Ans-1 $X = 200$

$X \sim Poi(\theta)$

$$\frac{X-\theta}{\sqrt{\theta}} \overset{\sim}{\sim} N(0,1)$$

$$P[x_1 < \theta < x_2] = 0.95$$

$$P[-1.96 < \theta < 1.96] = 0.95$$

$\Rightarrow$ 95% CI $= \theta \pm 1.96\sqrt{\theta}$

$$= 200 \pm 1.96 \cancel{\sqrt{\theta}} \sqrt{200}$$

$$= (172.281, 227.718)$$

Ans-2 $X_i = 1, 2, \ldots n$; iid RV

Mean $= \mu$

Variance $= \sigma^2$

$$\bar{X} = \sum_1^n \frac{X_i}{n}$$

$$S^2 = \frac{1}{n-1} \sum_1^n (x_i - \bar{x})^2$$

i) Mean

$$E(\bar{x}) = E\left(\frac{\Sigma x_i}{n}\right) = \frac{1}{n} E(\Sigma x_i)$$

$$= \Sigma[\underbrace{E(x_i)}_{\mu}]$$

$$= \frac{n\mu}{n} = \mu$$

Variance

$$V(\bar{x}) = V\left(\frac{\Sigma x_i}{n}\right) = \frac{1}{n^2} V(\Sigma x_i)$$

$$= \frac{\Sigma V(x_i)}{n^2}$$

$$= \frac{n\sigma^2}{n^2}$$

$$= \frac{\sigma^2}{n}$$

ii) $$E(s^2) = E\left(\frac{1}{(n-1)} \Sigma(x_i - \bar{x})^2\right]$$

$$= \frac{1}{(n-1)} E\left[\Sigma(x_i - \bar{x})^2\right]$$

$$= \frac{1}{(n-1)} \Sigma\left[\sigma^2 + \mu^2 - \left(\frac{\sigma^2}{n} + \mu^2\right)\right]$$

$$= \frac{1}{(n-1)} \Sigma\left[\sigma^2 \left(\frac{n-1}{n}\right)\right]$$

$$= \sigma^2$$

Ans - 8 i) $E(\hat{\theta}) = \theta$

ii) $E(\hat{\theta}) - \theta \neq 0$

iii) $\theta = \text{Var} + \text{bias}^2$

Ans 3 X: No. of claims
$X \sim Bin(4, p)$

No. of claims	0	1	2	3	4
No of policies	86	75	16	2	1

i) $L = \prod_{i=1}^{180} P(X = n)$

$= [P(X=0)]^{86} \cdot [P(X=1)]^{75} \cdot [P(X=2)]^{16} \cdot [P(X=3)]^{2} \cdot [P(X=4)]$

$L = \text{constant} \times (1-p)^{603}\, p^{117}$

$\ln L = \text{constant} + 603 \ln(1-p) + 117 \ln p$

$\Rightarrow 603p = 117(1-p)$

$p = \underline{\underline{0.1625}}$

Ans-4 i) PDF of random variable is given by:

$f(x) = \frac{cB^3}{(x+B)^4}; \; x > 0, \; B > 0$ —— (1)

$f(x) = \frac{\alpha \lambda^\alpha}{(\lambda + x)^{\alpha+1}}$ —— (2) (From table book)

By (1) & (2)
$c = 3$

Mean of x :-

$$E(x) = \frac{B}{C-1}$$

$$E(x) = \frac{B}{2}$$

Variance :-

$$V(x) = \frac{CB^2}{(C-1)^2(C-2)} = \frac{3B^2}{4 \cdot 1}$$

$$V(x) = \frac{3B^2}{4}$$

ii) Using method of moments

$$\Rightarrow E(x) = \sum_{i=1}^{n} \frac{X_i}{n}$$

$$\Rightarrow \frac{B}{C-1} = \bar{x}_n$$

$$\hat{B} = 2\bar{x}_n$$

To verify unbiasedness

$$\text{Bias} = E[g(x)] - \theta$$

$$= E(\hat{B}) - B$$

$$= E(2\bar{x}_n) - B$$

$$= 2E\left(\frac{\sum X_{in}}{n}\right) - B$$

$$= \frac{2}{n} \sum E(X_{in}) - B$$

$$= \frac{2nB}{2n} - B = 0$$

1) MSE = $Var(\hat{B})$ + $Bias^2$

MSE = $V(\bar{2x_n})$

$= 4V\left(\frac{\sum x_i}{n}\right)$

$= \frac{4}{n^2}\sum\left[V(x_i)\right]$

$= \frac{4}{n^2}\sum\left(\frac{3B^2}{4}\right)$

$= \frac{3nB^2}{n^2}$

$= \frac{3B^2}{n}$