1) X claim size of home insurance $\quad X \sim normal(800, 100^2)$
Y " " " Core " $\quad Y \sim normal(1200, 300^2)$

$P(Y_1+Y_2+Y_3-X_1-X_2-X_3-X_4 > 800)$

$\therefore N \sim (400, 310000)$

$P(Z > 0.718) = 1 - P(Z < 0.718) = 0.23576$

2) N = no of claim $\quad N \sim -ve\ bin(3, 0.9)$
X = claim amount $\quad X \sim gama(6, 2)$

$M_Y(t) = M_N \log(M_X t)$
$M_X(t) = \left(1-\frac{t}{2}\right)^{-6}$ $\quad M_Y(t) = \left(\frac{0.9\left(1-t/2\right)^{-6}}{1-(0.1)\left(1-t/2\right)^{+6}}\right)^3$

$E(N) = 3.33$ $\quad V(N) = 0.37$ $\quad E(X) = 3$ $\quad V(X) = 1.5$

$V(Y) = 8.33$ $\quad SD(Y) = 2.88$

4) $G_N(t) = \left(\frac{p}{1-qt}\right)^k$ $\quad p+q=1$ $\quad \therefore -ve\ bin$

$P(V=y) = \frac{\sqrt{k+y}}{\sqrt{y}\sqrt{k}} p^y q^y$

6) $X \sim Poi(\lambda)$ $\quad Y \sim Poi(\mu)$
$M_X(t) = e^{\lambda(e^t-1)}$ $\quad M_Y(t) = e^{\mu(e^t-1)}$

$M_{X-Y}(t) = E\left(e^{t(X-Y)}\right) = \frac{E(e^{tX})}{E(e^{tY})} = e^{\lambda-\mu(e^t-1)}$

$M_{X+Y}(t) = E(e^{tX})E(e^{tY}) = \cancel{e^t}\ e^{\lambda+\mu(e^t-1)}$

8) $\alpha = 3 \qquad \lambda = 2 \qquad X \sim gama(3, 2)$

$E(y / x = n) = 3x + 1 \qquad Var(y | x = n) = 2x^2 + 5$

$E(y) = E(3n + 1) = 3\left(\frac{3}{2}\right) + 1 = \frac{11}{2}$

$v(y) = E(2x^2 + 5) + v(3x + 1) = 2(3) + 5 + 9\left(\frac{3}{4}\right) = 17.75$

9) $Z = X + Y$

$E(X) = 3 \qquad E(Y) = 4$

$\sigma_x = 2 \qquad \sigma_y = 1$

$Cov(x, z) = Cov(x, x + y) = Cov(x, x) + Cov(x, y)$

$= v(x) + (-0.6) = 3.4$

$var(z) = v(x) + v(y) \cancel{+} 2 cov(x, y)$

$= 4 + 1 \cancel{+} 2(0.6) = \cancel{6.2}\ 3.8$

10) $f(x, y) = k n^{-\alpha} e^{-y/\beta}$

$1 = k \int_1^{\alpha} \int_1^{\alpha} n^{-\alpha} e^{-y/\beta}\, dn\, dy = k\left(\frac{1}{\alpha+1}\right)\left[-\beta e^{-y/\beta}\right)$

$k = \cancel{2} \quad \frac{\alpha + 1}{\beta e^{-y/\beta}}$