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Q3) $X \sim Bin(4, p)$

No. of claims	0	1	2	3	4
No. of pue	86	75	16	2	1

$$MLE = (P(x=0))^{86} (P(x=1))^{75} (P(x=2))^{16} P(x=3)^{2} P(x=4)^{1}$$

$$= \cancel{{}^4C_0 p^{86} + {}^4C_1 p^{75}(1-p)}$$

$$= ({}^4C_0 (1-p)^4)^{86} ({}^4C_1 p^1 (1-p)^3)^{75} ({}^4C_2 p^2 (1-p)^2)^{16}$$

$$({}^4C_3 p^3 (1-p))^2 ({}^4C_4 p^4 (1-p)^0)^1$$

$$= \text{constan } (1-p)^{344} (p)^{75} (1-p)^{225} (p^{32}) (1-p)^{32} (p)^6 (1-p)^2 \cdot p^4$$

$$L(\theta) = \text{constn } p^{117} \cdot (1-p)^{603}$$

taking log both sides

$$\ln L(\theta) = \log \text{cons} + 117 \log p + 603 \log(1-p)$$

diff. both sides

$$\frac{d}{dx} \ln(L(\theta)) = 0 + \frac{117}{p} + \frac{603}{1-p}(-1) = 0$$

$$\therefore \frac{117}{p} = \frac{603}{1-p}$$

$$117 - 117p = 603p$$

$$\boxed{p = 0.225} \quad p = 0.1625$$

$$\therefore q = 0.8375$$

ii) Goodness of fit Test

H_0: Distribution follows Bin (4, 0.225)

H_1: Given n does not follow Bin (4, 0.225)

No. of claim	0	1	2	3	4
No. of Policie	86	75	16	2	1
Expect. freq.	64.9	75.4	32.8	6.34	0.46

Chi square Test

~~No. of claim~~	0	1	2	3+
Observed	86	75	16	3
Expected	~~68.96~~	~~68.72~~	~~37.28~~	6.86

$\cancel{6.85 + 0.0021 + 8.6 + 2.7 = 17.62}$

$df = 4 - 1 - 1 = 2$

$\cancel{\chi^2_2}$

The value 17.62 exceeds ~~TC~~ 5.991 so we can say that we have insufficient evidence to reject H_0

we have
ject H_0 at

unknown param

hat $E(g(x))=0$

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Q7) i) $\bar{X}_{Public} = 30$ $\quad \bar{X}_{Priv} = 35.05$

$S^2_{Public} = 64.31$ $\quad S^2_{priv} = 67.4$

ii) Condition that needs to be satisfied is that the both the samples show be independent from each other.

Test

H_0 $\quad \mu_{Public} = \mu_{private}$

H_1 $\quad \mu_{private} > \mu_{public}$ $\Rightarrow$ ~~$\mu_{priv}-\mu_{pub}>0$~~

Test statistic $\dfrac{(\bar{X}_{pri} - \bar{X}_{pub}) - (\mu_{pri} - \mu_{pub})}{S_p^2 \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim t_{n_1+n_2-2}$

$$S_p^2 = \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}$$

$$= \frac{(64.31)(9) + (67.4)9}{18} = 65.85$$

$$\frac{(35.05-30)-0}{\sqrt{65.85\left(\frac{1}{10}+\frac{1}{10}\right)}}$$

$$= \frac{5.05}{5.13} = 0.984$$

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This is less than 1.734. So we have insufficient evidence to reject H_0 at 5% level

Q8) i) Unbiased Estimator

If $g[X]$ is Estimator of unknown param θ then for it is desirable that $E(g(X)) = \theta$

This property is called unbiasedness and the estimator who follow this property is called unbiased estimator

ii) Bias

If $E(g(X)) \neq \theta$ then this estimator will be called as biased estimator

iii) Mean square Error

To measure the efficiency of the estimators MSE is calculated.

$$MSE(\tilde{\theta}) = [E(\bar{\theta} - \theta)^2]$$
$$= Var(\tilde{\theta}) - (Bias(\theta))^2$$

iv) $\tilde{\theta}$ is more efficient than $\hat{\theta}$ because MSE is the quantity which measures the efficiency of a estimator greater the MSE lower is the efficiency

$2.5750 \cdot \sqrt{\frac{48.667}{10}}$

55.61

$n = 1000$

$20)$

$1 - P(Z < 1.825)$

0.034

d by α and it
when it is true.

v) The estimators would be termed as consistent when their MSE decrease with increase in sample size.

vi)
- Method of Moments
- Maximum Likelihood Estimation

(Q9) i) In real life population variance is unknown so we use S^2 is sample variance

$$t_k \sim \frac{Z}{\sqrt{\frac{c}{k}}} = \frac{\bar{x}-\mu}{s/\sqrt{n}} \sim t_{n-1}$$

ii) Mean $E(t_k) = 0$

$$Var(t_k) = \frac{k}{k-2}$$

iii) a) $n = 10$, $\bar{x} = 50$, $s^2 = 48.667$

From the tables we find that $t_{0.025, 9} = 2.262$

$$0.95 = \left(-2.262 < \frac{\bar{x}-\mu}{s/\sqrt{n}} < 2.262\right)$$

$$0.95 = \left(\bar{x} - 2.262\frac{s}{\sqrt{n}} < \mu < \bar{x} + 2.262\frac{s}{\sqrt{n}}\right)$$

$(45.01, 54.99)$

b) $t_{0.005, 9} = 2.5750$

$$\mu = 50 \pm 2.5750 \sqrt{\frac{48.667}{10}}$$

$$44.32, 55.61$$

Q10) $X \sim Poi(3)$ $n=1000$

$\Sigma X \sim Poi(3n)$

$\Sigma X \sim Poi(3000)$

$\therefore \Sigma X \sim N(3000, 3000)$

i) $P(x > 3100 \text{ when } \lambda = 3000)$

$P(x > 3100)$

$\Rightarrow P\left(Z > \frac{3100 - 3000}{\sqrt{3000}}\right)$

$= P(Z > 1.825) = 1 - P(Z < 1.825)$

$= 0.034$

Type I error is denoted by α and it says H_0 is rejected when it is true.

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i) Type II error is denoted β and it occurs when H_0 is ~~not~~ accepted when it is false.

ii) The power of test is reject H_0 when it is false ~~and~~, so it equals $1-\beta$

$$\therefore \quad 1-\beta = 1-\beta(x<3100 \text{ when } \lambda=\mu)$$

$$= 1-P\left(x < \frac{3100-\mu}{\sqrt{\mu}}\right)$$

iv) $n = 2900$

$$0.99 = \left(-2.5758 < \frac{\hat{\lambda}-\lambda}{\sqrt{\hat{\lambda}/n}} < 2.5758\right)$$

$$\cancel{0.99 \quad \left(-2.5758\sqrt{\frac{\hat{\lambda}}{n}} < \hat{\lambda}\,\lambda <\right.}$$

$$\hat{\lambda} - 2.5758\sqrt{\frac{\hat{\lambda}}{n}} < \lambda < \hat{\lambda} + 2.5758\sqrt{\frac{\hat{\lambda}}{n}}$$

$$\cancel{30} = (2.9172, 3.0828)$$

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Q11) $\Sigma X = 213200$ Sln $\bar{X} = 17767$

$\Sigma X^2 = 4919860000$ $S = 10144$

$S^2 = 102900736$

$\Sigma X_N = 213200 - 11000 - 48000 + 27500 + 31500$

$\Sigma X_N = 213200$

$\Sigma X_N^2 = 4919860000 - (11000)^2 - (48000)^2 + (27500)^2 + (31500)^2$

$= 4243360000$

$\bar{X}_n = \frac{\Sigma X_n}{12} = 17767$

$S^2 = \frac{1}{n-1}\left(\Sigma X^2 - n\bar{X}\right) = \frac{1}{11}\left(4243360000 - (12)(1776\right.$

$\therefore\ S = 19640$

Q1) $X \sim Poi(\theta)$

then $\frac{X-\theta}{\sqrt{\theta}} \sim N(0,1)$

Q2) iv) $(50 < S^2 < 150)$

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$>0,\ \beta>0$

$f(x) = \dfrac{\alpha\lambda^{\alpha}}{(\lambda+x)^{\alpha+1}}$

Pareto dist^n

$\dfrac{\beta}{2}$

$\dfrac{3\beta^2}{2^2(1)} = \dfrac{3}{4}\beta^2$

$\hat{\beta} = 2\bar{x}$

it is unbiased

tor

istency increas

Q2) i) $E(\bar{x}) = E\left(\dfrac{\sum_1^n x_i}{n}\right)$

$= \dfrac{1}{n}\sum_i^n E(x_i) = \dfrac{n\mu}{n} = \mu$

$Var(\bar{x}) = Var\left(\dfrac{\sum_i^n x_i}{n}\right)$

$= \dfrac{1}{n^2} n\sigma^2 = \dfrac{\sigma^2}{n}$

ii) $E(s^2) = \dfrac{1}{n-1}\sum(x_i - \bar{x})^2$

$= E\left(\dfrac{1}{n-1}\sum(x_i - \bar{x})^2\right)$

$\dfrac{1}{n-1} E\left(\sum x_i^2 - n\bar{x}^2\right)$

$\dfrac{1}{n-1}\left(\sum E(x_i)^2 - n E(\bar{x}^2)\right)$

$\dfrac{1}{n-1}\left(n\sigma^2 + n\bar{x}^2 - \left(\cancel{n}\dfrac{\sigma^2}{\cancel{n}} + n\bar{x}^2\right)\right)$

$\dfrac{1}{\cancel{n-1}}\left(\cancel{(n-1)}\sigma^2\right) = \boxed{\sigma^2}$

iii) $\dfrac{(n-1)s^2}{\sigma^2} \sim \chi^2_{n-1}$

$Var\left(\dfrac{(n-1)s^2}{\sigma^2}\right) = Var\left(\chi^2_{n-1}\right)$

$\dfrac{(n-1)^2}{\sigma^4} Var\left(\dfrac{s^2}{\sigma^2}\right) = 2(n-1)$

$\boxed{Var(s^2) = \dfrac{2\sigma^2}{n-1}}$

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Q4) $f(x) = \frac{C\beta^3}{(x+\beta)^4}$; $x>0,\ \beta>0$

$f(x) = \frac{\alpha\lambda^\alpha}{(\lambda+x)^{\alpha+1}}$

i) $C=3$ Pareto distn

$E(X) = \frac{\lambda}{\alpha-1} = \frac{\lambda}{2} = \frac{\beta}{2}$

$V(X) = \frac{\alpha\lambda^2}{(\alpha-1)^2(\alpha-2)} = \frac{3\beta^2}{(2)^2(1)} = \frac{3}{4}\beta^2$

ii) $E(X) = \bar{X}$

$\therefore \frac{\beta}{2} = \bar{X}$

Hence $\hat{\beta} = 2\bar{X}$

Unbiasedness

$E(\hat{\beta}) - \beta$

$E(2\bar{x}) - \beta$

$2\mu - \beta$

$\beta - \beta = \underline{\underline{0}}$ Hence it is unbiased

Consistency

$\hat{\beta} = \frac{2\sum_{i=1}^{n} x_i}{n}$

$\because$ n is in denominator $\therefore$

As n increases consistency increases

$n\to\infty$

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Q1) $X \sim \text{Poi}(X)$; $\bar{X} = 200$

$\bar{X} \pm 1.96 \sqrt{\frac{\theta}{n}}$

Q5) $U(a, b)$

$E[x] = \frac{a+b}{2}$

$\text{Var}[x] = \frac{(b-a)^2}{12}$ $\qquad \sqrt{12}\, s = b-a$

$b = a + 2\sqrt{3}s$

$a = \bar{x} - \sqrt{3}$

$\bar{x} = \frac{a + a + 2\sqrt{3}s}{2}$

$a = \bar{x} - \sqrt{3}s$

b) $a = b - 2\sqrt{3}s$

$\bar{x} = \frac{b - 2\sqrt{3}s + b}{2} = b - \sqrt{3}s$

$\therefore b = \bar{x} + \sqrt{3}s$

c) Sample $= 2, 4, 6, 8, 10$

$\bar{x} = 6$

$S = 3.16$

$a = 6 - \sqrt{3}(3.16) = 0.527$

$b = 6 + \sqrt{3}(3.16) = 11.47$

d) $U(0, b)$ $\qquad x = \frac{1}{b}$

$L = \prod_{1}^{n} \frac{1}{b} = \frac{1}{b^n}$

$\log L = -n \log b$

$b \to \infty$