

HOMEWORK - 1 & 2

HW - 1: Lagrange's Interpolation.

1.	x	7	8	9	10	9.5
	$f(x)$	3	1	1	9	?

$$f(9.5) = \frac{(9.5 - 8)(9.5 - 9)(9.5 - 10)}{(7 - 8)(7 - 9)(7 - 10)} (3) +$$

$$+ \frac{(9.5 - 7)(9.5 - 9)(9.5 - 10)(1)}{(8 - 7)(8 - 9)(8 - 10)} +$$

$$\frac{(9.5 - 7)(9.5 - 8)(9.5 - 10)(1)}{(9 - 7)(9 - 8)(9 - 10)} +$$

$$\frac{(9.5 - 7)(9.5 - 8)(9.5 - 9)(9)}{(10 - 7)(10 - 8)(10 - 9)}$$

$$= \frac{(1.5)(0.5)(+0.5)(3)}{(+1)(+2)(+3)} + \frac{(2.5)(0.5)(+0.5)(1)}{(1)(-1)(+2)}$$

$$+ \frac{(2.5)(1.5)(+0.5)(1)}{(2)(1)(+1)} +$$

$$\frac{(2.5)(1.5)(0.5)(9)}{(3)(2)(1)}$$

$$= 0.1875 + (-0.3125) +$$

$$0.9375 + 2.2125$$

$$= \boxed{3.625}$$

2.	x	3	7	9	10	6
	$f(x)$	168	120	72	63	?

$$f(6) = \frac{(6-7)(6-9)(6-10)(168)}{(3-7)(3-9)(3-10)} +$$

$$\frac{(6-3)(6-9)(6-10)(120)}{(7-3)(7-9)(7-10)} + \frac{(6-3)}{(6-3)}$$

(63)

$$\frac{(6-3)(6-7)(6-10)(72)}{(9-3)(9-7)(9-10)} + \frac{(6-3)(6-7)(6-10)}{(10-3)(10-7)(10-9)}$$

$$= \frac{(-1)(+3)(-4)(168)}{(+4)(+6)(+7)} + \frac{(3)(-3)(+4)(120)}{(4)(+2)(-3)}$$

$$+ \frac{(3)(-1)(-4)(72)}{(6)(2)(1)} + \frac{(3)(+1)(-3)(63)}{(7)(3)(1)}$$

$$= 12 + 180 - 72 + 972 - 21 = 147$$

$$= \boxed{147}$$

3.	x	1	2	7	8	6
	$f(x)$	4	5	5	4	?

$$f(6) = \frac{(6-2)(6-7)(6-8)(4)}{(1-2)(1-7)(1-8)} + \frac{(6-1)(6-7)(6-8)(5)}{(2-1)(2-7)(2-8)}$$

$$+ \frac{(6-1)(6-2)(6-8)(5)}{(7-1)(7-2)(7-8)} + \frac{(6-1)(6-2)(6-7)(4)}{(8-1)(8-2)(8-7)}$$

$$= \frac{(4)(\cancel{1})(+2)(4)}{(\cancel{-1})(+6)(-7)} + \frac{(\cancel{5})(+1)(+2)(5)}{(1)(\cancel{+5})(+6)}$$

$$+ \frac{(5)(4)(+2)(\cancel{5})}{(6)(\cancel{5})(+1)} + \frac{(5)(4)(-1)(4)}{(7)(6)(1)}$$

$$= -0.76190476 + 1.6666667$$

$$6.666667 - 1.9047619$$

$$= \boxed{5.667}$$

4.	x	1	2	3	4	7
	$b(n)$	2	4	8	16	128

$b(5) = ?$

$$y(s) = \frac{(s-2)(s-3)(s-4)(s-7)(2)}{(1-2)(1-3)(1-4)(1-7)} +$$

$$\frac{(s-1)(s-3)(s-4)(s-7)(4)}{(2-1)(2-3)(2-4)(2-7)} + \frac{(s-1)(s-2)(s-4)(s-7)(16)}{(4-1)(4-2)(4-3)(4-7)}$$

$$\frac{(s-1)(s-2)(s-4)(s-7)(8)}{(3-1)(3-2)(3-4)(3-7)} + \frac{(s-1)(s-2)(s-3)(s-7)(16)}{(7-1)(7-2)(7-3)(7-4)}$$

$$+ \frac{(s-1)(s-2)(s-3)(s-4)(128)}{(7-1)(7-2)(7-3)(7-4)}$$

$$= \frac{(3)(2)(1)(-2)(2)}{(+1)(+1)(+3)(+6)} + \frac{(4)(2)(1)(+2)(4)}{(1)(+1)(+2)(+5)}$$

$$+ \frac{(4)(3)(1)(-2)(8)}{(2)(1)(+1)(+4)} + \frac{(4)(3)(2)(+2)(16)}{(3)(2)(1)(+3)} +$$

$$\frac{(4)(3)(2)(1)(128)}{(6)(5)(4)(3)}$$

$$= -0.6666667 + 6.4 - 24 + 42.6666667 + 8.5333333$$

$$= \boxed{32.933}$$

5.	x	10	20	14	16	15
f(x)	0.25	0.2	0.15	0.10	?	

$$f(15) = \frac{(15-20)(15-14)(15-16)}{(10-20)(10-14)(10-16)} (0.25) + \frac{(15-10)(15-14)(15-16)}{(20-10)(20-14)(20-16)} (0.2)$$

$$+ \frac{(15-10)(15-20)(15-16)}{(14-10)(14-20)(14-16)} (0.15) + \frac{(15-10)(15-20)(15-14)}{(16-10)(16-20)(16-14)} (0.1)$$

$$= \frac{(+5)(+1)(+1)}{(+10)(+4)(-6)} (0.25) + \frac{(+5)(1)(-1)}{(10)(6)(4)} (0.2) +$$

$$\frac{(5)(+5)(+1)}{(4)(+6)(+2)} (0.15) + \frac{(5)(+5)(1)}{(6)(+4)(2)} (0.1)$$

$$= -0.0052083 - 0.0041667 + 0.078125 + 0.0520833 = \boxed{0.120833}$$

HW-2 : Regula Falsi Method

1. $f(x) = x^3 + x - 1 = 0 \quad 0.5 < x < 1$

x	y
0.5	-0.375

0.6	-0.184	→ -ve
-----	--------	-------

0.7	0.043	→ +ve
-----	-------	-------

$$x_1 = 0.6 \quad x_2 = 0.7$$

$$y_1 = -0.184 \quad y_2 = 0.043$$

$$x_3 = \frac{(0.6)(0.043) - (0.7)(-0.184)}{(0.043) - (-0.184)}$$

$$= 0.681057$$

$$y_3 = -0.0030418 \quad \text{replace } y_2$$

$$x_4 = \frac{(0.681057)(0.043) - (0.7)(-0.0030418)}{(0.043) - (-0.0030418)}$$

$$x_4 = 0.68230484 \rightarrow \underline{\underline{\text{root}}}$$

$$y_4 = -0.0000550366$$

2. $f(x) = x^3 - 2x + 0.5 = 0 \quad 0 < x < 1$

x	y
0	0.5
0.1	0.801 0.801
0.2	0.108 $\rightarrow y_1$
0.3	-0.073 $\rightarrow y_2$

$x_1 = 0.2 \quad y_1 = 0.108$
 $x_2 = 0.3 \quad y_2 = -0.073$

$$x_3 = \frac{(0.2)(-0.073)(+0.3)(0.108)}{(-0.073) - (0.108)}$$

$x = 0.259663506$

$y_3 = -0.0018201574 \rightarrow \text{replace } y_2$

$x_4 = \text{root} = 0.2586748727$

$y_4 = -0.00004111$

3. $f(x) \quad x \in (0, 2.5) \quad x^3 + 2x - 20 = 0 \quad 2 < x < 2.5$

x	y
2.4	-1.376 $\rightarrow y_1$
2.5	0.625 $\rightarrow y_2$

$x_1 = 2.4 \quad y_1 = -1.376$
 $x_2 = 2.5 \quad y_2 = 0.625$

$$x_3 = \frac{(2.4)(0.625) - (2.5)(-1.376)}{0.625 - (-1.376)}$$

$$= 2.468765$$

$$y_3 = -0.0158270149 \rightarrow \text{replace } y_1$$

$$x_4 = 2.469537039 \rightarrow \underline{\underline{\text{root}}}$$

$$y_4 = -0.00017479.$$

$$4. \quad f(x) = 3x - \cos x - 1 = 0 \quad 0 < x < 1$$

$$x \quad y$$

$$0.6 \quad -0.1999451694 \rightarrow y_1$$

$$0.7 \quad 0.1000746303 \rightarrow y_2$$

$$x_1 = 0.6110000000$$

$$x_2 = 0.7$$

$$x_3 = \frac{(0.6)(0.1000746303) - (0.7)(-0.1999451694)}{(0.1000746) - (-0.1999451694)}$$

$$x_3 = 0.66664399 \rightarrow \underline{\underline{\text{root}}}$$

$$y_3 = -0.00000033$$

$$5. \quad f(x) = x^3 - 3x - 5 \quad 2 < x < 2.5$$

$$\begin{array}{ccc} x & & y \\ x_1 \leftarrow 2.2 & -0.952 & \rightarrow y_1 \\ x_2 \leftarrow 2.3 & 0.267 & \rightarrow y_2 \end{array}$$

$$x_3 = \frac{(2.2)(0.267) - (2.3)(-0.952)}{0.267 - (-0.952)}$$

$$x_3 = 2.2780968007$$

$$y_3 = -0.0115944071 \rightarrow \text{replaces } y_1$$

$$x_4 = \frac{(2.2780968007)(0.267) - (2.3)(-0.0115944)}{(0.267) - (-0.0115944)}$$

$$x_4 = 2.278011107 \rightarrow \underline{\underline{\text{root}}}$$

$$y_4 = -0.00009661$$

$$6. \quad f(x) = x \log_{10} x - 1.2 = 0 \quad 2 < x < 3$$

$$\begin{array}{ccc} x & & y \\ x_1 \leftarrow 2.7 & -0.0353178 & \rightarrow y_1 \\ x_2 \leftarrow 2.8 & 0.0520424 & \rightarrow y_2 \end{array}$$

$$x_3 = \frac{(2.7)(0.0520424) - (2.8)(-0.0353178)}{0.0520424 - (-0.0353178)}$$

$$x_3 = 2.792154169$$

$$y_3 = +0.04573149 \rightarrow \text{replaces } y_2$$

$$x_4 = 2.740456302 \rightarrow \underline{\text{root}}$$

$$y_4 = -0.00016552$$

$$7. \quad x^3 - 4x + 1 = 0$$

$$\begin{array}{r} x \\ 0.4 \\ 0.2 \\ 0.208 \end{array} \quad \begin{array}{r} y \\ -0.536 \\ 0.208 \\ -0.536 \end{array}$$

$$x_1 = 0.4 \quad -0.536 \rightarrow y_1$$

$$x_2 = 0.2 \quad 0.208 \rightarrow y_2$$

$$x_3 = \frac{(0.4)(0.208) - (-0.536)(0.2)}{0.208 - (-0.536)}$$

$$x_3 = 0.255914$$

$$y_3 = -0.0068956 \rightarrow \text{replaces } y_1$$

$$x_2 = 0.255914$$

$$y_3 = -0.0068956$$

$$x_{20} = 0.2$$

$$y_2 = 0.208$$

$$x_4 = \frac{(0.208)(0.255914) - (0.2)(-0.0068956)}{0.255914 - (-0.0068956)}$$

$$x_4 = 0.254119 \rightarrow \underline{\text{root}}$$

$$y_4 = -0.000069028$$

$$8. f(x) = x^3 + 2x^2 - 8 = 0$$

$$\begin{array}{lcl} x_1 \leftarrow 1.5 & -0.125 & \rightarrow y_1 \\ x_2 \leftarrow 1.6 & 1.216 & \rightarrow y_2 \end{array}$$

$$x_3 = \frac{(1.5)(1.216) - (1.6)(-0.125)}{1.216 - (-0.125)}$$

$$= 1.5093214019$$

$$y_3 = -0.00558654 \rightarrow \text{replaces } y_1$$

now

$$\cancel{x_4 = (1.5093214019)(-0.00558654)}$$

$$x_4 = \frac{(1.5093214019)(1.216) - (1.6)(-0.00558654)}{1.216 - (-0.00558654)}$$

$$(x_4 = 1.509736089) \rightarrow \underline{\underline{\text{root}}}$$

$$y_4 = 0.0002477$$

9. $f(x) = x^3 - 9x + 1 = 0$

$$\begin{array}{ccc} x & & y \\ x_1 \leftarrow 0.1 & 0.101 & \rightarrow y_1 \\ x_2 \leftarrow 0.2 & -0.792 & \rightarrow y_2 \end{array}$$

$$x_3 = \frac{(0.1)(-0.792) - (0.2)(0.101)}{(0.1) - 0.792 - 0.101}$$

$$x_3 = 0.1113101904 \rightarrow \underline{\text{root}}$$

$$y_3 = -0.000472585$$

10. $f(x) = x^3 - x - 4 = 0$

$$\begin{array}{ccc} x & & y \\ 1.6 & -1.504 & \\ x_1 \leftarrow 1.7 & -0.787 & \rightarrow y_1 \\ x_2 \leftarrow 1.8 & 0.032 & \rightarrow y_2 \end{array}$$

$$x_3 = \frac{(1.7)(0.032) - (1.8)(-0.787)}{0.032 - (-0.787)}$$

$$x_3 = 1.7960927961$$

$$y_3 = -0.0019884399 \rightarrow \text{replaces } y_1$$

$$x_4 = \frac{(1.7960927961)(0.032) - (1.8)(-0.0019884399)}{0.032 - (-0.0019884399)}$$

$$x_4 = 1.796321381 \rightarrow \underline{\text{root}}$$

$$y_3 = -0.0000045$$

Formulas used:

• For HW-1

Lagrange's interpolation formula:

$$f(x) = \frac{(x - x_1)(x - x_2) \dots (x - x_n)(y_0) +$$

$$(x_0 - x_1)(x_0 - x_2) \dots (x_0 - x_n)$$

$$\frac{(x - x_0)(x_0 - x_2) \dots (x - x_n)(y_1) + \dots +$$

$$(x_1 - x_0)(x_1 - x_2) \dots (x_1 - x_n)$$

$$\frac{(x - x_0)(x_0 - x_2) \dots (x - x_{n-1})(y_n)}{(x_n - x_0)(x_n - x_1) \dots (x_n - x_{n-1})}$$

• For HW-2

Regula Falsi Method (False position):

$$\begin{array}{lcl} x_1 = \langle \text{value} \rangle & \nearrow & y_1 = +ve \langle \text{value} \rangle \\ x_2 = \langle \text{value} \rangle & \searrow & y_2 = -ve \langle \text{value} \rangle \\ & \downarrow & \end{array}$$

cross multiply

$$x_3 = \frac{(x_1)(y_2) - (x_2)(y_1)}{y_2 - y_1}$$

$$x_3 = \langle \text{value} \rangle$$

$y_3 \rightarrow y$ +ve replace x_1 ; if -ve replace x_2 & y_2 & so on.
 $y_n = 0.000 \dots \rightarrow x_n$ will be root.