Prob & Stats : assignment : 1

Q1) i) Data in A.O : 4, 7, 9, 9, 11, 15, 18, 18, 18, ~~25~~ 25

$\therefore$ Mean $= \frac{4+7+9+9+11+15+18+18+18+25}{10} = \frac{134}{10} = 13.4$

$\therefore$ Median $= \cancel{\frac{5^{th}\ obs + 6^{th}\ obs}{2}}$ ~~5th obs~~ 13

$\therefore$ Mode $= 18$

• IQR =

$q_1 = 8$

$q_2 = 13$

$q_3 = 18$

$\therefore q_3 - q_1 = 18 - 8$
$= 10$

ii) Mean $(\bar{x}) = \frac{0(5) + 1\times 11 + 2\times 26 + 3\times 15 + 4\times 3}{5+11+26+15+3} = \frac{120}{60} = 2$

Median : $n = 60$

$= \frac{30^{th} + 31^{st}}{2} = \frac{2+2}{2} = 2$

IQR : $q_2 = 2$

$q_1 = 1$

$q_3 = 3$

IQR : $q_3 - q_1$
$= 3 - 1$
$= 2$

S.D

x	f	$\bar{x}$	$(x-\bar{x})^2$	$f(x-\bar{x})^2$
0	5	2	4	20
1	11	2	1	11
2	26	2	0	0
3	15	2	1	15
4	3	2	4	12

$$S.D = \sqrt{\frac{\sum f(x-\bar{x})^2}{n-1}} = \sqrt{\frac{58}{59}} = 0.99149$$

2) i) $P(X=0) = \dfrac{e^{-\lambda}\lambda^{x}}{x!} = \dfrac{e^{-0.5}\lambda^{0}}{0!} = e^{-0.5} = 0.6065.$

ii) let y = no. of years with a claim

Then $Y \sim$ Binomial $(3, 0.3935)$

$P(Y=1) \rightarrow {}^{3}C_{1} \times (0.3935)^{1}(0.6065)^{2}$

$= 0.434$

iii) $X \sim \exp(0.5)$

$P(X>2) = \int_{2}^{\infty} \lambda e^{-\lambda x}\,dx$

$= 0.5 \int_{2}^{\infty} e^{-0.5x}$

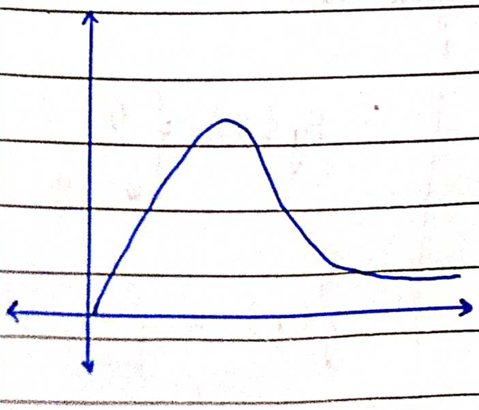
$= 0.368.$

3) i) $X \sim$ Gamma $(2, 1/2)$

$E(X) = \alpha/\lambda = 2 \times 2 = 4$

$V(X) = \alpha/\lambda^{2} = 2 \times 4 = 8.$

ii)



$P(x) = \dfrac{\lambda^{\alpha} e^{-\lambda x} x^{\alpha-1}}{\Gamma\alpha}$

increases in the begining and then decreases when $\alpha = 2$, $\lambda = 1/2$

4) CDF of Gamma Distribution

$$\begin{cases} 0 & x < 0 \\ \int_0^x \lambda^{\alpha} x^{\alpha-1} e^{-\lambda x} , & x > 0. \end{cases}$$

5) $P(L_1) = 0.4$; $P(L_2) = 0.5$; $P(L_3) = 0.1$

A $\rightarrow$ Package arriving late

$P(L_2/A) = ?$

$P(A/L_1) = 0.2$; $P(A/L_2) = 0.4$; $P(A/L_3) = 0.1$

$$\therefore P[L_2/A] = \frac{P(L_2) \times P(A/L_2)}{P(L_1) \times P(A/L_1) + P(L_2) \times P(A/L_2) + P(L_3) \times P(A/L_3)}$$

$$= \frac{0.5 \times 0.4}{(0.4)(0.2) + (0.5)(0.4) + (0.1)(0.1)}$$

$$= \frac{0.2}{0.29} = \underline{\underline{0.6896}}.$$

6) $X \sim v(1, 2)$

$$y = \frac{3}{6-X}$$

$F_y(Y) = P(Y \leq y)$

$= P\left(\frac{3}{6-x} \leq y\right) \longrightarrow$ ~~$P(3 \leq y(6-x))$~~

$= P\left(\frac{6-x}{3} \geq y\right)$

$= P(x - 6 \leq -3y$

$= P(X \leq 6 - 3y)$

$$\therefore F_y(Y) \begin{cases} 0 & y > 2 \\ 6-3y & 1 \leq y \leq 2 \end{cases}$$

$$f'(y) = \begin{cases} 0 & y > 2 \\ -3 & 1 < y \leq 2 \end{cases}$$

7) i) $F(3) = F(X \le 3)$

$= 0.05 + 0.15 + 0.35$

$= 0.55$

ii) $E(X) = 1(0.05) + 2(0.15) + 3(0.35) + 4(0.4) + 5(0.05)$

$= 3.25$

$E(X^2) = 1(0.05) + 4(0.15) + 9(0.35) + 16(0.4) + 25(0.05)$

$= 11.45$

iii) $\therefore V(X) = E(X^2) - [E(X)]^2$

$= 11.45 - (3.25)^2$

$= 0.8875$

iv) Co-eff of skewness is $E(X-\bar{x})^3$

8) $X \sim B(15, 0.2)$

$P(X<3) \rightarrow P(X=0) + P(X=1) + P(X=2)$

$= {}^{15}C_0 (0.2)^0 (0.8)^{15} + {}^{15}C_1 (0.2)^1 (0.8)^{14} + {}^{15}C_2 (0.2)^2 (0.$

$= 0.39802.$

9) So, he won two prizes in 6 weeks

So, $P(X=7) = {}^6C_2 (0.01)^3 (0.99)^4$

$= 0.00001 4409$

10) $X \sim P(0.4)$

$P(X>2) = 1 - P(X \le 2)$

$= 1 - 0.99207$

$= 0.0793$

11) P(Next three boys) $= \frac{1}{2^3} = \frac{1}{8}$

12) $X \sim Exp(10)$

$P(X < 0.1) = \int_0^{0.1} \lambda e^{-\lambda x}\, dx$

$= 10 \int_0^{0.1} e^{-10x}\, dx = \cancel{10}\left[\frac{e^{-10x}}{-\cancel{10}}\right]_0^{0.1} = 1 - e^{-1}$

$= \underline{\underline{0.6321}}$

13) $X \sim U(75, 120)$

$P(x) = \int_{80}^{100} \frac{1}{b-a}\, dx$

Req prob $= \frac{1}{45}[100-80] = \frac{20}{45} = \frac{4}{9}$

14) $X \sim Gamma(5, 0.4)$

$\alpha = 5 \quad ; \quad \lambda = 0.4$

$\because X \sim Gamma(\alpha, \lambda)$

$\therefore 2\lambda X \sim \chi^2_{2\alpha = r}$

$\therefore 2 \times 0.4 \times X \sim \chi^2_{10}$

$\therefore 0.8X \sim \chi^2_{10}$

$P(X < 22.89) = ?$

$\therefore P(2\lambda X < 22.89 \times 2\lambda)$

$\therefore P(\chi^2_{10} < 22.89 \times 0.8)$

$\therefore P(\chi^2_{10} < 18.312) = 0.004898.$

15) i) $P(x) = f(x) = \frac{k}{(x+5)^4} \qquad x > 0$

$\int_0^{\infty} f(x)\, dx = 1$

$\therefore k \int_0^{\infty} (x+5)^{-4}\, dx = 1$

$\therefore \frac{k}{-3} \left[(x+5)^{-3}\right]_0^{\infty} = 1$

$\therefore \frac{k}{-3} \left[0 - 5^{-3}\right] = 1$

$\therefore 5^{-3} k = 3$

$\therefore k = 375.$

2) $f(10) = P(X \le 3) \rightarrow \int_0^{30} f(x)\, dx \rightarrow 375 \int_0^{30} (x+5)^{-4}\, dx$

$= \underline{0.962963}$

3) $E(X) = \int_0^{\infty} x\, f(x)\, dx$

$= 375 \int_0^{\infty} x \cdot (x+5)^{-4}\, dx$

On solving the eqn, we get

$E(X) = 2.5.$