

Assignment 1

Q1. $d^{(4)} = 8\%$

a) $\delta = ?$

$$\left(1 - \frac{d^{(4)}}{4}\right)^4 = e^{-\delta}$$

$$0.92236816 = e^{-\delta}$$

$$\ln(0.92236816) = -\delta$$

$$\therefore \delta = 0.08081082927$$

$$= 8.081082\%$$

b) $i = ?$

$$1 + i = \left(1 - \frac{d^{(4)}}{4}\right)^{-4}$$

$$\therefore i = 8.416578\%$$

c) $d^{(12)} = ?$

$$\left(1 - \frac{d^{(12)}}{12}\right)^{12} = \left(1 - \frac{d^{(4)}}{4}\right)^4$$

$$1 - \frac{d^{(12)}}{12} = 0.9932883884$$

$$d^{(12)} = 8.05393392\%$$

Q2. $S(t) = 0.004t + 0.0002t^2$

i) $1000 = PV.e^{-\int_0^{10} 0.004t + 0.0002t^2 dt}$

$= PV.e^{-\left[0.002t^2 + \frac{0.0002t^3}{3}\right]_0^{10}}$

$= PV.e^{-\left[0.2 + \frac{0.2}{3}\right]}$

$PV = 1305.605172.765.928.3384$

ii) $1305.605172 = 1000 \times (1+i)^{10}$

$(1+i)^{10} = 0.7659283384$

$\therefore i = 0.02702540389$

Q3. i) $1-d = \frac{1}{1+i}$

$1 - \frac{1}{1+i} = d$

$\frac{1+i-1}{1+i} = d$

$\frac{i}{1+i} = d$

$i \times \frac{1}{1+i} = d$

$iv = d$

ii) $d^{(2)} = ?$

a) $\frac{d^{(4)}}{4} = 2.25\%$

$$\left(1 - \frac{d^{(4)}}{4}\right)^4 = \left(1 - \frac{d^{(2)}}{2}\right)^2$$

$$0.95550625 = 1 - \frac{d^{(2)}}{2}$$

$$\therefore d^{(2)} = 0.0889875\%$$

$$= 8.89875\%$$

b) $d^{(1/2)} = 5\%$

$$\left(1 - \frac{d^{(1/2)}}{1/2}\right)^{1/2} = \left(1 - \frac{d^{(2)}}{2}\right)^2$$

$$\therefore d^{(2)} = 5.199250715\%$$

iii) $100 \left(1 + i\right)^{\frac{91}{365}} = FV$

$$FV = 101.2238407$$

$$\frac{101.2238407}{100} = \frac{1}{(1 - dt)}$$

$$0.9879095608 = (1 - dt)^{\frac{91}{365}}$$

$$d \times \frac{91}{365} = 0.1209043925$$

$$\therefore d = 4.849461895\%$$

99. i) Question repeated.

ii) $i = 0.08$

a) $d^{(12)} = ?$

$$\left(1 - \frac{d^{(12)}}{12}\right)^{-12} = (1 + i)$$

$$1 - \frac{d^{(12)}}{12} = 0.993607102$$

$$d^{(12)} = 7.671477614 \%$$

b) $\left(1 + \frac{i^{(365)}}{365}\right)^{365} = (1 + i)$

$$i^{(365)} = 7.696915541 \%$$

c. $(1 + i) = e^{\delta}$

$$\delta = 7.696104114 \%$$

d) $i^{(1/2)} = ?$

$$\left(1 + \frac{i^{(1/2)}}{1/2}\right)^{1/2} = (1 + i)$$

$$i^{(1/2)} = 8.32 \%$$

Q5. i) $d^{(4)} = 6\%$.

a) $i^{(2)} = ?$

$$\left(1 - \frac{d^{(4)}}{4}\right)^{-4} = \left(1 + \frac{i^{(2)}}{2}\right)^2$$

$$i^{(2)} = 6.137751552\%$$

b) $d^{(12)} = ?$

$$\left(1 - \frac{d^{(4)}}{4}\right)^4 = \left(1 - \frac{d^{(12)}}{12}\right)^{12}$$

$$d^{(12)} = 6.030252528\%$$

$$2,20,000 = 2,00,000 (1 + i)$$

$$i = 10\%$$

$$2,00,000 \left(1 + i\right)^{\frac{93}{365}} = FV$$

$$FV = 2,04,916.3564$$

$\therefore$ The sum paid by Amit on 17 July, 2005 to terminate the contract is 2,04,916.3564.

60. i) Let the product be of £100.

Cash payment at $t_0 = 70$.

Credit payment at $t_{0.5} = 75$.

$$70 = 75(1-d)^{1/2}.$$

$$1 - \left(\frac{70}{75}\right)^2 = d.$$

$$\therefore d = 12.888889\%.$$

ii) a) $(1-d) = \left(1 - d^{(12)}\right)^{12}.$

$$\therefore d^{(12)} = 13.7195439\%.$$

b) $i^{(2)} = ?$

$$\left(\frac{1 + i^{(2)}}{2}\right)^2 = (1-d)$$

$$i^{(2)} = 14.28571429\%.$$

c) $\delta = ?$

$$(1-d) = e^{-\delta}$$

$$\ln(1-d) = -\delta$$

$$-\ln(1-d) = \delta$$

$$\delta = 13.795743\%.$$

20,000 $\rightarrow$ 26,000 in 17 months.

$$\frac{26,000}{20,000} = (1+i)^{\frac{17}{12}}$$

$$i = 20.34570672\%$$

i) $i^{(4)} = ?$

$$\left(1 + \frac{i^{(4)}}{4}\right)^4 = (1+i)$$

$$i^{(4)} = 18.95525456\%$$

ii) $d^{(2)} = ?$

$$\left(1 - \frac{d^{(2)}}{2}\right)^{-2} = (1+i)$$

$$d^{(2)} = 17.68823531\%$$

iii) $1 + \frac{17}{12}i =$ Simple interest.

$$\frac{26,000}{20,000} = (1 + ni)$$

$$1.3 = 1 + \frac{17}{12}i$$

$$i = 21.17647059\%$$

iv). The interest rate decreases as p^{th} increases in nominal rates of interest convertible p^{th} ly. Also simple interest is more than compound if the time period is more than 1 year.

8.

$$i = 8\%$$

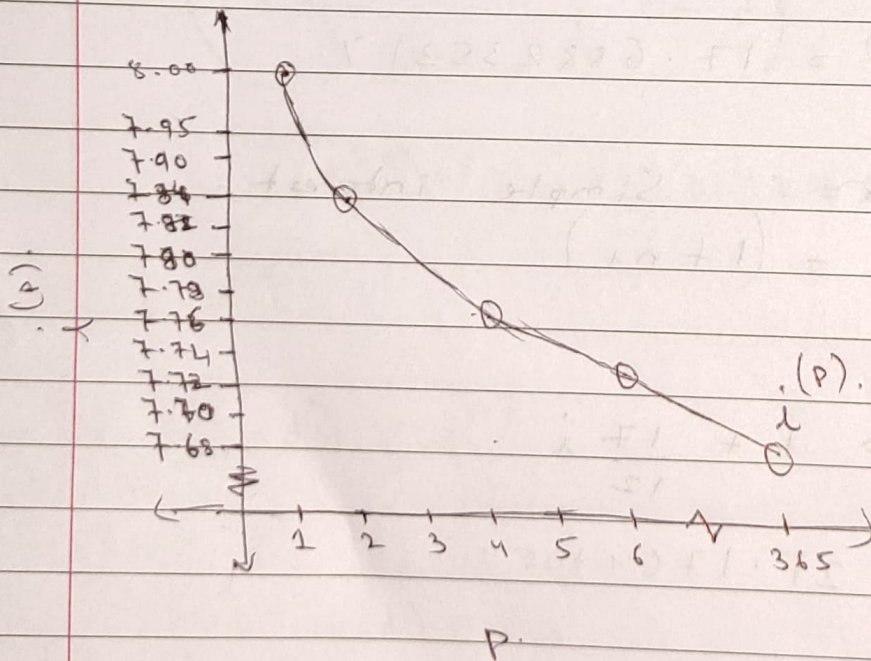
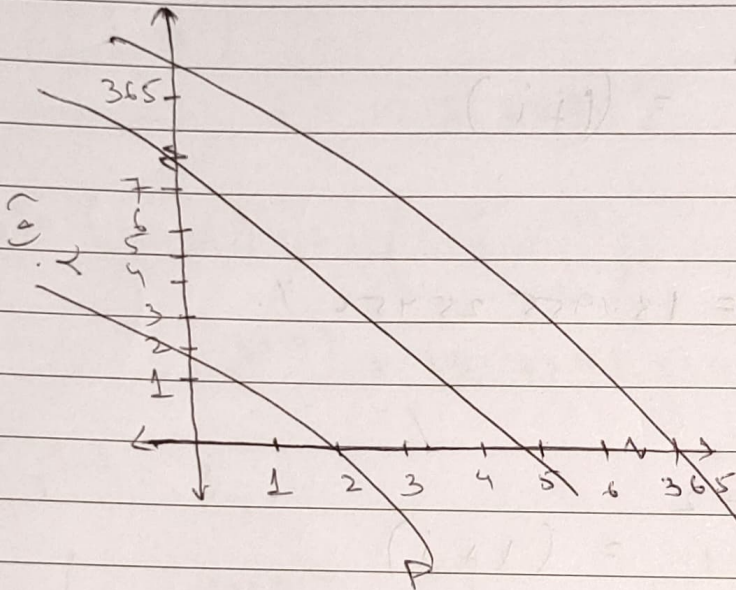
$$\lambda^{(2)} = 7.846096908$$

$$\lambda^{(4)} = 7.770618763$$

$$\lambda^{(6)} = 7.745674178$$

$$\lambda^{(12)} = 7.720836132$$

$$\lambda^{(365)} = 7.696915541$$



The nominal rate of interest decreases as they are convertible p thly, where p is increasing.

9. i) Force of interest refers to nominal rate of interest compounded infinite number of times or every moment per time period.

ii) $i^{(2)} = 0.0775$.

$$i = ?$$

$$(1+i) = \left(1 + \frac{i^{(2)}}{2}\right)^2$$

$$i = 0.0790015625.$$

$$\delta = ?$$

$$\ln(1+i) = \delta$$

$$\delta = 0.07603613738.$$

$$d^{(12)} = ?$$

$$\left(1 - \frac{d^{(12)}}{12}\right)^{-12} = (1+i)$$

$$d^{(12)} = 0.0757957468.$$

$$i^{(1/2)} = ?$$

$$\left(1 + \frac{i^{(1/2)}}{1/2}\right)^{1/2} = (1+i)$$

$$i^{(1/2)} = 0.08212218594.$$

$$Q11. \quad 50,000 (1-d)^n = PV.$$

$$50,000 (1-0.18)^{9/12} = PV.$$

$$PV = 43085.42623$$

$\therefore$ The amount initially lent to the borrower was 43085.42623.

$$b) i) \quad \delta = 6\%. \quad d^{(12)} = ?$$

$$\left(1 - \frac{d^{(12)}}{12}\right)^{12} = e^{-\delta}$$

$$d^{(12)} = 0.05985024969 \\ = 5.985024969\%$$

$$ii) \quad d = 6\%. \quad i^{(12)} = ? \quad i^{(4)} = ?$$

$$(1-d)^{-1} = \left(1 + \frac{i^{(12)}}{12}\right)^{12}$$

$$i^{(12)} = 6.203520182\%$$

$$i^{(4)} = 6.235645164\%$$

$$c. \quad \cancel{i} < \cancel{i^{(12)}} < \delta < \cancel{d}.$$

$$i > i^{(4)} > \delta > d.$$

Q12

$$\begin{aligned}
 FV &= 7049 (A_{0,2}) (A_{2,4.5}) (A_{4.5,6}) \\
 &= 7049 \left(1 + \frac{0.06}{12} \right)^{12 \times 2} \left(1 + 2.5 \times 0.06 \right)^{12 \times 1.5} \\
 &\quad \left(1 - \frac{0.06}{12} \right)^{12 \times 1.5}
 \end{aligned}$$

$$FV = 8348.842852.$$

∴ The accumulated amount after 6 years is 8348.842852.

Q13

$$2 = (1+i)^n$$

$$\ln(2) = n \ln(1+i)$$

$$\ln(2) = n \ln(1.1)$$

$$\frac{\ln(2)}{\ln(1.1)} = n$$

$$\ln(1.1)$$

$$n = 7.2725$$

∴ It would take around 7.27 years to double an investment at 10% rate of interest.