

Homework - 1 Lagrange's Interpolation:-

x	7	8	9	10
f(x)	3	1	1	9

find y at $x = 9.5$

$$f(x) = \frac{(9.5-8)(9.5-9)(9.5-10)}{(7-8)(7-9)(7-10)} (3) +$$

$$\frac{(9.5-7)(9.5-9)(9.5-10)}{(8-7)(8-9)(8-10)} (1) +$$

$$\frac{(9.5-7)(9.5-8)(9.5-10)}{(9-7)(9-8)(9-10)} (1) +$$

$$\frac{(9.5-7)(9.5-8)(9.5-9)}{(10-7)(10-8)(10-9)} (9)$$

$$= \frac{1.5 \times 0.5 \times -0.5}{-1 \times -2 \times -3} + \frac{(2.5)(0.5)(-0.5)}{(1)(-1)(-2)}$$

$$+ \frac{(2.5)(1.5)(-0.5)}{(2)(1)(-1)} + \frac{(2.5)(1.5)(0.5)}{(3)(2)(1)}$$

$$= 0.1875 - 0.3125 + 0.9375 + 2.8125$$

$$= 3.625 \text{ Ans.}$$

$$\begin{array}{cccccc}
 2) & x & 3 & 7 & 9 & 10 \\
 f(x) & & 168 & 120 & 72 & 63 \\
 \text{at } x = 6.
 \end{array}$$

$$f(6) = \frac{(6-7)(6-9)(6-10)}{(3-7)(3-9)(3-10)} (168) +$$

$$+ \frac{(6-3)(6-9)(6-10)}{(7-3)(7-9)(7-10)} (120) +$$

$$+ \frac{(6-3)(6-7)(6-10)}{(9-3)(9-7)(9-10)} (72) +$$

$$\frac{(6-3)(6-7)(6-9)}{(10-3)(10-7)(10-9)} (63)$$

$$\begin{aligned}
 f(6) &= \frac{72 \times \cancel{1} \times \cancel{1} \times \cancel{1}}{\cancel{1} \times \cancel{1} \times \cancel{1}} \times 168 + \frac{3 \times \cancel{1} \times \cancel{1} \times \cancel{1}}{\cancel{1} \times \cancel{1} \times \cancel{1}} \times 120 \\
 &+ \frac{3 \times \cancel{1} \times \cancel{1} \times \cancel{1}}{\cancel{1} \times \cancel{1} \times \cancel{1}} (72) + \frac{3 \times \cancel{1} \times \cancel{1} \times \cancel{1}}{\cancel{1} \times \cancel{1} \times \cancel{1}} \times 63
 \end{aligned}$$

$$= 12 + 180 - 72 + 27$$

$$= 147 \text{ Ans.}$$

3)	x	1	2	7	8
	$f(x)$	4	5	5	4

Find $b(6)$

$$b(6) = \frac{(6-2)(6-7)(6-8)(4)}{(1-2)(1-7)(1-8)} +$$

$$\frac{(6-1)(6-7)(6-8)(5)}{(2-1)(2-7)(2-8)} +$$

$$\frac{(6-1)(6-2)(6-8)(5)}{(7-1)(7-2)(7-8)} +$$

$$\frac{(6-1)(6-2)(6-7)(4)}{(7-1)(8-2)(8-7)}$$

$$= \frac{4 \times \cancel{1} \times -2 \times 4}{\cancel{1} \times -6 \times -7} + \frac{5 \times \cancel{1} \times -2 \times 5}{1 \times \times 5 \times -6} +$$

$$\frac{6 \times 5 \times 4 \times \cancel{2} \times 8}{6 \times \cancel{8} \times \times 1} + \frac{5 \times 4 \times -1 \times 4}{7 \times 6 \times 1}$$

$$= -\frac{32}{42} + \frac{50}{30} + \frac{40}{6} - \frac{80}{42}$$

$$= -0.76190 + 1.66667 + 6.66667 - 1.90476$$

$$= 5.666678 \text{ Ans.}$$

4)	x	1	2	3	4	7
	b(x)	2	4	8	16	128

find $b(5)$

$$b(5) = \frac{(5-2)(5-3)(5-4)(5-7)(2)}{(1-2)(1-3)(1-4)(1-7)} +$$

$$\frac{(5-1)(5-2)(5-4)(5-7)(4)}{(2-1)(2-3)(2-4)(2-7)} +$$

$$\frac{(5-1)(5-2)(5-4)(5-7)(8)}{(3-1)(3-2)(3-4)(3-7)} +$$

$$\frac{(5-1)(5-2)(5-3)(5-7)(16)}{(4-1)(4-2)(4-3)(4-7)} +$$

$$\frac{(5-1)(5-2)(5-3)(5-4)(128)}{(7-1)(7-2)(7-3)(7-4)}$$

$$b(5) = \frac{\cancel{3} \times 2 \times 1 \times \cancel{2} \times 2}{\cancel{1} \times \cancel{2} \times \cancel{3} \times -6} + \frac{4 \times \cancel{2} \times 1 \times \cancel{2} \times 4}{1 \times -1 \times \cancel{2} \times -5} +$$

$$\frac{\cancel{4} \times 3 \times 1 \times -2 \times 8}{2 \times 1 \times \cancel{1} \times \cancel{4}} + \frac{4 \times \cancel{3} \times \cancel{2} \times 2 \times 16}{\cancel{3} \times \cancel{2} \times 1 \times \cancel{3}} +$$

$$\frac{\cancel{4} \times \cancel{3} \times 2 \times 1 \times 128}{6 \times 5 \times \cancel{4} \times \cancel{3}}$$

$$f(5) = -\frac{4}{6} + \frac{32}{5} - \frac{48}{2} + \frac{128}{3} + \frac{128}{30}$$

$$= 28.6667 \text{ Ans.}$$

5)	x	10	20	14	16
	$f(x)$	0.25	0.2	0.15	0.10

Find $f(15)$

$$f(15) = \frac{(15-20)(15-14)(15-16)(0.25)}{(10-20)(10-14)(10-16)} +$$

$$\frac{(15-10)(15-14)(15-16)(0.2)}{(20-10)(20-14)(20-16)} +$$

$$\frac{(15-10)(15-20)(15-16)(0.15)}{(14-10)(14-20)(14-16)} +$$

$$\frac{(15-10)(15-20)(15-14)(0.10)}{(16-10)(16-20)(16-14)}$$

$$= \frac{5 \times 1 \times -1 \times 0.25}{-10 \times -4 \times -6} + \frac{5 \times 1 \times -1 \times 0.2}{10 \times 6 \times 4}$$

$$+ \frac{5 \times -5 \times -1 \times 0.15}{4 \times -6 \times -2} + \frac{5 \times -5 \times 1 \times 0.10}{6 \times -4 \times 2}$$

$$= -0.005208 + 0.004166 + 0.07812 + 0.05208$$

$$= 0.073954 \text{ Ans.}$$

$$1) \quad x^3 + x - 1 = 0$$

$$0.5 \leq 1.$$

x	0.5	0.6	0.7	0.8	0.9	1
y	-0.375	-0.184	0.043	0.312	0.629	1

$$X = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1}$$

$$X_0 = \frac{(0.6)(0.043) - (0.7)(-0.184)}{0.043 + 0.184}$$

$$X_0 = \frac{0.68106}{1.227} \quad Y_0 = -0.0030$$

$$2) \quad x^3 - 2x + 0.5 = 0$$

$$x \quad 0 \quad 1 \quad X_0 = \frac{-0.5}{-1.0} = 0.5$$

$$y \quad 0.5 \quad -0.5 \quad Y_0 = -0.375$$

$$X_3 = \frac{0.25}{0.875} = 0.28571 \quad Y_3 = -0.048$$

$$(0.28571)(1/3)(-0.048)(Y_3) \quad X_4 = 0.26068$$

$$Y_4 = -0.0036$$

$$3) \quad x^3 + 2x - 20 = 0$$

x	2	2.1	2.2	2.3	2.4 (x _L)	2.5 (x _H)
y	-8	-6.539	-4.952	-3.23	-1.37	0.625

$$x_0 = 2.4687$$

$$y_0 = -0.0158$$

$$x_2 = 2.5$$

$$y_{20} = 0.625$$

$$x_3 = 2.469$$

$$y_3 = -0.0001752$$

$$4) \quad 3x - \cos x - 1 = 0$$

$$x \quad 0 \quad 1$$

$$y \quad -2 \quad 0.000152$$

$$x_0 = \frac{2}{3.000152} = 0.666632 \text{ Angew.}$$

$$y_0 = -0.000033642$$

$$5) \quad x^3 - 3x - 5$$

$$x \quad 2.1 \quad 2.2 \quad 2.3$$

$$y \quad -2.039 \quad -0.952 \quad 0.267$$

$$x_0 = 2.2781$$

$$x_3 = 2.2790 \text{ Angew.}$$

$$y_0 = -0.011994$$

$$y_3 = -0.000236$$

$$6) \quad x \log x = 1.2$$

$$x \quad 2 \quad 3$$

$$y \quad -0.99794 \quad 0.23136$$

$$7) \quad x^3 - 4x + 1 = 0$$

$$x \quad 1 \quad 2$$

$$y \quad -2 \quad 1$$

$$x_0 = \frac{1+4}{3} = 5/3 \approx 1.67$$

$$x_2 = 2 \quad y_2 = 1$$

$$x_3 = 1.836 \quad y_3 = -0.1799$$

$$x_4 = 1.8591 \quad y_4 = -0.017$$

$$8) \quad x^3 + 2x^2 - 8 = 0$$

$$x \quad 0 \quad 1 \quad 2$$

$$y \quad -5 \quad 8$$

$$x_0 = -\frac{18}{13} = 1.3846$$

$$x_2 = 2$$

$$y_2 = 8$$

$$x_3 = -0.34752$$

$$y_3 = 8$$

$$x_4 = 1.5039$$

$$y = -0.025176$$

$$x_7 = 1.5026$$

$$y_7 = -0.00071765$$

$$9) \quad x^3 - 9x + 1$$

$$x \quad 0 \quad 1$$

$$y \quad 1 \quad -7$$

$$x_0 = 0.125 \quad y_0 = -0.123$$

$$x_2 = 0 \quad y_2 = 1$$

$$x_3 = 0.11131 \quad y_3 = -0.0001$$

Ans.