-0.3}{\sqrt{4 \times 1}} = \frac{0.3}{2} = 0.15$

$E(xZ) =$

ii $V(Z) = V(x) E(Y) + [E(x)]^2 V(Y)$

$= 4(4) + 3^2(1)$

$= 25$

10. $\int_1^\infty \int_1^\infty K x^{-\alpha} e^{-y/\beta} \, dy \, d\cancel{x}$

$K \int_1^\infty \left[ x^{-\alpha} y - \beta e^{-y/\beta} \right]_1^\infty dx = K \int_1^\infty - \left[ x^{-\alpha} - \beta e^{-1/\beta} \right] dx$

$= -K \left[ \frac{x^{-2\alpha}}{2} - \beta e^{-1/\beta} x \right]_1^\infty = K \left[ \frac{\cancel{x} 1^\alpha}{2} - \beta e^{-1/\beta} \right] = 1$

$= K \left[ \frac{1^\alpha - 2\beta e^{-1/\beta}}{2} \right] = 1$

$K = \frac{2}{1^\alpha - 2\beta e^{-1/\beta}}$