Date ....

Expt. No. .... Page No. ....

Probability & Statistics
Assignment - 2

① $X =$ claim size on a home insurance

$X \sim \text{Normal}(800, 100^2)$

$Y =$ claim size on a car insurance

$Y \sim \text{Normal}(1200, 300^2)$

$$P(Y_1 + Y_2 + Y_3 > X_1 + X_2 + X_3 + X_4 + 800)$$
$$= P[(Y_1 + Y_2 + Y_3) - (X_1 + X_2 + X_3 + X_4) > 800]$$

$$(Y_1 + Y_2 + Y_3) - (X_1 + X_2 + X_3 + X_4) \sim N[3(1200) - 4(800), (3 \times 300^2 + 4 \times 100^2)$$
$$\sim N(400, 310000)$$

$$P[(Y_1 + Y_2 + Y_3) - (X_1 + X_2 + X_3 + X_4) > 800]$$
$$= P\left(Z > \frac{800 - 400}{\sqrt{310000}}\right) = P(Z > 0.718)$$
$$= 1 - P(Z < 0.718)$$
$$\simeq 1 - 0.76424$$
$$\simeq 0.23576$$

② $N =$ number of claims

$N \sim \text{Negative Binomial}(3, 0.9)$

Teacher's Signature : ______________

Date

Expt. No. Page No.

X = Claim amount

$X \sim$ Gamma $(6, 2)$

Y = Total claim amount

$$M_Y(t) = M_N[\log M_X(t)]$$

$$M_X(t) = \left(1 - \frac{t}{2}\right)^{-6}$$

$$M_Y(t) = M_N\left[\log\left(1 - \frac{t}{2}\right)^{-6}\right]$$

$$\therefore \left[M_N(t) = \left(\frac{pe^t}{1-qe^t}\right)^t\right]$$

$$M_Y(t) = \left(\frac{pe^{\log\left(1-\frac{t}{2}\right)-6}}{1-qe^{\log\left(1-\frac{t}{2}\right)^{-6}}}\right)^3$$

$$M_Y(t) = \left[\frac{p\left(1-\frac{t}{2}\right)^{-6}}{1-q\left(1-\frac{t}{2}\right)^{-6}}\right]^3$$

$$M_Y(t) = \left[\frac{0.9\left(1-\frac{t}{2}\right)^{-6}}{1-(0.1)\left(1-\frac{t}{2}\right)^{-6}}\right]^3$$

Teacher's Signature :

Date

Expt. No.   Page No.

$$E(N) = \frac{3}{0.9} = \frac{10}{3}$$

$$V(N) = \frac{k(0.1)}{(0.9)^2} = \frac{3}{0.9}\left(\frac{1}{9}\right) = \frac{10}{27}$$

$$E(x) = \frac{\alpha}{\lambda} = \frac{6}{2} = 3 \qquad V(x) = \frac{\alpha}{\lambda^2} = \frac{6}{4} = \frac{3}{2}$$

$$V(Y) = E(N)V(x) + E(x)^2 V(N)$$

$$= \frac{10}{3}\left(\frac{3}{2}\right) + 9\left(\frac{10}{27}\right)$$

$$= 5 + \frac{10}{3} = \frac{25}{3}$$

$$S.D(Y) = \sqrt{\frac{25}{3}} = 2.886$$

③ $f_x(n) = \lambda e^{-\lambda(n-\alpha)}$

$$M_x(t) = E(e^{tn}) = \int_{\alpha}^{\infty} e^{tn} \lambda e^{-\lambda(n-\alpha)}\, dn$$

$$= \lambda \int_{\alpha}^{\infty} e^{tn} e^{-\lambda n} e^{\lambda\alpha}\, dn$$

$$= \lambda e^{\lambda\alpha} \int_{\alpha}^{\infty} e^{n(t-\lambda)}\, dn$$

$$= \frac{\lambda e^{\lambda\alpha}}{t-\lambda}\left[e^{(t-\lambda)n}\right]_0^{\infty} = \frac{\lambda e^{\lambda\alpha}}{t-\lambda}\left[e^{-(\lambda-t)n}\right]_0^{\infty}$$

Teacher's Signature :

Date

Expt. No. Page No.

$$M_x(t) = \frac{\lambda e^{\lambda\alpha}}{t-\lambda}\left[0 - e^{-\lambda\alpha} \cdot e^{t\alpha}\right]$$

$$M_x(t) = \frac{\lambda e^{t\alpha}}{\lambda - t}$$

ii) $M'_x(t) = \lambda(e^{t\alpha})(\lambda-t)^{-2}(-1)(-1) + \lambda(\lambda-t)^{-1}(e^{t\alpha})(\alpha)$

$$= \frac{\lambda e^{t\alpha}}{(\lambda-t)^{t2}} + \frac{\lambda e^{t\alpha}\alpha}{\lambda - t}$$

$$E(x) = M'_x(0) = \frac{\lambda(1)}{\lambda^2} + \frac{\lambda\alpha}{\lambda}$$

$$= \frac{1+\lambda\alpha}{\lambda} = \frac{1+\lambda\alpha}{\lambda}$$

$$M''_x(t) = \lambda e^{t\alpha}(-2)(\lambda-t)^{-3}(-1) + \lambda(\lambda-t)^{-2}(e^{t\alpha})(\alpha)$$
$$+ \lambda\alpha e^{t\alpha}(\lambda-t)^{-2}(-1)(-1)$$
$$+ \lambda\alpha(\lambda-t)^{-1}(e^{t\alpha})(\alpha)$$

$$= \frac{2\lambda e^{t\alpha}}{(\lambda-t)^3} + \frac{\lambda\alpha e^{t\alpha}}{(\lambda-t)^2} + \frac{\lambda\alpha e^{t\alpha}}{(\lambda-t)^2} + \frac{\alpha^2\lambda e^{t\alpha}}{\lambda - t}$$

$$= \left(\frac{2\lambda e^{t\alpha}}{(\lambda-t)^3}\right) + \left(\frac{2\lambda\alpha e^{t\alpha}}{(\lambda-t)^2}\right) + \left(\frac{\lambda\alpha^2 e^{t\alpha}}{\lambda - t}\right)$$

$$E(x^2) = M''_x(0) = \frac{2\lambda(1)}{\lambda^2} + \frac{2\lambda\alpha}{\lambda^2} + \frac{\lambda\alpha^2}{\lambda}$$

$$= \frac{2}{\lambda^2} + \frac{2\alpha}{\lambda} + \alpha^2$$

Teacher's Signature :

Date ....

Expt. No. .... Page No. ....

$$V(X) = E(X^2) - (E(X))^2$$

$$= \alpha^2 + \frac{2\alpha}{\lambda} + \frac{2}{\lambda^2} - \left(\frac{1 + \lambda^2\alpha^2 + 2\alpha\lambda}{\lambda^2}\right)$$

$$= \cancel{\alpha^2} + \frac{2\cancel{\alpha}}{\cancel{\lambda}} + \frac{2}{\lambda^2} - \frac{1}{\lambda^2} - \cancel{\alpha^2} - \frac{2\cancel{\alpha}}{\cancel{\lambda}}$$

$$V(X) = \frac{1}{\lambda^2}$$

(4) $G_v(t) = \left(\frac{p}{1-qt}\right)^k$, $p+q=1$

∴ Given distribution is negative binomial type 2

$$P(V=4) = \frac{\Gamma(k+4)}{\Gamma 5 \; \Gamma k} p^4 q^4$$

(5) $f(x,y) = ax^2 + axy \qquad 0 < x < 5$

$10 < y < 20$

$$\Rightarrow 1 = \int_{10}^{20}\int_0^5 ax^2 + axy \, dx \, dy$$

$$1 = \int_{10}^{20} \left[\frac{ax^3}{3}\right]_0^5 + \left[ay\frac{x^2}{2}\right]_0^5 dy$$

$$1 = \int_{10}^{20} \frac{125a}{3} + \frac{25}{2} ay \, dy$$

Teacher's Signature : ....

Expt. No. Page No. Date

$$1 = \frac{125a}{3}\left[y\right]_{10}^{20} + \frac{25a}{2}\left[\frac{y^2}{2}\right]_{10}^{20}$$

$$1 = \frac{1250a}{3} + 1875a$$

$$a = \frac{3}{6875}$$

$$f_X(x) = \int_{10}^{20} (x^2 + xy)\, a\, dy$$

$$= \frac{3}{6875}\int_{10}^{20} (x^2 + xy)\, dy$$

$$= \frac{3}{6875}\left[x^2\left[y\right]_{10}^{20} + x\left[\frac{y^2}{2}\right]_{10}^{20}\right]$$

$$= \frac{3}{6875}(10x^2 + 150x)$$

$$E\left(\frac{Y}{X}\right) = y f\left(\frac{Y}{X}\right) = y\int_{10} \frac{f(x,y)}{f_X(x)}$$

$$= \frac{\int_{10}^{20} y\left(\frac{3}{6875}\right)(x^2 + xy)\, dy}{\frac{3}{6875}(10x^2 + 150x)}$$

$$= \frac{1}{10}\int_{10}^{20} \frac{y(x+y)}{(x+15)}\, dy$$

Teacher's Signature :

$$= \frac{1}{10(x+15)}\left[\left(x\frac{y^2}{2}+\frac{y^3}{3}\right)_{10}^{20}\right]$$

$$= \frac{1}{10(x+15)}\left[150x+\frac{7000}{3}\right]$$

$$= \frac{1}{x+15}\left(15x+\frac{700}{3}\right) \Rightarrow \frac{45x+700}{3(x+15)}$$

$$E\left(\frac{Y}{X}\right) = \frac{45x+700}{3(x+15)}$$

$$P\left(Y>\frac{1}{3}x+10\right) \Rightarrow$$

$$f_Y(y) = \int_0^5 \frac{3}{6875}(x^2+xy)\,dx$$

$$= \frac{3}{6875}\left[\frac{x^3}{3}+\frac{x^2y}{2}\right]_0^5$$

$$= \frac{3}{6875}\left[\frac{125}{3}+\frac{25y}{2}\right]$$

$$= \frac{1}{6875}\left[\frac{250+75y}{2}\right]$$

$$P\left(Y>\frac{1}{3}x+10\right) = \int_{\frac{1}{3}x+10}^{20} \frac{1}{6875}\frac{(250+75y)}{2}\,dy$$

$$= \frac{1}{6875(2)}\left[250\left[y\right]_{\frac{1}{3}x+10}^{20} + 75\left[\frac{y^2}{2}\right]_{\frac{x}{3}+10}^{20}\right]$$

Date

Expt. No. Page No.

$$= \frac{1}{6875(2)}\left[5000 - \frac{250x}{3} - 2500 + 15000 - \frac{75x^2}{3} - 7500 - \frac{1500x}{3}\right]$$

$$= \frac{1}{6875(2)}\left[10000 - \frac{1750x}{3} - \frac{25x^2}{3}\right]$$

$$X \sim Poi(\lambda) \qquad Y \sim Poi(\mu)$$

⑥ $M_X(t) = e^{\lambda(e^t-1)}$ $\qquad M_Y(t) = e^{\mu(e^t-1)}$

$$M_{X-Y}(t) = E\left[e^{t(X-Y)}\right]$$

$$= E\left[e^{Xt - Yt}\right]$$

$$= \frac{E(e^{Xt})}{E(e^{Yt})} = \frac{e^{\lambda(e^t-1)}}{e^{\mu(e^t-1)}} = e^{(\lambda-\mu)(e^t-1)}$$

$$X - Y \sim Poi(\lambda - \mu)$$

$\therefore$ Hence Proved

$$M_{X+Y}(t) = E\left(e^{t(X+Y)}\right) = E\left(e^{tX+Yt}\right)$$

$$= E\left(e^{tX}\right) \cdot E\left(e^{tY}\right)$$

$$= e^{\lambda(e^t-1)} \cdot e^{\mu(e^t-1)}$$

$$= e^{(\lambda-\mu)(e^t-1)}$$

$$X - Y \sim Poi(\lambda + \mu)$$

$\therefore$ Hence Proved

Teacher's Signature :

Date ....

Expt. No. .... Page No. ....

7. $Var\left(\frac{X}{Y}\right)=(2) = E\left(\frac{X^2}{Y}\right)=2 - \left[E\left(\frac{X}{Y}\right)=2\right)\Big]^2$

$$= \sum_{n} n^2 \frac{p(x=n, y=2)}{p(y=2)} - \left(\sum_{*} n \frac{p(x=n, y=2)}{p(y=2)}\right)^2$$

$$= \frac{1^2(0)+2^2(0.3)+3^2(0.1)+4^2(0.2)}{0.6}$$

$$- \left[\frac{1(0)+2(0.3)+3(0.1)+4(0.3^2)}{0.6}\right]^2$$

$$= \frac{53}{6} - \frac{289}{36} = \frac{29}{36}$$

ii) $f(u,v) = \frac{48}{67}(2uv - v^2), \; 0<v<1, \; \frac{u}{2}<v<2$

$$E(U/V=v) = \int_0^1 u f(U/V=v)\, du$$

$$= \int_0^1 u \frac{f(u, v=v)}{f(v=v)}\, dn \quad — ①$$

$$\Rightarrow f_v(V=v) = \int_0^1 \frac{48}{67}(2uv - u^2)\, du$$

$$= \frac{48}{67}\left[\frac{2u^2v}{2} - \frac{u^3}{3}\right]_0^1$$

$$= \frac{48}{67}\left[v - \frac{1}{3}\right]$$

$$= \frac{48[3v-1]}{67(3)} = \frac{16}{67}(3v-1)$$

Teacher's Signature : ....

Topic .... Date ....

$$E(U/V=v) = \frac{\int_0^1 u\left(\frac{48}{67}\right)(2uv-u^2)\,du}{\frac{16}{67}(3v-1)}$$

$$= \frac{3}{(3v-1)}\int_0^1 (2u^2v - u^3)\,du$$

$$= \frac{3}{(3v-1)}\left[\frac{2u^3v}{3} - \frac{u^4}{4}\right]_0^1$$

$$= \frac{3}{(3v-1)}\left[\frac{2v}{3} - \frac{1}{4}\right]$$

$$E(U/V=v) = \frac{(8v-3)}{4(3v-1)}$$

(8) $X \sim \text{Gamma}(3,2)$

$E(Y/X=x) = 3x+1$ $\quad$ $Var(Y/X=x) = 2x^2+5$

$E[E(Y/X=x)] = E(Y)$

$$E(Y) = E(3x+1)$$
$$= 3E(x)+1$$
$$= 3\left(\frac{3}{2}\right)+1$$
$$= \frac{11}{2}$$

$Var[Var(Y/X=x)] = V(Y)$

$$V(Y) = E[Var(Y/X=x)] + V[E(Y/X=x)]$$
$$= E(2x^2+5) + V(3x+1)$$
$$= 2E(X^2)+5+9V(X)$$
$$= 2(3)+5+9\left(\frac{3}{4}\right)$$
$$= 6+5+6.75$$
$$= 17.75$$

Topic .... Date ....

9. $Z = X + Y$

$E(X) = 3 \quad E(Y) = 4$

$\sqrt{V(X)} = 2 \quad \sqrt{V(Y)} = 1$

i) $Cov(X, Z) = Cov(X, X+Y) \quad Cov(X, Y) = -0.3$

$= Cov(X, X) + Cov(X, Y)$

$= V(X) + (-0.6)$

$= 4 - 0.6 = 3.4$

ii) $Var(Z) = Var(X+Y)$

$= V(X) + V(X) + 2 Cov(X, Y)$

$= 4 + 1 + 2(-0.6)$

$= 5 - 1.2$

$= 3.8$

10. $f(x, y) = k x^{-\alpha} e^{-y/\beta}, \quad 1 < x < \infty$

$1 < y < \infty$

$\alpha > 1, \quad \beta > 0$

$1 = k \int_1^\infty x^{-\alpha} \, dx \int_1^\infty e^{-y/\beta} \, dy$

$1 = k \left[\frac{x^{-\alpha+1}}{-\alpha+1}\right]_1^\infty \left[\frac{e^{-y/\beta}}{-1/\beta}\right]_1^\infty$

$1 = k \left[0 - \frac{1}{-\alpha+1}\right] \left[0 + \beta (e^{-1/\beta})\right]$

$1 = k \left[\frac{1}{-\alpha+1}\right] \left[-\beta e^{-1/\beta}\right]$

$k = \frac{-1+\alpha}{\beta e^{-1/\beta}} = \frac{\alpha - 1}{\beta e^{-1/\beta}} = k$

CRYSTAL™ S.F.P.I.

Teacher's Signature ....