$e^{2/w} \times 2\log \frac{dt}{dw}$

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Calculus

1. $\int_{1/50}^{2} \frac{e^{2/w}}{w^2} dw$: let $w^2 = t$: $w^2 = t$ $\quad \frac{2w = dt}{dw}$ $\quad w = t/w$ $\quad dw = \frac{dt}{2w}$

$\rightarrow \int_{1/50}^{2} \frac{e^{2w/t}}{t} \frac{dt}{2\sqrt{t}}$

Q.2 $\int \frac{2t^3+1}{(t^4+2t)^3} dt$ $\quad \frac{d[t^4+2t]}{dt} = 4t^3+2$
$= 2[2t^3+1]$

$= \frac{1}{2}\int (t^4+2t)^{-3} \times 2(2t^3+1)\, dt = \frac{1}{2}\int (t^4+2t)^{-3} \cdot (4t^3+2)\, dt$

By: $\int [f(x)]^n \cdot f'(x)\, dx = \frac{f(x)^{n+1}}{n+1} + c$

$I = \frac{1}{2} \times \frac{[t^4+2t^3]^{-2}}{-2} = \frac{-1}{4(t^4+2t^3)^2} + c$

Q.3 $f'(x) = x^4+3x-9$

$\frac{d}{dx} f(x) = f'(x)$ $\quad \therefore \int f'(x)\, dx = f(x)$

$\int x^4+3x-9\, dx = \frac{x^5}{5} + \frac{3x^2}{2} - 9x + c$

Q.4 $f(x) = \begin{cases} 6 & x > 1 \\ 3x^2 & x \leq 1 \end{cases}$

$\therefore \int_{-2}^{3} f(x)\, dx = \int_{-2}^{1} f(x)\, dx + \int_{1}^{3} f(x)\, dx$

$= \int_{-2}^{1} 6\, dx + \int_{1}^{3} 3x^2\, dx = [6x]_{-2}^{1} + \left[\cancel{\frac{3x^3}{3}}\, x^3\right]_{1}^{3}$

$\{6(1)-6(-2)\} + \cancel{3}[27-1] \rightarrow (6+12) + \cancel{26} = 44$

Q5 $\int 3[8y-1] \cdot e^{4y^2-y}\, dy$ let $4y^2-y=t$

$8y-1=\frac{dt}{dy}$

$\therefore (8y-1)\,dy = dt$

$= 3\int e^t \times (1)\, dt$

$= e^t \times 3 + C$

$= 3e^{4y^2-y} + C$

6.) $f''(x) = 15\sqrt{x} + 5x^3 + 6$ $f(1) = -5/4$ $f(4) = 404$

$f''(x) = \frac{d}{dx} f'(x)$ $\therefore f'(x) = \int f''(x)\, dx$

$= \int 15\sqrt{x} + 5x^3 + 6\, dx = \frac{\cancel{15}^{5} x^{3/2}}{\cancel{3}/2} + \frac{5x^4}{4} + 6x + C$

$f'(x) = 10x^{3/2} + \frac{5x^4}{4} + 6x + C$ $f'(x) = \frac{d}{dx} f(x)$ $\therefore f(x) = \int f'(x)\, dx$

$f(x) = \int 10x^{3/2} + \frac{5x^4}{4} + 6x + C\, dx = \frac{\cancel{10}^{2}\, x^{5/2}}{5/2} + \frac{\cancel{5}}{4} \cdot \frac{x^5}{\cancel{5}} + \frac{\cancel{6}^{3} x^2}{\cancel{2}} + Cx + K$

$\therefore f(x) = 4x^{5/2} + x^5/4 + 3x^2 + Cx + K$

$f(1) = 4(1)^2\sqrt{1} + \frac{(1)^5}{4} + 3(1) + 1(C) + K = -5/4$

$= \cancel{4}$

$\frac{4+1}{\cancel{4}}$ $7 + \frac{1}{4} + C + K = \frac{-5}{4}$ $\therefore C+K = \frac{-5}{4} - \frac{1}{4} - 7 = \frac{-\cancel{34}^{17}}{\cancel{4}\,2}$

$f(4)$ $4(4)^2\sqrt{4} + \frac{(4)^4 \cdot \cancel{(4)}}{\cancel{4}} + 3(4)^2 + 4C + K = 404$

$= 64(2) + 256 + 48 + 4C + K = 404$

$4C + K = 404 - 256 - 48 - 128$

$4C + \cancel{K} = -28$

$- \; C + \cancel{K} = -17/2$

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$3C = -28 + \frac{17}{2}$ $\therefore \cancel{3}C = \frac{-\cancel{39}^{13}}{2}$ $\boxed{C = \frac{-13}{2}}$

$-28 = 4C + K$

$\frac{-13}{2} + K = \frac{-17}{2}$ $\therefore K = \frac{-17+13}{2} = -2$

$\therefore f(x) = 4x^{5/2} + \frac{x^5}{4} + 3x^2 - \frac{13x}{2} - 2$

Q8. $\frac{dr}{d\theta} = \frac{r^2}{\theta}$, $r(1) = 2$ $\rightarrow \frac{1}{r^2} dr = \frac{1}{\theta} d\theta$

Int BS $\int \frac{1}{r^2} dr = \int \frac{1}{\theta} d\theta$: $\frac{-1}{r} = \log\theta + c$.

$\therefore 0 = \log\theta + \frac{1}{r} + c$, when $r(1) = 2$

$2 = \log\theta + (1/1) + c$ $\therefore$ $\boxed{\log\theta + c = 2}$

Q9: $\frac{dv}{dt} = 9.8 - 0.196v$ $\therefore \frac{dv}{dt} + 0.196v = 9.8$.

$\therefore \boxed{P = 0.196}$ $\boxed{Q = 9.8}$

$IF = e^{\int P dt} \rightarrow e^{\int 0.196 dt} = e^{0.196t}$

Soln: $\cancel{v[0.19} \; v[e^{0.196t}] = \int 0.98 \times e^{0.196t} dt$

$v \times e^{0.196t} = \cancel{0.98}^{9.8} \times \left[\frac{e^{0.196t}}{0.196} + c \right]$

$ve^{0.196t} = \cancel{9.8} \cdot e^{0.196t} \left[\frac{\cancel{0.98}\,9.8}{0.196}^{50} + \frac{c}{e^{0.196t}} \right]$

$\therefore \boxed{v = 50 + ce^{-0.196t}}$

Q.10 $t \cdot \frac{dy}{dx} + 2y = t^2 - t + 1$; $\frac{dy}{dt} + \frac{2}{t} y = t^2 - t - 1$

$\downarrow_P$ $\downarrow_Q$

IF: $e^{\int P dt} = e^{\int 2/t \, dt} = e^{2\log_e t}$

$\therefore \boxed{IF = t^2}$

Soln: $y(IF) = \int Q(IF) dt$, $yt^2 = \int (t^2 - t - 1) t^2 dt$

$\rightarrow yt^2 = \int t^4 - t^3 - t^2 dt = \frac{t^5}{5} - \frac{t^4}{4} - \frac{t^3}{3} + c = y \cdot t^2$

$y=1$, $\therefore \cancel{y \cdot t^2} = \cancel{t^2} \left[\frac{t^3}{5} - \frac{t^2}{4} - \frac{t^1}{3} + \frac{c}{t^2} \right]$

$\therefore y(1) = 1/2 = \frac{t^3}{5} - \frac{t^2}{4} - \frac{t}{3} + \frac{c}{t^2}$

Q.12. $f(x) = x^3 - 10x^2 + 6 \quad , \; \underline{x=3}$

$f'(x) = 3x^2 - 20x$

$f''(x) = 6x - 20$

$f'''(x) = 6$

$f^{IV}(x) = 0$

$f(3) = 27 - 10(9) + 6 = -57$

$f'(3) = 3(9) - 20(3) = -33$

$f''(3) = 6(3) - 20 = -2$

$f'''(3) = 6$

$f^{IV}(3) = 0$

Taylor series: $f(x) + \frac{f'(x)(x-k)}{1!} + \frac{f''(x)(x-k)^2}{2!} + \frac{f'''(x)(x-k)^3}{3!} + \dots$

$\therefore f(x) = x^3 - 10x^2 + 6 = -57 + \frac{-33[x-3]}{1!} + \frac{-2[x-3]^2}{2!} + \frac{6[x-3]^3}{3!} + \frac{0[x-3]^4}{4!}$

$\Rightarrow \therefore f(x) = x^3 - 10x^2 + 6 = \cancel{-57} \; \{-57 - 33 - 2 + 6\} \times \left[\sum_{n=0}^{4} \frac{(x-3)^n}{n!}\right]$

Q.13 $f(x) = (1+x)^\mu = \cancel{(1+x)}$

$f'(x) = \mu[1+x]^{\mu-1}$

$f''(x) = \mu(\mu-1)[1+x]^{\mu-2}$

$f'''(x) = \mu(\mu-1)(\mu-2)(1+x)^{\mu-3}$

$f^{IV}(x) = \mu(\mu-1)(\mu-2)(\mu-3)(1+x)^{\mu-4}$

$f(0) = (1+0)^\mu = 1$

$f'(0) = \mu[1]^{\mu-1} = \mu$

$f''(0) = \mu(\mu-1)[1]^{\mu-2} = \mu(\mu-1)$

$f'''(0) = \mu(\mu-1)(\mu-2)(1)^{\mu-3} =$

$f^{IV}(0) = \mu(\mu-1)(\mu-2)(\mu-3)(1)$

$\rightarrow$ Maclaurian Series: $f(0) + \frac{f'(0)(x-0)}{1} + \frac{f''(0)(x-0)^2}{2!}$

$+ \frac{f'''(0)(x-0)^3}{3!} + \frac{f^{IV}(0)(x-0)^4}{4!}$

$f(x) = \cancel{1 + \mu x + \frac{\mu(\mu-1)x^2}{2!} + \frac{\mu(\mu-1)x^3}{3!} + x^4 \mu(\mu-1)}$

$f(x) = 1 + \frac{\mu(x)^1}{1!} + \frac{\mu(\mu-1)(x^2)}{2!} + \frac{\mu(\mu-1)(\mu-2)x^3}{3!} + \frac{x^4[\mu(\mu-1)(\mu-2)(\mu-3)]}{4!}$

$= \sum_{n=0}^{\infty} \frac{x^n}{n!} \times \boxed{\cancel{(\mu-n)^n \mu}} \rightarrow 1$

or $f(x) = 1 + \frac{\mu(x)}{1!} + \frac{\mu(\mu-1)(x^2)}{2!} + \frac{\mu(\mu-1)(\mu-2)(x^3)}{3!} + \dots$

Q.14 $f(x) = \sqrt{1+x}$

$f'(x) = 1/2\sqrt{1+x}$

$f''(x) = \dfrac{-1}{4(1+x)^{3/2}}$

$\cancel{f'''(x)}$

$\cancel{f^{iv}(x)}$

$f'''(x) = \dfrac{-1}{4}\left(\dfrac{-3}{2}\right) \times \dfrac{1}{(1+x)^{5/2}}$

$f^{iv}(x) = \dfrac{3}{8}\left(\dfrac{-5}{2}\right) \times \dfrac{1}{(1+x)^{7/2}}$

$f(0) = \sqrt{1} = 1$

$f'(0) = 1/2\sqrt{1} = 1/2$

$f''(0) = \dfrac{-1}{4(1)} = \dfrac{-1}{4}$

$f'''(0) = \dfrac{3}{8} \times \dfrac{1}{(1)} = \dfrac{3}{8}$

$f^{iv}(0) = \dfrac{-15}{16} \times \left(\dfrac{1}{1}\right) = \dfrac{-15}{16}$

$f(x) = f(0) + \dfrac{f'(0)(x)}{1!} + \dfrac{f''(0)(x^2)}{2!} + \dfrac{f'''(0)(x^3)}{3!} + \dfrac{f^{iv}(0)(x^4)}{4!}$

$\sqrt{1+x} = 1 + \dfrac{1}{2}x + \left(\dfrac{-1}{4}\right)\dfrac{x^2}{2!} + \left(\dfrac{3}{8}\right)\dfrac{x^3}{3!} + \dfrac{-15}{16}\left(\dfrac{x^4}{4!}\right) + \ldots$

$= \sum_{n=1}^{\infty} \dfrac{[1+2^n]}{2^n} \times \dfrac{x^n}{n!}$

$\cancel{\therefore f(x) = 1 + \dfrac{1}{2}x - \dfrac{1x^2}{8} + \dfrac{3x^3}{16} - \dfrac{5x^4}{128}}$

15. $f(x) = e^{-6x}$

$f'(x) = e^{-6x}(-6)$

$f''(x) = -6(-6)e^{-6x}$

$f'''(x) = 36(-6)e^{-6x}$

$f^{iv}(x) = \cancel{-12} -216(-6)e^{-6x}$

$f(-4) = e^{24}$

$f'(-4) = -6e^{24}$

$f''(-4) = 36e^{24}$

$f'''(-4) = -216e^{24}$

$f^{iv}(-4) = 1272e^{24}$

$f(x) = f(-4) + \dfrac{f'(-4)(x+4)}{1!} + \dfrac{f''(-4)(x+4)^2}{2!} + \dfrac{f'''(-4)(x+4)^3}{3!} + \ldots$

$e^{-6x} = e^{24} + \dfrac{-6e^{24}(x+4)}{1!} + \dfrac{(-6)^2(x+4)^2 e^{24}}{2!} + \dfrac{(-6)^3 e^{24}(x+4)^3}{3!}$

$\rightarrow e^{24}\left\{\sum_{n=0}^{\infty} \dfrac{(-6)^n \cdot (x+4)^n}{n!}\right\}$

Q16 $f(x) = \log(3+4x)$

$$f'(x) = \frac{1}{(3+4x)}(4)$$

$$f''(x) = 4 \times \frac{1}{(3+4x)^2}(-1)(4)$$

$$f'''(x) = \frac{4(4)(-1)\times(-2)(4)}{(3+4x)^3}$$

$$f^{iv}(x) = \frac{(4)(4)(4)(2)(-3)(4)}{(3+4x)^4}$$

$f(0) = \log(3)$

$$f'(0) = \frac{4}{3}$$

$$f''(0) = \frac{(4)^2(-1)}{(3)^2}$$

$$f'''(0) = \frac{(4)^3(-1)(-2)}{(3)^3}$$

$$f^{iv}(0) = \frac{(4)^4(2)(-3)}{(3+\cancel{4x})^4}.$$

$$\therefore f(x) = \log 3 + \frac{4}{3}(x-0) + \left(\frac{-16}{9}\right)\frac{(x-0)^2}{2!} + \frac{(4)^3(2)}{(3)^3}\frac{(x-0)^3}{3!} + \frac{(4)^4(2)(-3)}{(3)^4}\frac{(x-0)^4}{4!}$$

$$f(x) = \log 3 + \sum_{n=1}^{\infty}\left[\left(\frac{4}{3}\right)^n \frac{(x-0)^n}{n!} \times (-1)^{(n+1)}\right]$$

$$\therefore f(x) = \log 3 + \sum_{n=1}^{\infty}\left\{\left(\frac{4}{3}\right)^n (-1)^{(n+1)} \cdot \frac{(x-0)^n}{n!}\right\}$$