

Homework1. $y = f(x)$ point $\rightarrow$

Lagrange's Interpolation

7	8	9	10
3	1	1	9

 $f(9.5)$

$$f(9.5) = \frac{(9.5-8)(9.5-9)(9.5-10)(3)}{(7-8)(7-9)(7-10)} + \frac{(9.5-7)(9.5-9)(9.5-10)(1)}{(8-7)(8-9)(8-10)} \\ + \frac{(9.5-7)(9.5-8)(9.5-10)(1)}{(9-7)(9-8)(9-10)} + \frac{(9.5-7)(9.5-8)(9.5-9)(9)}{(10-7)(10-8)(10-9)} \\ \Rightarrow 0.1875 + (-0.3125) + (0.3125) + (2.8125) \\ \Rightarrow \boxed{3}$$

2.

2	7	9	10
108	120	72	63

 $f(6)$

Lagrange's interpolation

$$f(6) \Rightarrow \frac{(6-7)(6-9)(6-10)(108)}{(2-7)(2-9)(2-10)} + \frac{3(6-2)(6-9)(6-10)}{(7-2)(7-9)(7-10)} + \frac{12(6-2)(6-7)(6-10)}{(9-2)(9-7)(9-10)} + \frac{3(6-2)(6-7)(6-9)}{(10-2)(10-7)(10-9)} \\ \Rightarrow \boxed{142.7142857}$$

3.

1	2	7	8
4	5	5	4

 $f(6)$

$$f(6) \Rightarrow \frac{8(4)}{-42} + \frac{10(5)}{30} + \frac{40(5)}{+30} + \frac{-20(4)}{40}$$

$$\Rightarrow \boxed{5.6667}$$

4.5

10	20	14	16
0.25	0.20	0.15	0.10

 $f(15)$

$$f(15) \Rightarrow \frac{5(0.25)}{-240} + \frac{-5(0.20)}{240} + \frac{25(0.15)}{48} + \frac{+25(0.10)}{+48} \\ \Rightarrow \boxed{0.1208333}$$

4.

1	2	3	4	7
2	4	8	16	128

$f(5)$

$$f(5) = \frac{-12(2)}{363} + \frac{-16(4)}{210} + \frac{-24(8)}{8} + \frac{+28(16)}{218} + \frac{+24(128)}{300}$$

$$\Rightarrow \boxed{41.4667}$$

2. 1. $x^3 + x - 1 = 0$

$$y = x^3 + x - 1$$

0.5 and 1.

x_0	0.5	x_1	0.6	x_2	0.7
y	-0.375		-0.184		0.043

$$x_n \Rightarrow \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1} \Rightarrow$$

$$x_n \Rightarrow \boxed{0.681057268}$$

$$x_n \Rightarrow \frac{(0.6810 \times 0.043) + (0.7)(-0.184)}{0.043 - 0.184}$$

$$\Rightarrow \boxed{0.258675} \text{ and } y_n \Rightarrow \boxed{-0.0000041}$$

$\therefore$ Root is 0.258675

2. $x^3 + 2x + 0.5 = 0$

0 and 1

x	x_0	x_1	x_2	x_3
	0	0.1	0.2	0.3
y	0.5	0.701	0.908	1.127

$$x_n \Rightarrow \frac{(1.127)(0.2) - (0.3)(0.908)}{1.127 - 0.908}$$

$$\Rightarrow \boxed{-0.214611872}$$

$$y_n \Rightarrow \boxed{0.060916991}$$

$$x_n = \frac{(0.6091699) - (-0.21461872)(0.10)}{0.6091699 - 1.376(0.21461872)}$$

$$y_n \Rightarrow -0.00017$$

$\therefore$ Root is 2.46954.

3. $x^3 + 2x - 20 = 0$ 2 and 2.5

x	2.0	2.1	2.2	2.3	2.4	2.5
y	-8	-6.539	-4.952	-3.233	-1.376	0.625

$$x_n = \frac{(0.625 \times 2.4) + (1.376)(2.5)}{0.625 + 1.376} \Rightarrow \boxed{2.46878}$$

$$y_n = -0.015535266$$

$$x_n \Rightarrow \frac{0.625(2.46878) + (0.01582)(2.5)}{0.625 + 20.01882} \Rightarrow \boxed{2.46954}$$

$$\therefore y_n \Rightarrow \boxed{-0.000076} \text{ and root is } 2.46954.$$

4. $3x - 105x - 1 = 0$

x	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7
y	-1	-11.2	-21.4	-31.6	-41.8	-52	-62.2	-72.4

$$x_n \Rightarrow \frac{(-72.4)(0.6) - (0.7)(-62.2)}{-72.4 - (-62.2)} \Rightarrow 0.6666$$

$$y_n \Rightarrow \boxed{-0.00000003}$$

$$\therefore \text{Root is } \boxed{-0.6666}.$$

5. $x^3 - 3x - 5 = 0$

between 2 and 2.5

x	2.0	2.1	2.2	2.3	2.4
y	-3	-2.039	-0.952	0.267	1.625

$$x_n = \frac{0.267(2.2) - (-0.952)(2.3)}{0.267 - (-0.952)} \Rightarrow \boxed{2.77809}$$

$$y_n = \boxed{-0.01159}$$

$$x_1 = 2.77809$$

$$y_1 = -0.01159$$

$$x_2 = 2.3$$

$$y_2 = 0.267$$

$$x'_n = \frac{(0.267)(2.77809) + (2.3)(0.01159)}{0.203 + 0.01159} \Rightarrow \boxed{2.27901}$$

$$y'_n = \boxed{0.0001} \text{ and } \therefore \text{root is } \boxed{2.27901}$$

6. $x \log_{10} x - 1.2 = 0$

between 2 & 2

x	2	2.1	2.2	2.3	2.4	2.5	2.6	2.7
y	-0.597	-0.52329	-0.4667	-0.36806	-0.28749	-0.20514	-0.12106	-0.0353

2.8
0.05205

$$x_1 \Rightarrow 2.7$$

$$y_1 \Rightarrow -0.0353$$

$$x_2 \Rightarrow 2.8$$

$$y_2 \Rightarrow +0.05205$$

$$x_n \Rightarrow \frac{0.05205(2.7) - (2.8)(-0.0353)}{0.05205 - (-0.0353)} \Rightarrow \boxed{2.7404}$$

$$y_n \Rightarrow -0.00014 \text{ and root is } 2.7404$$

7. $x^3 - 4x + 1 = 0$

0

x	0	0.1	0.2	0.3
y	1	0.60	0.205	-0.173

$$x_1 = 0.2$$

$$y_1 \Rightarrow 0.205$$

$$x_2 = 0.3$$

$$y_2 = -0.173$$

$$x_n \Rightarrow \frac{-0.173(0.3) - (0.2)(0.205)}{-0.173 - 0.205} \Rightarrow \boxed{0.2546}$$

$$y_n \Rightarrow \boxed{-0.00187}$$

$$x_1 = 0.2$$

$$x_2 = 0.2546$$

$$y_1 = 0.0008$$

$$y_2 = 0.00187$$

$$y_n = x'_n \rightarrow \boxed{0.2541}$$

$$\text{root is } 0.254$$

$$\text{and } y''_n \Rightarrow \boxed{-0.0002}$$

$$8. \quad x^3 + 2x^2 - 8 = 0$$

x	0	1	2
y	-8	-5	8

$$x_1 = 1$$

$$y_1 = -5$$

$$x_2 = 2$$

$$y_2 = 8$$

$$x_n \Rightarrow \boxed{1.3846}$$

$$\text{and } y_n \Rightarrow \boxed{-1.5112}$$

$$x_1 = 1.4824$$

$$y_1 = -0.3475$$

$$x_2 = 2$$

$$y_2 = 8$$

$$x''_n = \frac{8(1.4824) - (2)(-0.3475)}{8 + 0.3475} \Rightarrow \boxed{-1.5039}$$

$$y''_n \Rightarrow \boxed{0.0747}$$

$$x_1 = 1.5033$$

$$y_1 = -0.0747$$

$$x_2 = 2$$

$$y_2 = 8$$

$$x'''_n \Rightarrow \frac{8(1.5033) - (2)(-0.0747)}{8 - (-0.0747)} = \boxed{1.5085}$$

$$\text{and } y'''_n \Rightarrow \boxed{-0.0158}$$

9. $\lambda^3 - 9\lambda + 1 = 0$

λ	0	0.1	0.2
y	1	0.101	-0.792

$$\lambda_n \Rightarrow \frac{0.192(0.1) - (0.2)(0.101)}{-0.792 - 0.101} \Rightarrow \boxed{0.1113}$$

$$y_n \Rightarrow \boxed{0.00041} \quad \text{and use it in } \boxed{0.1113}$$

10. $\lambda^3 - \lambda - 4 = 0$

λ	0	1	2
y	-4	-4	2

$$\lambda_n \Rightarrow \frac{2(1) - (4)(2)}{2 - (-4)} \Rightarrow \boxed{1.3334}$$

$$y_n \Rightarrow \boxed{-2.9629}$$

$$\lambda_1 = 1.3334$$

$$\lambda_2 = 2$$

$$y_1 = -2.9629$$

$$y_2 = 2$$

$$\lambda_n \Rightarrow \frac{2(1.3334) - (2)(-2.9629)}{2 - (-2.9629)} \Rightarrow \boxed{1.7313}$$

$$y_n \Rightarrow \boxed{-0.5415}$$

$$\lambda_1 = 1.7313$$

$$\lambda_2 = 2$$

$$y_1 = -0.5415$$

$$y_2 = 2$$

$$\lambda_n \Rightarrow \frac{2(1.7313) - (2)(-0.5415)}{2 - (-0.5415)} \Rightarrow \boxed{1.2885}$$

$$y_n \Rightarrow \boxed{0.6680}$$

$$\lambda_1 = 1.2885$$

$$y_1 = \boxed{0.6680}$$

$$\lambda_2 = 2$$

$$y_2 = 2$$

$$\lambda_n \Rightarrow \frac{2(1.2885) + 2(0.6680)}{2 + (0.6680)} \Rightarrow \boxed{1.7954}$$