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Assignment 1

1) i) $d^{(4)} = 8\%$

$\delta = \ln\left(\frac{1}{1-d}\right) \Rightarrow \delta = \ln\left(\frac{1}{1-\frac{d^{(p)}}{p}}\right)^p$

$\Rightarrow \delta = p\ln\left(\frac{1}{1-\frac{d^{(p)}}{p}}\right) \Rightarrow \delta = 4\ln\left(\frac{1}{1-\frac{d^{(4)}}{4}}\right)$

$\Rightarrow \delta = 4\ln\left(\frac{1}{1-\frac{0.08}{4}}\right) \Rightarrow \delta = 4\ln\left(\frac{1}{1-0.02}\right)$

$\Rightarrow \delta = 4\ln\left(\frac{1}{0.98}\right) \Rightarrow \delta = 4(0.0202027073 2)$

$\Rightarrow \delta = 0.08081082927.$

$\Rightarrow \delta = 8.08108\%$ (ans)

ii) $d^{(4)} = 8\%$

$1-d = \left(1-\frac{d^{(p)}}{p}\right)^p \Rightarrow 1-d = \left(1-\frac{0.08}{4}\right)^4$

$\Rightarrow 1-d = (0.98)^4 \Rightarrow 1-d = 0.92236816$

$\Rightarrow d = 0.07763184$

$i = \frac{d}{1-d} \Rightarrow i = \frac{0.07763184}{1-0.07763184}$

$i = 0.08416578473$

$\Rightarrow i = 8.41658\%$ (ans)

iii) $\left(1-\frac{d^{(4)}}{4}\right)^4 = \left(1-\frac{d^{(12)}}{12}\right)^{12}$

Raising both sides by $1/12$.

$\left(1-\frac{d^{(4)}}{4}\right)^{1/3} = \left(1-\frac{d^{(12)}}{12}\right) \Rightarrow (1-0.02)^{1/3} = 1-\frac{d^{(12)}}{12}$

$= 0.9932883884 = 1-\frac{d^{12}}{12}$

$\Rightarrow d^{(12)} = 0.08053933945$

$d^{(12)} = 8.05393\%$

2 $\delta(t) = 0.004t + 0.0002t^2$

i) $PV = ?$ $AV = 1000$

$\delta(t) = 0.004t + 0.0002t^2$

$PV = AV\, e^{-\int_0^{10} 0.04t + 0.0002t^2\, dt}$

$\Rightarrow PV = 1000\, e^{-\left[\frac{0.04t^2}{2} + \frac{0.0002t^3}{3}\right]_0^{10}}$

$\Rightarrow PV = 1000\, e^{-\left(0.002\times 100 + \frac{0.0002(1000)}{3}\right)}$

$\Rightarrow PV = 1000\, e^{-(0.266667)}$

$\Rightarrow PV = 765.9280831$

ii) $AV = PV(1+i)^{10}$

$1000 = 765.9280831(1+i)^{10}$

$1.305605607 = (1+i)^{10}$

$1.027025438 = 1+i$

$0.027025438 = i$

$2.70254\% = i$

3 i) $1+i = \frac{1}{1-d} \Rightarrow 1-d = \frac{1}{1+i}$

$\Rightarrow 1 - \frac{1}{1+i} = d \Rightarrow \frac{1+i-1}{1+i} = d$

$\frac{i}{1+i} = d \Rightarrow i \cdot \frac{1}{1+i} = d$

$\Rightarrow iv = d$

ii) a) $\frac{d^4}{4} = 2.25\%$

$\left(1 - \frac{d^4}{4}\right)^4 = \left(1 - \frac{d^2}{2}\right)^2$

$\Rightarrow (1 - 0.0225)^4 = \left(1 - \frac{d^2}{2}\right)^2$

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$0.95550625 = 1 - \frac{d^2}{2}$

$\Rightarrow 0.04449375 = \frac{d^2}{2} \Rightarrow 0.0889875 = d^2$

$\Rightarrow 8.89875\% = d^2$ (ans)

b) $d^{(0.5)} = 5\%$

$\left(\frac{1 - d^{(0.5)}}{0.5}\right)^{0.5} = \left(\frac{1 - d^{(2)}}{2}\right)^2$

$\Rightarrow \left(1 - \frac{0.05}{0.5}\right)^{0.5} = \left(1 - \frac{d^2}{2}\right)^2 \Rightarrow 0.9740037464 = 1 - \frac{d^2}{2}$

$0.0519925071 5 = d^{(2)}$

$\Rightarrow 5.19925\% = d^{(2)}$ (ans)

iii) Let bill amount be ₹100

$\therefore AV = 100(1+0.05)^{91/365}$

$= 100(1.012238407)$

$= 101.2238407$

$(1 - nd)$ ~~100~~ $101.2238407 = 100$

$(1 - d) = 0.9879095607$

$d = 1.2090439 28$

4i) Explained in 3(i)

ii a) $i = 0.08 \Rightarrow d = \frac{0.08}{1+0.08} \Rightarrow d = 0.07407407$

$1 - d = \left(1 - \frac{d^{(12)}}{12}\right)^{12}$

$\Rightarrow (1 - 0.07407407) = \left(1 - \frac{d^{(12)}}{12}\right)^{12}$

$$0.9936071024 = 1 - \frac{d^{(12)}}{12}$$

$$\Rightarrow 0.07671477177 = d^{(12)}$$

$$7.671477\% = d^{(12)} \text{ (ans)}$$

b) $$1+i = \left(1 + \frac{i^{(365)}}{365}\right)^{365}$$

$$\Rightarrow (1.08)^{1/365} = 1 + \frac{i^{(365)}}{365}$$

$$\Rightarrow 1.000210874 = 1 + \frac{i^{(365)}}{365}$$

$$\Rightarrow 0.07696915541 = i^{(365)}$$

$$\Rightarrow 7.696915\% = i^{(365)} \text{ (ans)}$$

c) $$\delta = \ln(1+i)$$

$$\Rightarrow \delta = \ln(1+0.08) \Rightarrow \delta = \ln(1.08)$$

$$\delta = 0.07696104114$$

$$\delta = 7.696104\% \text{ (ans)}$$

d) $$(1+i) = \left(1 + \frac{i^{(0.5)}}{0.5}\right)^{0.5}$$

$$1.1664 = 1 + \frac{i^{(0.5)}}{0.5} \Rightarrow 0.0832 = i^{(0.5)}$$

$$8.32\% = i^{(0.5)} \text{ (ans)}$$

5i $d^{(4)} = 6\%$

a) $$\left(1 + \frac{i^{(2)}}{2}\right)^2 = \frac{1}{\left(1 - \frac{d^{(4)}}{4}\right)^4}$$

$$\Rightarrow \left(1 + \frac{i^{(2)}}{2}\right)^2 = \frac{1}{\left(1 - \frac{0.06}{4}\right)^4}$$

$$\left(1 + \frac{i^{(2)}}{2}\right)^2 = \frac{1}{0.9413365506}$$

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$$\left(1+\frac{i^{(2)}}{2}\right)^2 = 1.062319315$$

$$i^{(2)} = 0.0613775 1552$$

$$\therefore i^{(2)} = 6.1377515\% \text{ (ans)}$$

b) $$\left(1-\frac{d^{(4)}}{4}\right)^4 = \left(1-\frac{d^{(12)}}{12}\right)^{12}$$

$$\Rightarrow \left(1-\frac{0.06}{4}\right)^4 = \left(1-\frac{d^{(12)}}{12}\right)^{12}$$

$$\Rightarrow \left(1-\frac{0.06}{4}\right)^{1/3} = \left(1-\frac{d^{(12)}}{12}\right)^{12}$$

$$\Rightarrow 0.9949747896 = \left(1-\frac{d^{(12)}}{12}\right)$$

$$\Rightarrow 0.06030252528 = d^{12}$$

$$\Rightarrow 6.0302525\% = d^{(12)} \text{ (ans)}$$

ii) Interest to be paid for 1 year = 220000 − 200000
= 20000

$\therefore$ Rate of interest or i = $\frac{20000}{200000}$ = 10%.

Loan is repaid on 17th July 2005.
No. of days of loan = 15 + 31 + 30 + 17 = 93 days

Amount to be paid after 93 days = $200000\,(1+0.1)^{93/365}$
= 204916.3564

6 i) Let the retail price of the toy be ₹100

Payment if by cash = 100 − 30% of 100
= ₹70.

Payment if by credit = 100 − 25% of 100
₹75

For rate of discount

$$70 = 75(1-d)^{0.5}$$

$$\frac{70}{75} = (1-d)^{0.5} \Rightarrow$$

$$d = 12.88889\%.$$

ii)
$$1-d = \left(1 - \frac{d^{(12)}}{12}\right)^{12}.$$

$$\Rightarrow \left(1 - 0.1288889\right)^{1/12} = 1 - \frac{d^{(12)}}{12}$$

$$d^{(12)} = 13.71954\%.$$

$$\left(1 + \frac{i^{(2)}}{2}\right)^2 = \frac{1}{1-d}$$

$$\left(1 + \frac{i^{(2)}}{2}\right)^2 = \frac{1}{1-0.1288889}$$

$$i^{(2)} = 14.28571\%.$$

$$\delta = \ln(1+i) \Rightarrow \delta = \ln\left(\frac{1}{1-d}\right)$$

$$\Rightarrow \delta = \ln(1.14795933)$$

$$\delta = 13.798587\%.$$

7 i)
$$AV = PV\left(1 + \frac{i^{(4)}}{4}\right)^4$$

$$26000 = 20000\left(1 + \frac{i^{(4)}}{4}\right)^{17/12 \times 4}$$

$$\Rightarrow \frac{26000}{20000} = \left(1 + \frac{i^{(4)}}{4}\right)^{17/3}$$

$$\Rightarrow i^{(4)} = 18.955254 56\%.$$

ii)
$$\left(1 - \frac{d^{(2)}}{2}\right)^2 = \frac{1}{\left(1 + \frac{i^{(4)}}{4}\right)^4}$$

$$\left(1-\frac{d^{(2)}}{2}\right)^2 = \left(\frac{1}{1.047388136}\right)^4$$

$$\Rightarrow \left(1-\frac{d^{(2)}}{2}\right) = 0.977116125$$

$$d^{(2)} = 4.5767775\ \%$$

$$d^{(2)} = 17.68823517\ \%. \quad \text{(ans)}$$

iii) $$26000 = 20000\,(1+ni)$$

$$\frac{26000}{20000} = \left(1+\frac{17}{12}i\right) \Rightarrow 0.3 = \frac{17i}{12}$$

$$i = 21.17647\ \%. \quad \text{(ans)}$$

iv) The interest rates keep getting lower as the frequency of payment increases.

$$d < d^{(12)} < i^{(4)} < i$$

The simple rate of interest has to be higher than the compound rate of discount to maintain the same amount.

8) The relationship between $i^{(p)}$ and p can be represented by the formula

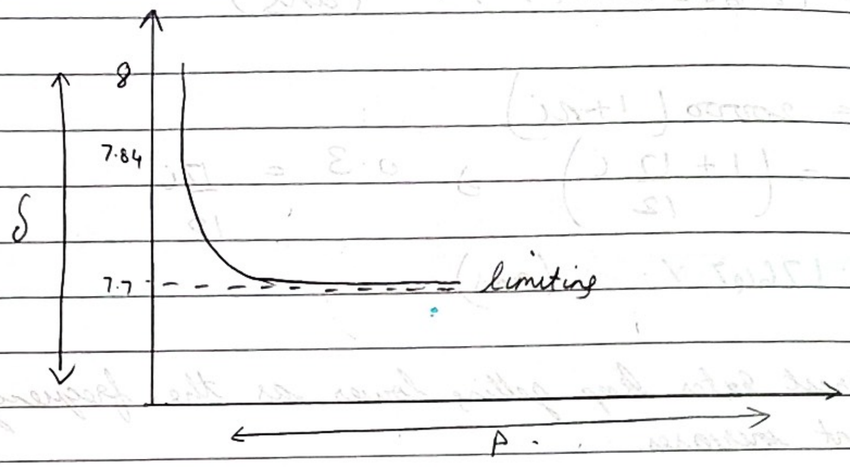
$$i^{(p)} = p\left[(1+i)^{1/p} - 1\right].$$

$$i = 8\%$$

Taking $p = 1, 2, 3, 4, 5, 10, 15, 50, 365$

p	$i^{(p)}$
1	0.08
2	0.0784696908
3	0.07795670402
4	0.07770618763
5	0.07755639199
10	0.07725795243
15	0.07715881262
50	0.07702030156
365	0.07696915541

As p increases the value of $i^{(p)}$ decreases. There is a limit to compounding. The limiting case is when $p \to \infty$, there is an instantaneous change and it keeps compounding till delta (δ).



A9 i) Force of interest refers to a nominal interest rate or a discount rate compounded infinite number of times per time period. If $i^{(p)}$ is the interest rate compounded p times a year, then the limit of $i^{(p)}$ tends to infinity would be the force of interest

Force of interest $= \delta = \lim_{p \to \infty} i^{(p)}$

ii $i^{(2)} = 0.0775$.

$$(1+i) = \left(1 + \frac{i^{(2)}}{2}\right)^2 \Rightarrow (1+i) = \left(1 + \frac{0.0775}{2}\right)^2$$

$\Rightarrow$ $1+i = 1.079001563$

$i = 0.079001563$ $\Rightarrow$ ~~$i =$~~

$i = 7.9001563\ \%$.

$\delta = \ln(1+i)$

$= \ln(1 + 0.079001563)$

$= 0.076036134 84$

$\delta = 7.6036134684\ \%$.

$$\left(1 + \frac{i^{(2)}}{2}\right)^2 = \frac{1}{\left(1 - \frac{d^{(12)}}{12}\right)^{12}}$$

$$\Rightarrow 1.079001563 = \left(1 - \frac{d^{(12)}}{12}\right)^{-12}$$

$$\Rightarrow 1.006356462 = 1 - \frac{d^{(12)}}{12}$$

$$\Rightarrow 0.9936836877 = 1 - \frac{d^{(12)}}{12}$$

$$\Rightarrow d^{(12)} = 7.5795\ 76\ \%.$$

$$\left(1 + \frac{i^{(2)}}{2}\right)^2 = \left(1 + \frac{i^{(0.5)}}{0.5}\right)^{0.5}.$$

$$\Rightarrow 1.079001563 = \left(1 + \frac{i^{(0.5)}}{0.5}\right)^{0.5}$$

$$\Rightarrow 8.212218648\ \% = i^{(0.5)}$$

10) $$\delta(t) = \begin{cases} 0.08 & \text{for } 0 \le t \le 5 \\ 0.06 & \text{for } 5 \le t \le 10 \\ 0.04 & \text{for } t \ge 10 \end{cases}$$

$$V(t) = e^{-\delta} \text{ or } \frac{1}{A(t)}$$

$$= \frac{1}{A(0,5)\,A(5,10)\,A(10,t)}$$

$$= e^{-\int_0^5 0.08\,dt} \times e^{-\int_5^{10} 0.06\,dt} \times e^{-\int_{10}^{t} 0.04\,dt}$$

$$\Rightarrow e^{-(0.08t)_0^5} \times e^{-(0.06t)_5^{10}} \times e^{-(0.04t)_{10}^{t}}$$

$$\Rightarrow e^{-0.4} \times e^{-(0.06 - 0.3)} \times e^{-(0.04t - 0.4)}$$

$$\Rightarrow e^{-0.4} \times e^{-0.3} \times e^{0.4 - 0.04t}$$

$$\Rightarrow e^{-0.04t - 0.3}$$

$$\therefore V(t) = e^{-(0.04t + 0.3)}$$

11 a) $PV = AV \times V(t)$

$PV = c\,(1-d)^n$

$PV = 50000\,(1-0.18)^{9/12}$

$PV = 50000\,(0.8617085266)$

$PV = 43085.42623$

b i) $\delta = 6\%$ $\quad \delta = \ln(1+i)$

$\Rightarrow \delta = \ln\left(\frac{1}{1-d}\right)$

$\Rightarrow 0.06 = \ln\left(\frac{1}{\left(1-\frac{d^{(12)}}{12}\right)^{12}}\right)$

$\Rightarrow e^{0.06} = \frac{1}{\left(1-\frac{d^{(12)}}{12}\right)^{12}}$

$d^{(12)} = 0.05985024969$

$d^{(12)} = 5.985024969\%$

ii) $d^{(4)} = 0.06$ $\quad i^{(12)} = ?$

$\left(1+\frac{i^{(12)}}{12}\right)^{12} = \frac{1}{\left(1-\frac{d^{(4)}}{4}\right)^4}$

$\Rightarrow \left(1+\frac{i^{(12)}}{12}\right)^{12} = 1.062319315$

$\Rightarrow i^{(12)} = 0.0606070886$

$i^{(12)} = 6.060708865\%$

c) $d < \delta < i^{(4)} < i$

The interest d is payable at the begining of the time period and so its numeric percentage would be lower than of i which is paid at the end of the period.

d) Same as 4(i)

12) $AV = 7049 \times A(0,2) \times A(2, 4\frac{1}{2}) \times A(4\frac{1}{2}, 6) \quad \frac{1}{}$

$= 7049 \times \left(1 + \frac{i^{(12)}}{12}\right)^{24} \times (1 + 10i) \times \left(1 - \frac{d^{(12)}}{12}\right)^{18}$

$= 7049 \times \left(1 + \frac{0.06}{12}\right)^{24} \times (1 + 0.15) \times \left(\frac{1}{1 - \frac{0.06}{12}}\right)^{18}$

$= 7049 \times 1.296233743 \times \frac{1.0904421325}{\cancel{1.0002785}13}$

~~= 9139.696623~~

$AV = 9999.893613$

13 Let the initial investment be x Rs.

$AV = 2x$.

i) $2x = x(1 + 0.1)^n$

$2 = \cancel{1+0}(1.1)^n$

Taking log on both side

$\log 2 = n \log 1.1$

$0.3010299957 = n \log 1.1$

$7.272540897 = n$.

ii) $d^{(12)} = 10\%$.

$x = 2x(1-d)^n$

$\frac{1}{2} = \left(1 - \frac{d^{(12)}}{12}\right)^{12n} \Rightarrow \cancel{\frac{1}{2}} =$

Taking log on both sides

$\ln \frac{1}{2} = 12n \ln \left(1 - \frac{d^{(12)}}{12}\right)$.

$\ln \frac{1}{2} = 12n \ln (0.9916667)$

$n = 6.902528119$ years.

—— x ——