

Homework - 1

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Lagrange

Q.14

x	7	8	9	10
$y=f(x)$	3	1	1	9

$$f(9.5) =$$

$$\frac{(9.5-8)(9.5-9)(9.5-10)(3)}{(7-8)(7-9)(7-10)} + \frac{''''''}{''''''}$$

$$\frac{(9.5-7)(9.5-9)(9.5-10)(3)}{(7-8)(7-9)(7-10)} + \frac{(9.5-7)(9.5-9)(9.5-10)(1)}{(8-7)(8-9)(8-10)}$$

$$\frac{(9.5-7)(9.5-8)(9.5-10)(1)}{(9-7)(9-8)(9-10)} + \frac{(9.5-7)(9.5-8)(9.5-9)(9)}{(10-7)(10-8)(10-9)}$$

$$\frac{(1.5)(0.5)(-0.5)(3)}{(-1)(-2)(-3)} + \frac{(2.5)(0.5)(-0.5)(1)}{(+1)(-1)(-2)}$$

$$\frac{(2.5)(1.5)(-0.5)}{(2)(1)(-1)} + \frac{(2.5)(1.5)(0.5)(9)}{(3)(2)(1)}$$

$$\frac{+1.5}{2} - \frac{(2.5)(0.5)(0.5)}{2} + \frac{(2.5)(1.5)(0.5)}{2} + \frac{(2.5)(1.5)(0.5)(3)}{2}$$

$$= \frac{57}{16} = 4.1875$$

Q.24

x	3	6	7	9	10
y	168	?	120	72	63

$$y_6 = \frac{(6-7)(6-9)(6-10)}{(3-7)(3-9)(3-10)} (168) + \frac{(6-3)(6-9)(6-10)}{(7-3)(7-9)(7-10)} (120)$$

$$+ \frac{(6-3)(6-9)(6-10)}{(9-3)(9-7)(9-10)} (72) + \frac{(6-3)(6-7)(6-9)}{(10-3)(10-7)(10-9)} (63)$$

$$= 12 + 180 - 72 + 27 = 147$$

The best estimate of function at position 6 is 147.

$$8.3) \begin{array}{|c|c|c|c|} \hline 1 & 2 & 7 & 8 \\ \hline 4 & 5 & 7 & 4 \\ \hline \end{array} \quad \sqrt{(6)} = ?$$

$$\frac{(6-2)(6-7)(6-8)(4)}{(1-2)(1-7)(1-8)} + \frac{(6-1)(6-7)(6-8)(5)}{(2-1)(2-7)(2-8)}$$

$$+ \frac{(6-1)(6-2)(6-8)(7)}{(7-1)(7-2)(7-8)} + \frac{(6-1)(6-2)(6-7)(4)}{(4-1)(4-2)(4-7)}$$

$$- \frac{(4)(+3)(+2)(4)}{(+1)(+6)(+7)} + \frac{(5)(+1)(\cancel{1})(5)}{(1)(\cancel{5})(\cancel{6})3}$$

$$\frac{(5)(4)(2)(7)}{(6)(5)(1)} + \frac{(5)(4)(1)(4)}{(3)(2)(3)}$$

$$\frac{-16}{21} + \frac{5}{3} + \frac{20}{3} - \frac{40}{21}$$

$$\approx \underline{\underline{5.6667}}$$

Q.44

x	1	2	3	4	7
$f(x)$	2	4	8	16	128

$$\begin{aligned}
 f(15) &= \frac{(5-2)(5-3)(5-4)(5-7)(2)}{(1-2)(1-3)(1-4)(1-7)} \\
 &+ \frac{(5-1)(5-3)(5-4)(5-7)(4)}{(2-1)(2-3)(2-4)(2-7)} \\
 &+ \frac{(5-1)(5-2)(5-4)(5-7)(8)}{(3-1)(3-2)(3-4)(3-7)} \\
 &+ \frac{(5-1)(5-2)(5-3)(5-7)(16)}{(4-1)(4-2)(4-3)(4-7)} \\
 &+ \frac{(5-1)(5-2)(5-3)(5-4)(128)}{(7-1)(7-2)(7-3)(7-4)}.
 \end{aligned}$$

$$= \frac{-2}{3} + \frac{32}{5} - 24 + \frac{128}{3} + \frac{128}{15}$$

$$= 32.9334$$

Q.5)

x	10	20	14	16
$f(x)$	0.25	0.20	0.15	0.10

$$\frac{1}{5} f(15) = \frac{(15-20)(15-14)(15-16)(0.25)}{(10-20)(10-14)(10-16)}$$

$$+ \frac{(15-10)(15-14)(15-16)(0.20)}{(20-10)(20-14)(20-16)}$$

$$+ \frac{(15-10)(15-20)(15-16)(0.15)}{(24-10)(14-20)(14-16)}$$

$$+ \frac{(15-10)(15-20)(15-14)(0.10)}{(16-10)(16-20)(16-14)}$$

$$= \frac{-1}{112} - \frac{1}{240} + \frac{5}{64} + \frac{5}{96}$$

$$= 0.1208$$

HOMEWORK-2]

Q.14 $x^3 + x - 1 = 0$ b/w 0.5 and 1
 $y = x^3 + x - 1$

$$x_n = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1}$$

x	y
0.5	-0.315
0.6	-0.184
0.7	0.043

$$x_1 = 0.6 \quad x_2 = 0.7$$

$$y_1 = -0.184 \quad y_2 = 0.043$$

$$x_n = \frac{(0.043)(0.6) + (0.7)(-0.184)}{0.043 - 0.184}$$

$$= 0.6810$$

$$r = -0.003042$$

$$x_1 = 0.6810$$

$$x_2 = 0.7$$

$$y_1 = 0.003042$$

$$y_2 = 0.043$$

$$x_n = \frac{(0.6810)(0.043) + (0.7)(0.003042)}{0.043 + 0.003042}$$

$$= 0.6825$$

$$y_n = -0.000045$$

Root $y'x^3 + x - 1 = 0$ is 0.6825 .

$$8.24x^3 + 2x + 0.5 = 0 \quad \text{b/w } 0 \text{ and } 1$$

$$y = x^3 + 2x + 0.5$$

x	y
0	0.5
0.1	0.301
0.2	0.108
0.3	-0.073

$$x_1 = 0.2 \quad x_2 = 0.3$$

$$y_1 = 0.108 \quad y_2 = -0.073$$

$$x_n = \frac{(-0.073)(0.2) + (0.3)(0.108)}{0.108 + (-0.073)}$$

$$= 0.259669$$

$$y_n = -0.001824$$

$$x_1 = 0.2$$

$$x_2 = 0.259664$$

$$y_1 = 0.108$$

$$y_2 = -0.001829$$

$$x_n = \frac{(-0.001829)(0.2) + (0.108)(0.259664)}{0.108 + (-0.001829)}$$

$$= 0.255675$$

$$y_n = 0.000041$$

Root of " $x^3 - 2x + 0.5$ " is $= 0.258675$.

8.43 $x^3 + 2x - 20 = 0$ b/w 2 and 2.5.
 $y = x^3 + 2x - 20$

x	y
2.0	-8
2.1	-6.534
2.2	-4.953
2.3	-3.233
2.4	-0.1625
2.5	0.625

$$x_1 = 2.4$$

$$x_2 = 2.5$$

$$y_1 = -0.1371$$

$$y_2 = 0.625$$

$$x_n = \frac{(0.625)(2.4) + (2.5)(-0.1371)}{-0.1371 + 0.625}$$

$$= 2.46878$$

$$y_n = -0.01582$$

$$x_1 = 2.46878$$

$$x_2 = 2.5$$

$$y_1 = -0.01582$$

$$y_2 = 0.625$$

$$x_n = \frac{(0.625)(2.46878) + (2.5)(-0.01582)}{0.625 + (-0.01582)}$$

$$= 2.469034$$

$$y_n = -0.000176$$

Root of eq. is ~~2.469034~~ 2.46954

Q.44 $3x - 105x - 1 = 0$ b/w 0 and 1

x	y
0	-2
0.1	-1.69998
0.2	-1.39996
0.3	-1.09993
0.4	-0.7999
0.5	-0.4999
0.6	-0.1999
0.7	0.10007

$$x_1 = 0.6 \quad x_2 = 0.1$$

$$y_1 = 0.1999 \quad y_2 = 10007$$

$$x_n = \frac{(0.10007)(0.6) + (0.1)(0.1999)}{0.1999 + 0.10007}$$

$$= 0.6666$$

$$y_n = -0.0000003$$

Root of equation is 0.6666

Q.54 $x^3 - 3x - 5 = 0$ b/w 0 and 2.5

x	y
2.0	-3
2.1	-2.039
2.2	-0.952
2.3	0.267
2.4	1.629

$$x_1 = 2.2$$

$$x_2 = 0.3$$

$$y_1 = -0.952$$

$$y_2 = 0.267$$

$$x_n = \frac{(0.267)(2.2) + (-0.952)(0.3)}{(-0.952) + 0.267}$$

$$= 2.27809$$

$$y_n = -0.01159$$

$$x_1 = 2.27804$$

$$x_2 = 2.3$$

$$y_1 = -0.01159$$

$$y_2 = 0.267$$

$$x_n = \frac{(0.267)(2.27804) + (2.3)(-0.01159)}{-0.01159 + 0.267}$$

$$x = 2.27901$$

$$y_n = 0.0001$$

$$\text{root of eq} = 2.27901$$

$$8.64 x \log_{10} x - 1.2 = 0$$

b/w 2 and 3.

x	y
2.0	-0.59794
2.1	-0.52339
2.2	-0.44667
2.3	-0.36806
2.4	-0.28749
2.5	-0.20514
2.6	-0.12106
2.7	-0.0353
2.8	-0.05205

$$x_1 = 2.7 \quad x_2 = 2.8$$

$$y_1 = 0.0353 \quad y_2 = 0.05205$$

$$x_n = 0.0502 \frac{(0.05205)(2.7) + (2.8)(0.0353)}{(0.0353) + (0.05205)}$$

$$= 2.7404$$

$$y_n = -0.00019$$

Root of eq = 2.7407.

Q.74. $x^3 - 4x + 1 = 0$ ~~Find~~

x	y
0	1
0.1	0.60
0.2	0.208
0.3	-0.173

$$x_n = \frac{(-0.173)(0.3) - (0.3)(0.208)}{-0.173 - 0.208}$$

$$= 0.2546$$

$$y_n = -0.00187$$

$$x_1 = 0.2$$

$$y_1 = 0.208$$

$$x_2 = 0.2546$$

$$y_2 = -0.00187$$

$$x'_n = \frac{(-0.00187)(0.2) - (0.208)(0.2546)}{0.208 - 0.00187}$$

$$= 0.254$$

$$y''_n = -0.00002$$

Root of equation is 0.254

$$Q. 84. x^3 + 2x^2 - 8 = 0$$

$$\therefore y = x^3 + 2x^2 - 8$$

x	y
0	-8
1	-5
2	8

$$x_1 = 1 \quad x_2 = 2$$

$$y_1 = -5 \quad y_2 = 8$$

$$x_n = \frac{8 + 5(2)}{8 + 5} = 1.3846$$

$$y_n = -1.5112$$

$$x'_n = 8(1.3846)$$

$$x_1 = 1.3846$$

$$x_2 = 2$$

$$y_1 = -1.5112$$

$$y_2 = 8$$

$$x'_n = \frac{(8)(-1.3846) + (2)(-1.5112)}{8 - 1.5112}$$

$$= 1.4824$$

$$y'_n = -0.3475$$

$$x''_n = \frac{(8)(1.5085) + (0.958)(2)}{8 + 0.038}$$

$$= 1.5093$$

$$y_n = +0.0033$$

$$x_1 = 1.8095 \quad x_2 = 2$$

$$y_1 = -0.0033 \quad y_2 = 8$$

$$x_n = \frac{8(1.8095) + 0.0033(2)}{8 + 0.0033}$$

$$= 1.5097$$

$$y_n = -0.0007$$

root of the eq = 1.5097

Q. 94 $x^3 - 9x + 1 = 0$

x	y
0	1
0.1	0.101
0.2	0.792

$$x_1 = 0.1 \quad x_2 = 0.2$$

$$y_1 = 0.101 \quad y_2 = 0.792$$

$$x_n = \frac{(-0.192)(0.1) - (0.2)(0.101)}{-0.792 - 0.101}$$

$$= 0.1113$$

$$y_n = 0.00041$$

Root of eq. is 0.1113

Q. 104 $x^3 - x - 4 = 0$

x	y
0	-4
1	-4
2	2

$$x_1 = 1 \quad x_2 = 2$$

$$y_1 = -4 \quad y_2 = 2$$

$$x_n = \frac{(1)(2) + (4)(2)}{2 + 4} = 1.3334$$

$$y_n = -2.9629$$

$$x_1 = 1.3334 \quad x_2 = 2.$$

$$y_1 = -2.9629 \quad y_2 = 2.$$

$$x_n = \frac{(2)(1.3334) + (2)(2.9629)}{2 + 2.9629}$$

$$= 1.7313$$

$$y_n = -0.5415$$

$$x_1 = 1.7313 \quad x_2 = 2.$$

$$y_1 = -0.5415 \quad y_2 = 2.$$

$$x_n = \frac{(2)(1.7313) + (2)(0.5415)}{2 + 0.6680}$$

$$= 1.7954$$

$$y_n = -0.0078$$

$$x_1 = 1.7954 \quad x_2 = 2.$$

$$y_1 = -0.0078 \quad y_2 = 2.$$

$$x_n = \frac{(2)(1.7954) + 2(0.0078)}{2 + 0.0078}$$

$$x_n = 1.7912$$

$$y_n = 0.00091$$

Root of equation is 1.7962