

Assignment solution

1)

$$a) \sum x_i = 850$$

$$\sum y_i = 1737$$

$$\sum x_i^2 = 925000$$

$$\sum y_i^2 = 372717$$

$$\sum x_i y_i = 182560$$

$$\begin{aligned} S_{xy} &= \sum x_i y_i - n(\bar{x})(\bar{y}) \\ &= 182560 - 10 \times \frac{850}{10} \times \frac{1737}{10} \\ &= 37915 \end{aligned}$$

$$\begin{aligned} S_{xx} &= \sum x_i^2 - n(\bar{x})^2 \\ &= 925000 - \frac{850^2}{10} \\ &= 82852750 \end{aligned}$$

$$\begin{aligned} S_{yy} &= \sum y_i^2 - n(\bar{y})^2 \\ &= 372717 - \frac{(1737)^2}{10} \\ &= 71000.1 \end{aligned}$$

$$\hat{\beta} = \frac{S_{xy}}{S_{xx}} = \frac{37915}{852750} = 0.040944$$

$$\hat{y} = \bar{y} + \hat{\beta} \bar{x}$$

$$\hat{y} = 170.21976$$

b) As $\hat{\beta}$ is close to zero hence, it is not significant for the model

2. $\sum x = 385.2$

i) $\sum x^2 = 12666.58$ $n = 12$

$\sum y = 1162.5$

$\sum y^2 = 119026.9$

$\sum xy = 38191.41$

$S_{xy} = \sum xy - n(\bar{x})(\bar{y})$

$= 38191.41 - \frac{385.2 \times 12666.58}{12}$

$= 875.16$

$S_{xx} = \sum x^2 - n(\bar{x})^2$

$= 12666.58 - \frac{385.2^2}{12}$

$= 301.66$

$S_{yy} = \sum y^2 - n(\bar{y})^2$

$= 6409.7125$

$\hat{\beta} = \frac{875.16}{301.66} = 2.9011$

$\hat{y} = \hat{\alpha} + \hat{\beta}\bar{x}$, $\hat{\alpha} = \bar{y} - \hat{\beta}\bar{x}$
 $= 3.7497$

The model, $y_i = 3.7497 + 2.9011x_i$

$$(i) \hat{\beta} \sim N(\beta, \sigma^2/s_{xx})$$

$$\frac{\hat{\beta} - \beta}{\sqrt{\sigma^2/s_{xx}}} \sim t_{n-2}$$

$$C.I \Rightarrow \hat{\beta} \pm t_{0.025, 10} \sqrt{\sigma^2/s_{xx}}$$

$$\hat{\sigma}^2 = \frac{1}{n-2} (S_{yy} - \frac{S_{xy}^2}{S_{xx}})$$

$$= \frac{1}{10}$$

$$= 387.0745$$

$$C.I = \hat{\beta} \pm t_{\alpha/2, n-2} \sqrt{\hat{\sigma}^2/s_{xx}}$$

$$C.I \Rightarrow 2.9011 \pm 2.228 \sqrt{\frac{387.0745}{301.66}}$$

$$= 2.9011 \pm 2.5238$$

$$= (0.3773, 5.4249)$$

$$(ii) Y_i \sim N\left(\hat{\alpha} + \hat{\beta}x_0, \left(\frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{xx}}\right)\hat{\sigma}^2\right)$$

$$E(\mu_0) = \hat{\alpha} + \hat{\beta}x_0$$

$$= 119.7936$$

$$C.I = \mu_0 \pm t_{\alpha, n-2} \sqrt{\left(\frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{xx}}\right)\hat{\sigma}^2}$$

$$= 119.7936 \pm 2.228 \sqrt{\frac{1}{12} + \frac{(40 - 32.1)^2}{301.66}} \hat{\sigma}^2$$

$$387.0745$$

$$C. t = 119.7936 \pm 22.1431$$

$$= (97.6504, 141.9366)$$