Calculus HW

1) $\lim\limits_{h \to 0} \dfrac{2(-3+h)^2 - 18}{h} = \dfrac{2(2)(-3+h)}{1} = 4(-3+h)$

$= \cancel{12(1} -12$ (Ans)

(or)

$\dfrac{2(h^2+9-6h) - 18}{h} = \dfrac{2h^2 + \cancel{18} - \cancel{18} - 12h}{h} = \dfrac{2\cancel{h}(h-6)}{\cancel{h}} = -12$ (Ans)

2) $g(x) = \begin{cases} 2x & x < 6 \\ x-1 & x \geq 6 \end{cases}$

a) for $x = 4$ $\quad f(4^-) = \cancel{2(4-h)} \lim\limits_{h \to 0} 2(4-h) = 8$

$f(4^+) = \lim\limits_{h \to 0} 2(4+h) = 8 \qquad f(4) = 2(4) = 8$

$\therefore$ Continuous at $x=4$ (Ans)

b) for $x = 6$ $\quad f(6^-) = \lim\limits_{h \to 0} 2(6-h) = 12 \quad f(6) = 6-1 = 5$

$\therefore$ as $f(6^-) \neq f(6)$
discontinuous at $x=6$ (Ans)

3) $\lim\limits_{x \to 9} \dfrac{x-9}{3-\sqrt{x}} = \lim\limits_{x \to 9} \dfrac{\cancel{x-9}(3+\sqrt{x})}{\cancel{9-x}} = -3-\sqrt{x} = -6$ (Ans)

4) $f(x) = \dfrac{4x+5}{9-3x}$

a) $x = -1$ $\quad f(-1^-) = \lim\limits_{h \to 0} \dfrac{4(-h-1)+5}{9-3(-1-h)} = \dfrac{-4h-4+5}{9+3+3h} = \dfrac{1-4h}{12+3h} = \dfrac{1}{12}$

$f(1) = \dfrac{9}{6} = \dfrac{3}{2} \quad \cancel{f(1^+)=} \quad \therefore$ as $f(-1^-) \neq f(1)$
discontinuous at $x=1$ (Ans)

b) $x = 0$ $\quad f(0) = \dfrac{5}{9} \qquad (0^+) = \lim\limits_{h \to 0} \dfrac{4(0+h)+5}{9-3(0+h)} = \dfrac{4h+5}{9-3h} = \dfrac{5}{9}$

$f(0^-) = \lim\limits_{h \to 0} \dfrac{4(0-h)+5}{\cancel{9(0}\ 9-3(0-h)} = \dfrac{-4h+5}{9+3h} = \dfrac{5}{9}$

$\therefore$ Continuous at $x=0$ (Ans)

c) $x=3$ $\quad f(3) = \frac{4(3)+5}{9-3(3)} = \frac{17}{0}$

$\therefore$ as $f(3)$ does not exist, discontinuous at $x=3$ (Ans)

5) $\lim_{x\to\alpha} \frac{6e^{4x} - e^{-2x}}{8e^{4x} - e^{2x} + 3e^{-x}}$ $\qquad$ Let $2x = y$

$\lim_{x\to\alpha} \frac{6e^{2y} - e^{-y}}{8e^{2y} - e^{y} + 3e^{-y/2}}$

$\Rightarrow \frac{6}{8} = \frac{3}{4}$ (Ans)

6) $g(y) = \begin{cases} y^2+5 & y < -2 \\ 1-3y & y \geq 2 \end{cases}$

a) $\lim_{y\to 6} g(y) = 1-3(6) = 1-18 = -17$ (Ans)

b) ~~lim~~ $\lim_{y\to -2} g(y) = 1-3(-2) = 7$

$\lim_{y\to 2^-} g(y) = (-2-h)^2 + 5 = -h^2 - 4 - 4h + 5 = 1$

$\lim_{y\to -2^+} g(y) = 1-3(-2+h) = 1+6-3h = 7$

$\therefore$ discontinuous at $x = -2$ (Ans)

7) $f(t) = (4t^2 - t)(t^3 - 8t^2 + 12)$

$f'(t) = (8t-1)(t^3 - 8t^2 + 12) + (3t^2 - 16t + 12)(4t^2 - t)$ (Ans)

8) $R(w) = \frac{3w + w^4}{2w^2 + 1}$

$R'(w) = \frac{(3+4w^3)(2w^2+1) - (4w)(3w+w^4)}{(2w^2+1)^2}$ (Ans)

9) $f(x) = 2e^{x} - 8^{x}$
$f'(x) = 2e^{x} - \log 8 \cdot 8^{x}$ (Ans)

10) $y = z^5 - e^z \ln(z)$
$y' = 5z^4 - e^z \ln z - \frac{e^z}{z}$ (Ans)

11) a) $f(x) = (6x^2 + 7x)^4$
$f'(x) = 4(6x^2+7x)^3(12x+7)$ (Ans)

b) $f(t) = 5 + e^{4t+t^7}$
$f'(t) = e^{4t+t^7}(4+7t^6)$ (Ans)

12) a) $z = \log(7-x^3)$
$z' = \frac{1}{7-x^3}(-3x^2)$

$\boxed{z'' = \frac{\cancel{-3}\,(-6x)(7-x^3)-(-3x^2)(-3x^2)}{(7-x^3)^2}}$ (Ans)

b) $Q(v) = \frac{2}{(6+2v-v^2)^4}$ $\quad Q'(v) = \frac{2(-4)}{(6+2v-v^2)^{+5}} = \frac{-8}{(6+2v-v^2)^{+5}}$

$\boxed{Q''(v) = \cancel{-8}\ \frac{40}{(6+2v-v^2)^6}}$ (Ans)

c) $f(t) = \log(1+t^2)$ $\quad f'(t) = \frac{1(2t)}{1+t^2}$

$\boxed{f''(t) = \frac{2(1+t^2)-(2t)(2t)}{(1+t^2)^2}}$ (Ans)

13) $f(x) = x^3 - 3x^2 - 9x + 12$
$f'(x) = 3x^2 - 6x - 9 = 0$ $\quad 3x^2 - 9x + 3x - 9 = 0$
$3x(x-3) + 3(x-3) = 0$ $\quad x = 3, -1$

$f''(x) = 6x - 6$ $\quad f''(3) = 12$ (+ve) (~~max~~ min value at $x=3$)
$f''(-1) = -12$ (-ve) (max value at $x=-1$)

$f(3) = 27 - 27 - 27 + 12 = -15$ (min ~~max~~ value)
$f(-1) = -1 - 3 + 9 + 12 = 17$ (max value) (Ans)

14) $f(x) = 4x^3 - 18x^2 + 24x - 7$

$f'(x) = 12x^2 - 36x + 24 = 0$

$12x^2 - 24x - 12x + 24 = 0$

$12x(x-2) - 12(x-2) = 0$

$x = 2, 1$

$f''(x) = 24x - 36$ $f''(2) = 12$ (min value at $x = 2$)

$f''(1) = -12$ (max value at $x = 1$)

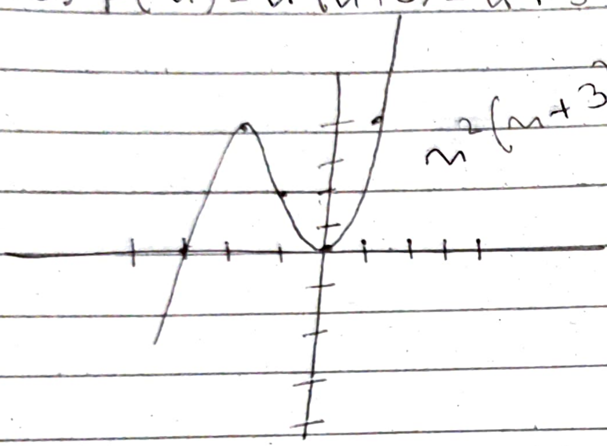
$f(2) = 32 - 72 + 48 - 7 = 1$ (min value)

$f(1) = 4 - 18 + 24 - 7 = 3$ (max value)

(Ans)

15) $f(x) = x^2(x+3) = x^3 + 3x^2$

x	0	1	-1	-2	3	-
$f(x)$	0	4	2	4	54	[illegible]



16) $f(x) = 3x^2 - 6x$

x	0	1	2	-1	-2	3	-3
$f(x)$	0	-3	0	9	24	9	45