1) i) 4, 7, 9, 9, 11, 15, 18, 18, 18, 25

Mean $= \frac{4+7+9+9+11+15+18+18+18+25}{10} = 13.4$

Median $= \left(\frac{n+1}{2}\right)^{th}$ term

$= \left(\frac{11}{2}\right)^{th}$ term

$= 5.5^{th}$ term $=$ 5th term + 6th term /2

$= \frac{11+15}{2} = 13$

Mode = 18

Inter quartile range =

$Q_1 = \left(\frac{N+1}{4}\right)^{th}$ term

$= \left(\frac{10+1}{4}\right)^{th}$ term

$= 2.75^{th}$ item

2nd term + 0.75 (3rd term - 2nd term)

$7 + \frac{3}{4}(9-\cancel{7})$

$7 + 3/2 = 17/2 = 8.5$

$Q_3 = 3\left(\frac{N+1}{4}\right)^{th}$ term

$= 8.25^{th}$ term

8th term $+ \frac{1}{4}$ [9th term − 8th term]

$18 + \frac{1}{4}\left[18-18\right] = 18$

$Q_3 - Q_1 = 18 - 8.5 = 9.5$ //

STANDARD DEVIATION

$(4-13.4)^2 = 88.36$
$(7-13.4)^2 = 40.96$
$(9-13.4)^2 = 19.36$
$(9-13.4)^2 = 19.36$
$(11-13.4)^2 = 5.76$
$(15-13.4)^2 = 2.56$
$(18-13.4)^2 = 21.16$
$(18-13.4)^2 = 21.16$
$(18-13.4)^2 = 21.16$
$(25-13.4)^2 = 134.56$ $= \frac{374.4}{10} = 37.44 = 6.118823416$

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ii)

No. of Households	No. of people	f x
0	5	0
1	11	11
2	26	52
3	15	45
4	3	12
		120

Mean = $\frac{\Sigma fx}{\Sigma f} = \frac{120}{10} = 12$

Mode = 2

Median = $\left(\frac{N+1}{2}\right) = \left(\frac{5+1}{2}\right)^{th}$ = 3rd term

= 26

Quartile Deviation =

$Q_1 = \left(\frac{N+1}{4}\right) = \left(\frac{6}{4}\right)^{th}$ = 1.5th term

1st term + 0.5 [2nd - 1st]

5 + 0.5 [11 - 5]

= 8

$Q_3 = 3\left(\frac{N+1}{4}\right)$ = 4.5 th term

4th term + 0.5 [5th term - 4th term]

$15 + 0.5\,[3-15]$

$15 + (-6) = 9$

$Q_3 - Q_1 = 9 - 8 = 1$

STANDARD DEVIATION

x_i	f_i	$f_i x_i$	$(x-\bar{x})^2$	$f_i(x-\bar{x})^2$
0	5	0	4	20
1	11	11	1	11
2	26	52	0	0
3	15	45	1	15
4	3	12	4	12
	$N = 60$	$\Sigma f_i x_i = 120$		58

$$\frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{120}{60} = 2$$

$$\sigma = \sqrt{\frac{\Sigma f_i (x_i - \bar{x})^2}{\Sigma f_i}} = \sqrt{\frac{58}{60}} = 0.98319208$$

2) $\lambda = 0.5$

i) $\frac{e^{-\lambda} \lambda^{x}}{x!} = 0.606530659$

$\frac{e^{-0.5} \cdot 0.5^{0}}{0!}$

ii) Using Binomial theorem

$^{n}C_{r}\, p^{r} q^{n-r}$

$^{n}C_{r} = {}^{3}C_{2} = \frac{3!}{2!(3-2)!} = \frac{3\times 2!}{2!} = 3$

$q^{n-r} = 0.606530659$

$p^{r} = 1-0.606530659$

$= 0.393469341$

$3 \times (0.606530659)^{2} \times 0.393469341$

$= 0.434247842$ //

iii) Since it is more than, S_x

$1-F_X(x) = e^{-\lambda x}$

Here $\lambda = 0.5$, $x = 2$

$= e^{-1} = 0.36787944$

3) $\alpha = 2 \quad \lambda = 1/2$

Rang $0 < x < \infty$

i) $E(X) = \alpha/\lambda = 2/\tfrac{1}{2} = 4$

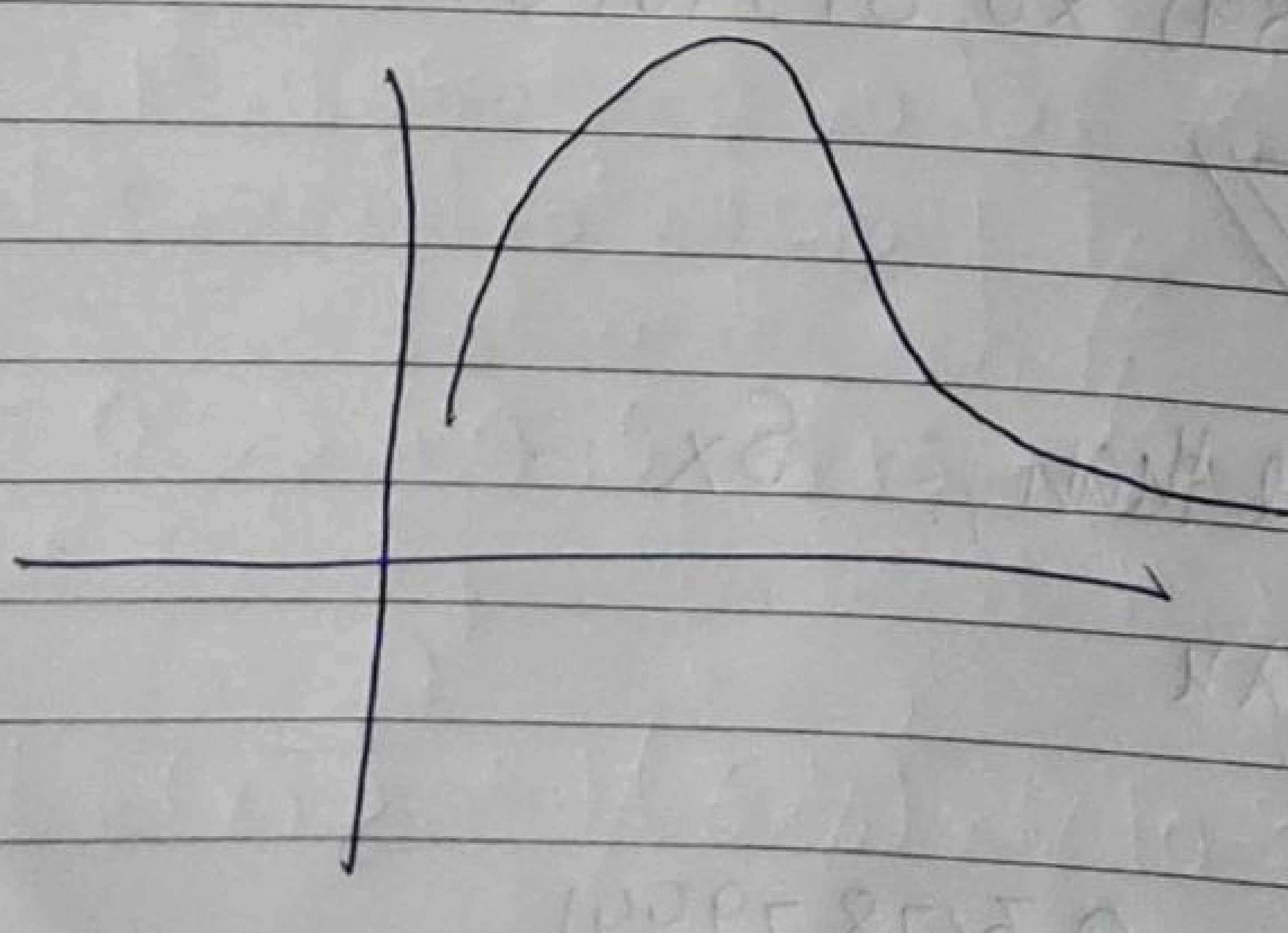
$\sqrt{\sigma(x)} = \sqrt{\alpha/\lambda^2} = \sqrt{2/(1/2)^2}$

$= \sqrt{2/\tfrac{1}{4}} = \sqrt{8} = 2.828427124 75$

ii) $2/\sqrt{\alpha}$

$= 2/\sqrt{2} = 1.414$

$\therefore$ Positively skewed



5) $P(L_1) = 40\%$

$P(L_2) = 50\%$

$P(L3) = 10\%$

$P(E/L_1) = 20\%$

$P(E/L_2) = 40\%$

$P(E/L_3) = 10\%$

$$\frac{P(L_2) \times P(E/L_2)}{P(L_1) \times P(E/L_1) + P(L_2) \times P(E/L_2) + P(L_3) \times P(E/L_3)}$$

$$= \frac{0.5 \times 0.4}{(0.4 \times 0.2) + (0.5 \times 0.4) + (0.1 \times 0.1)}$$

$$= \frac{0.2}{(0.08) + (0.2) + (0.01)} = \frac{0.2}{0.29} = 0.689655172$$

$= 68.9655172\%$

6) $Y = \frac{3}{6-x} = \frac{3}{6-1} = 3/5$

$Y = \frac{3}{6-x} = \frac{3}{6-2} = 3/4$

$$f_Y(y) = \frac{1}{3/5 - 3/4} = \frac{1}{3/20} = 20/3$$

7)

i) $F(3)$

$= F(X \leq 3) = 1+2+3$

$= 0.05 + 0.15 + 0.35$

$= 0.55$

ii) $E(x) = \cancel{0.05 + 0.15 + 0.35 + 0.4 + 0.05}$

$= \sum 1 \times 0.05 + 2 \times 0.15 + 3 \times 0.35 + 4 \times 0.4 + 5 \times 0.05$

$= 0.05 + 0.3 + 1.05 + 1.6 + \cancel{0.25}\ 0.25 = \cancel{5.5}\ 3.25$

iii) $V(x) = E(x^2) - [E(x)]^2$

$= 1^2 \times 0.05 + 2^2 \times 0.15 + 3^2 \times 0.35 + 4^2 \times 0.4 + 5^2 \times 0.05$

$= 0.05 + 0.6 + 3.15 + 6.4 + 1.25 = 11.45$

$[E(x)]^2 = (3.25)^2 = 10.5625$

$= E(x^2) - [E(x)]^2$

$= 11.45 - 10.5625 = 0.8875$

iv) Skewness

$E(x^3) - (E(X))^3 =$

$E(x^3) = 1^3 \times 0.05 + 2^3 \times 0.15 + 3^3 \times 0.35 + 4^3 \times 0.4 + 5^3 \times 0.05$

$= 0.05 + 1.2 + 9.45 + 25.6 + 6.25$
$= 42.55$

$[E(X)]^3 = (3.25)^3 = 34.328125$

$E(x^3) - [E(x)]^3 = 42.55 - 34.328125$
$= 8.221875$

q) Using Binomial theorem:

${}^nC_x \, p^x \, q^{n-x}$

${}^7C_3 \times 0.01^3 \times 0.99^{7-3}$

$35 \times 0.01^3 \times 0.96059601 = \cancel{0.336208603}$
$= 0.0000336208603 5$

10) $\lambda = 0.4$ [2/5]

$P(X>2) = 1 - P(X \leq 2)$

$P(X=0) = \frac{\lambda^x e^{-\lambda}}{x!} = \frac{(0.4)^0 e^{-0.4}}{0!} = e^{-0.4}$

$= 0.670320046$

$P(X=1) = \frac{\lambda^x e^{-\lambda}}{x!} = \frac{(0.4)^1 e^{-0.4}}{1!} = 0.4 \times e$

$= 0.268128017$

$P(X=2) = \cancel{(0.4)^2} \frac{\lambda^x e^{-\lambda}}{x!} = \frac{(0.4)^2 e^{(-0.4)}}{2!}$

$= 0.053625604$

$P(X \leq 2) = 0.992073668$

$P(X>2) = 1 - P(X \leq 2)$

$= (1 - 0.992073668)$

$= 0.007926332$

11) Probability of boy $= 1/2$

" " girl $= 1/2$

Probability of three boys $=$

$1/2 \times 1/2 \times 1/2 = 1/8$

Q12) $\lambda = 10$

$x = 0.1$

$\lambda e^{-\lambda x}$

$10 e^{-10 \times 0.1}$

$= 3.678794412$

Q13) $\frac{75+120}{2} = 97.5$

$x = 97.5$

$a = 80$

$b = 100$

$= \frac{x-a}{b-a} = \frac{97.5-80}{100-80} = \frac{17.5}{20}$

$= 0.875$

Q14) $X \sim gamma(5, 0.4)$ $P(X < 22.89)$

$\alpha = 5$ $\lambda = 0.4$

$2\lambda = 0.8$

$P(0.8X < 18.312)$

$0.8X \sim \chi^2_{2\times5}$

$P(\chi^2_{10} < 18.312)$

$P(\chi^2_{10} < 18.0) \rightarrow 0.9450$

$P(\chi^2_{10} < 18.312) \rightarrow ?$ 0.0079

0.5

$P(\chi^2_{10} < 18.5) \rightarrow 0.9529$

$0.5 \rightarrow 0.0079$

$1 \rightarrow \frac{0.0079}{0.5}$

$0.312 \rightarrow \frac{0.0079}{0.5} \times 0.312 = 0.0049296$

$= P(\chi^2_{10} < 18.312) = P(\chi^2_{10} < 18.0 + 0.0049296$

$= 0.9450 + 0.0049296$

$= \underline{0.9499296}$

15) i) $\int_0^\infty \frac{K}{(x+5)^4}\,dx = 1$ $\boxed{x > 0}$

$K\int_0^\infty \frac{1}{(x+5)^4}\,dx = 1$

$K\int_0^\infty \frac{(x+5)^{-3}}{-3} = 1$

$K\left[\frac{(x+5)^{-3}}{-3}\right]_0^\infty = 1$