Assignment -1

1) Evaluate the following limits:

$$\lim_{h \to 0} \frac{2(-3+h)^2 - 18}{h}$$

Solⁿ:-

$$= \lim_{h \to 0} \frac{2h^2 - 12h}{h}$$

$$= \lim_{h \to 0} \frac{2\cancel{h}(h-6)}{\cancel{h}}$$

$$= \lim_{h \to 0} 2(h-6)$$

$$= \lim_{h \to 0} (2h - 12)$$

$$= -12$$

$$\boxed{\lim_{h \to 0} \frac{2(-3+h)^2 - 18}{h} = -12}$$

2) Determine if the following function is continuous or discontinuous at (a) $x=4$
(b) $x=6$

$$g(x) = \begin{cases} 2x & x<6 \\ x-1 & x\geq 6 \end{cases}$$

a) $x=4$

Solⁿ

$g(4) = 2(4) = 8$

$\lim_{x\to 4^-} = 2(4) = 8$

$\lim_{x\to 4^+} = 2(4) = 8$

$\because \lim_{x\to 4^-} = \lim_{x\to 4^+} = 8$

$\therefore$ Limits do exists

And, $\lim_{x\to 4} g(x) = g(4) = 8$

$\therefore g(x) = 2x$ is continous at $x=4$

b) $x=6$

Solⁿ

$g(6) = x-1 = 6-1 = 5$

$\lim_{x\to 6^-} = 2(6) = 12$

$\lim_{x\to 6^+} = x-1 = 6-1 = 5$

$\because \lim_{x\to 6^-} \neq \lim_{x\to 6^+}$

$\therefore$ The $g(x)$ is not continous at $x=6$

3) Solve the following limits

$$\lim_{x \to 9} \frac{x-9}{3-\sqrt{x}}$$

$$\frac{x-9}{3-\sqrt{x}} = \frac{(x-9)(\sqrt{x}+3)}{(3-\sqrt{x})(\sqrt{x}+3} = -\sqrt{x}-3;$$

$$\lim_{x \to 9} (-\sqrt{x} - 3)$$

$$\lim_{x \to 9} (-\sqrt{x} - 3) = -\sqrt{9} - 3 = -6$$

$$\boxed{\therefore \lim_{x \to 9} \frac{x-9}{3-\sqrt{x}} = -6}$$

4) Determine if the following function is continuous or discontinuous at:

a) $x = -1$, b) $x = 0$, c) $x = 3$?

$$f(x) = \frac{4x+5}{9-3x}$$

Solⁿ:

a) $x = -1$

$$f(-1) = \frac{4(-1)+5}{9-3(-1)} = \frac{-4+5}{9+3} = \frac{1}{12}$$

$$\lim_{x \to -1^-} = \frac{4(-1)+5}{9-3(-1)} = \frac{1}{12}$$

$$\lim_{x \to -1^+} = \frac{4(-1)+5}{9-3(-1)} = \frac{1}{12}$$

$$\because \lim_{x \to -1^+} = \lim_{x \to -1^-} = \frac{1}{12}$$

And limits do exists

$$\lim_{x \to -1} f(x) = f(-1) = \frac{1}{12}$$

$\therefore$ $f(x) = \frac{4x+5}{9-3x}$ is Continuous at, $x = -1$

b) $x = 0$

$$f(0) = \frac{4(0)+5}{9-3(0)} = \frac{5}{9}$$

$$\lim_{x \to 0^+} = \frac{4(0)+5}{9-3(0)} = \frac{5}{9}$$

$$\lim_{x \to 0^-} = \frac{4(0)+5}{9-3(0)} = \frac{5}{9}$$

$$\therefore \lim_{x \to 0^+} = \lim_{x \to 0^-} = \frac{5}{9}$$

$\therefore$ limits do exists

And

$$\lim_{x \to 0} f(x) = f(0) = \frac{5}{9}$$

$\therefore$ $f(x) = \dfrac{4x+5}{9-3x}$ is continuous at $x = 0$

c) $x = 3$

$$f(3) = \frac{4x+5}{9-3x} = \frac{4(3)+5}{9-3(3)} = \frac{12+5}{0} = \text{not defined}$$

$\because f(3) =$ not defined

$\therefore f(x) = \frac{4x+5}{9-3x}$ is discontinous / not continous at $x = 3$

CLASSMATE

Date

Page

5) Evaluate each of the following limits

$$\lim_{x \to \infty} \frac{6e^{4x} - e^{-2x}}{8e^{4x} - e^{2x} + 3e^{-x}}$$

Sol^n:

$$= \lim_{x \to \infty} \frac{6e^{4x} - e^{-2x}}{8e^{4x} - e^{2x} + 3e^{-x}}$$

Divide by e^{4x}

$$= \lim_{x \to \infty} \frac{6 - e^{-6x}}{3e^{-5x} - e^{-2x} + 8}$$

The expression $-e^{-6x}$, $3e^{-5x}$ and $-e^{-2x}$ ~~all~~ all tends to zero as x approaches '∞';

$$= \frac{0+6}{0+0+8}$$

$$= \frac{6}{8}$$

$$= \frac{3}{4}$$

$$\therefore \lim_{x \to \infty} \frac{6e^{4x} - e^{-2x}}{8e^{4x} - e^{2x} + 3e^{-x}} = \frac{3}{4}$$

6) Given the function,

$$g(y) = \begin{cases} y^2 + 5 & \text{if } y < -2 \\ 1 - 3y & \text{if } y \geq -2 \end{cases}$$

Compute the following limits.

a) $\lim\limits_{y \to 6} g(y)$

$\lim\limits_{y \to 6} 1 - 3y$

$\therefore \lim\limits_{y \to 6} (-3y + 1) = -3 \times 6 + 1 = -17$

$$\boxed{\therefore \lim_{y \to 6} (-3y + 1) = -17}$$

b) $\lim_{y \to -2} g(y)$

$$\lim_{y \to -2} (1-3y) = 1-3(-2) = 1+6 = 7$$

$$\boxed{\therefore \lim_{y \to -2} (1-3y) = 7}$$

7) Use the Product Rule to find the derivative of:

$f(t) = (4t^2 - t)(t^3 - 8t^2 + 12)$

$\therefore f(t) = 4t^2(t^3 - 8t^2 + 12) - t(t^3 - 8t^2 + 12)$

$\therefore f(t) = 4t^5 - 32t^4 + 48t^2 - t^4 + 8t^3 - 12t$

$\therefore f(t) = 4t^5 - 33t^4 + 8t^3 + 48t^2 - 12t$

differentiating with respect to t

$\therefore f'(t) = 20t^4 - 132t^3 + 24t^2 + 96t - 12$

8) Use the Quotient Rule to find the derivative of

$$R(\omega) = \frac{3\omega + \omega^4}{2\omega^2 + 1}$$

Differentiating with respect to ω

$$R'(\omega) = \frac{(2\omega^2+1)\frac{d}{dx}(3\omega+\omega^4) - (3\omega+\omega^4)\frac{d}{dx}(2\omega^2+1)}{(2\omega^2+1)^2}$$

$$R'(\omega) = \frac{(2\omega^2+1)(3+4\omega) - (3\omega+\omega^4)(4\omega)}{(2\omega^2+1)^2}$$

$$= \frac{-4\omega(3\omega+\omega^4) + (1+2\omega^2)(3+4\omega^3)}{(1+2\omega^2)^2}$$

$$\therefore f'(\omega) = \frac{3 - 6\omega^2 + 4\omega^3 + 4\omega^5}{(2\omega^2+1)^2}$$

$$\therefore f'(\omega) = \frac{4\omega^5 + 4\omega^3 - 6\omega^2 + 3}{(2\omega^2+1)^2}$$

classmate

Date

Page

9) Differentiate :

$f(x) = 2e^x - 8^x$

differentiate with respect to x

$f'(x) = 2 \frac{d}{dx} e^x - 8^x \log(8)$

$= 2e^x - 8^x \log(8)$

$$\boxed{\therefore f'(x) = 2e^x - 8^x \log(8)}$$

10) $y = z^5 - e^z \log(z)$

differentiate with respect to z

$$\frac{dy}{dx} = \frac{d}{dx} z^5 - \left[e^z \frac{d}{dx} \log(z) + \log(z) \frac{d}{dx} e^z \right)$$

$$= 5z^4 - e^z \cdot \frac{1}{z} + \log z \cdot e^z$$

$$= 5z^4 + e^z \left(\log z - \frac{1}{z} \right)$$

$$\therefore \boxed{\frac{dy}{dx} = 5z^4 + e^z \left(\log z - \frac{1}{z} \right)}$$

11) Using chain rule differentiate the following:

a) $f(x) = (6x^2 + 7x)^4$

differentiate with respect to x

$\therefore f'(x) = 4(6x^2 + 7x)^3 \frac{d}{dx}(6x^2 + 7x)$

$\boxed{\therefore f'(x) = 4(6x^2 + 7x)^3 (12x + 7)}$

b) $f(t) = 5 + e^{4t + t^7}$

differentiate with respect to x

$\therefore f'(x) = e^{4t + t^7} \frac{d}{dx}(4t + t^7)$

$\therefore f'(x) = e^{4t + t^7} . (4 + 7t^6)$

$\boxed{\therefore f'(x) = e^{4t + t^7}(4 + 7t^6)}$

12) Determine the second derivative of

a) $z = \log(7 - x^3)$

Diff wtr x

$$\frac{dz}{dx} = \frac{1}{(7 - x^3)} \frac{d}{dx}(7 - x^3)$$

$$\therefore \frac{dz}{dx} = \frac{-3x^2}{7 - x^3}$$

differ~~ein~~ wrt x

$$\cancel{\therefore \frac{d^2z}{dx^2} = \frac{-6x}{}}$$

differentiating with respect to x

$$\therefore \frac{d^2z}{dx^2} = \frac{(7 - x^3)\frac{d}{dx}(-3x^2) - (-3x^2)\frac{d}{dx}(7 - x}{(7 - x^3)^2}$$

$$\therefore \frac{d^2z}{dx^2} = \frac{(7 - x^3)(-6x) + (3x^2)(-3x^2)}{(7 - x^3)^2}$$

$$\therefore \frac{d^2z}{dx^2} = \frac{6x^4 - 42x \cancel{+} - 9x^4}{(7 - x^3)^2}$$

$$\therefore \frac{d^2z}{dx^2} = \frac{-3x^4 - 42x}{(7 - x^3)^2}$$

$$\boxed{\therefore \frac{d^2z}{dx^2} = \frac{-3x(x^3 + 14)}{(7 - x^3)^2}}$$

b) $f(v) = \dfrac{2}{(6+2v-v^2)^4}$

Differentiating with respect to ~~x~~ v

$$\therefore \cancel{d}\, f'(v) = 2\left(\frac{d}{dx}\left(\frac{1}{(6+2v-v^2)^4}\right)\right)$$

$$\therefore f'(v) = 2\left[\frac{-4\left(d/dv\,(6+2v-v^2)\right)}{(6+2v-v^2)^5}\right]$$

$$\therefore f'(v) = \frac{8}{(6+2v-v^2)^5}\left(\frac{d}{dv}(6) + 2\left(\frac{d}{dv}(v)\right) - \frac{d}{dv}(v^2)\right)$$

$$\therefore f'(v) = \frac{-8\left(-\left(\frac{d}{dv}(v^2)\right) + 2\right)}{(6+2v-v^2)^5}$$

$$\therefore f'(v) = \frac{16(-1+v)}{(6+2v-v^2)^5}$$

differentiating with respect to v

$$\therefore f''(v) = 16\left(\frac{d}{dv}\left(\frac{-1+v}{(6+2v-v^2)^5}\right)\right)$$

$$\therefore f''(v) = 16\left(\frac{\frac{d}{dv}(-1+v)}{(6+2v-v^2)^5} + (-1+v)\left(\frac{d}{dv}\left(\frac{1}{(6+2v-v^2)^5}\right)\right)\right)$$

classmate

Date
Page

$$f''(v) = 16\left(\frac{d/dv(v)}{(6+2v-v^2)^5} + (-1+v)\left(\frac{d}{dv}\left(\frac{1}{(6+2v-v^2)^5}\right)\right)\right)$$

$$\therefore f''(v) = 16\left(\frac{1}{(6+2v-v^2)^5} - \frac{5(-1+v)\left(2\left(d/dv(v)\right) - d/dv\left(v^2\right)\right)}{(6+2v-v^2)^6}\right)$$

$$\therefore f''(v) = 16\left(\frac{1}{(6+2v-v^2)^5} - \frac{5(-1+v)(2-2v)}{(6+2v-v^2)^6}\right)$$

$$\boxed{\therefore f''(v) = \frac{16(16-18v+9v^2)}{(6+2v-v^2)^6}}$$

c) $f(t) = \log(1+t^2)$

differentiating with respect to t

$$f'(t) = \frac{1}{(1+t^2)}\frac{d}{dt}(1+t^2)$$

$$f'(t) = \frac{1}{(1+t^2)} \cdot 2t = \frac{2t}{(1+t^2)}.$$

diff wrt t

$$f''(t) = \frac{(1+t^2)\frac{d}{dt}(2t) - 2t\frac{d}{dt}(1+t^2)}{(1+t^2)^2}$$

$$f''(t) = \frac{(2+2t^2) - 4t^2}{(1+t^2)^2} = \frac{(2-2t^2)}{(1+t^2)^2}$$

~~$\therefore f''(t) =$~~

$$\boxed{\therefore f''(t) = \frac{2(1-t^2)}{(1+t^2)^2}}$$

13) Find the maximum and minimum the function.

$$x^3 - 3x^2 - 9x + 12$$

diff. w.r.t x

$\therefore f'(x) = 3x^2 - 6x - 9$

when $f'(x) = 0$

$3x^2 - 6x - 9 = 0$

$3(x^2 - 2x - 3) = 0$

$x^2 - 2x - 3 = 0$

$x^2 - 3x + x - 3 = 0$

$\therefore x(x-3) + 1(x-3) = 0$

$\therefore (x-3)(x+1) = 0$

$\therefore x = 3$ & $x = -1$

$f''(x) = 6x - 6$

$\therefore f''(3) = 6(3) - 6 = 18 - 6 = 12$

$f''(-1) = 6(-1) - 6 = -6 - 6 = -12$

$\therefore f(x)$ is maximum at $x = -1$

& minimum at $x = 3$

14) Find the maximum and minimum value of the function.

$$4x^3 - 18x^2 + 24x - 7$$

diff. wrt x

$f'(x) = 12x^2 - 36x + 24$

when $f'(x) = 0$

$\therefore \quad x^2 - 3x + 2 = 0$

$\therefore \quad x^2 - 2x - x + 2 = 0$

$\therefore \quad x(x-2) - 1(x-2) = 0$

$\therefore \quad (x-2)(x-1) = 0$

$\therefore \quad x = 2 \quad \& \quad x = 1$

$f''(x) = 24x - 36$

$f''(2) = 24(2) - 36 = 48 - 36 = 12$

$f''(1) = 24(1) - 36 = 24 - 36 = -12$

$\therefore$ $f(x)$ is maximum at $x = 1$

and minimum at $x = 2$

15) Sketch the curve for the following function.

$f(x) = x^2(x+3)$

$f(x) = x^3 + 3x^2$

diff wrt x

$f'(x) = 3x^2 + 6x$

when $f'(x) = 0$

$3x(x+2) = 0$

$\therefore x + 2 = 0$

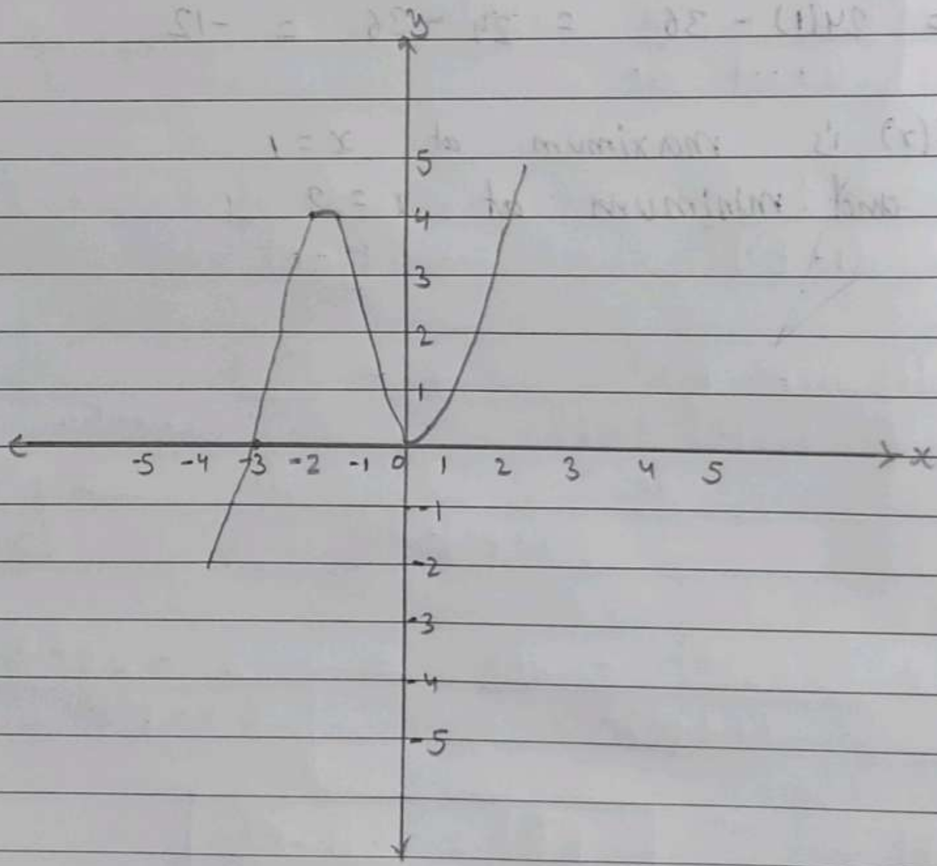
$\therefore x = -2$

$f''(x) = 6x + 6$

$f''(-2) = 6(-2) + 6 = -12 + 6 = -6$

$\therefore f(x)$ is maximum at $x = -2$

and minimum at $x = 0$



16) Sketch a detailed graph of the following.

$f(x) = 3x^2 - 6x$

diff wrt x

$f'(x) = \cancel{3}\, 6x - 6$

When $f'(x) = 0$

$6x - 6 = 0$

$\therefore x - 1 = 0$

$\therefore x = 1$

$f''(x) = 6$

$f''(1) = 6$

$\therefore f(x)$ is minimum at $x = 1$